

$$a) C_D = c_d + \left( \frac{C_L^2}{\pi R} \right) \equiv C_{Di} \quad , \quad \frac{C_L}{C_D} = \frac{C_L}{c_d + C_L^2/\pi R}$$

$$\text{Maximize } \frac{C_L}{C_D} \rightarrow \text{set } \frac{d}{dC_L} \left( \frac{C_L}{C_D} \right) = 0$$

$$\frac{d}{dC_L} \left( \frac{C_L}{c_d + C_L^2/\pi R} \right) = \frac{(c_d + C_L^2/\pi R) - C_L(2C_L/\pi R)}{(c_d + C_L^2/\pi R)^2} = \frac{c_d - C_L^2/\pi R}{(\quad)^2} = 0$$

$$\Rightarrow c_d = \frac{C_L^2}{\pi R} \quad \text{or} \quad \boxed{C_L = \sqrt{c_d \pi R}} \quad \text{at max } \frac{C_L}{C_D}$$

$$\text{At this point, } \boxed{C_D = c_d + \frac{(c_d \pi R)^2}{\pi R} = 2c_d} \quad C_{Di} = c_d$$

$$b) \text{ Set } \frac{d}{dC_L} \left( \frac{C_L^{3/2}}{C_D} \right) = 0$$

$$\frac{d}{dC_L} \left( \frac{C_L^{3/2}}{c_d + C_L^2/\pi R} \right) = \frac{\frac{3}{2} C_L^{1/2} (c_d + C_L^2/\pi R) - C_L^{3/2} (2C_L/\pi R)}{(c_d + C_L^2/\pi R)^2} = C_L^{1/2} \frac{\frac{3}{2} c_d - \frac{1}{2} \frac{C_L^2}{\pi R}}{(\quad)^2} = 0$$

$$\Rightarrow 3c_d = \frac{C_L^2}{\pi R} \quad \text{or} \quad \boxed{C_L = \sqrt{3c_d \pi R}} \quad \text{at max } \frac{C_L}{C_D}$$

$$\text{At this point, } \boxed{C_D = c_d + \frac{(3c_d \pi R)^2}{\pi R} = 4c_d} \quad C_{Di} = 3c_d$$