Unified problems M1 - M3
Solutions

M1 ... Minimum mass structure with a ... strength requirement, need to ... maximize σ_f/ρ

... Compute σ_f/ρ for available materials

	σ_f/MPa	ρ/kg/m^3	kPa/kg/m^3
... steel	220	7900	27.8
... Al	380	2800	125
... Ti	850	4500	188
... CFRP	700	1500	467
... Wood	30	600	50
... SiC	300	3000	100

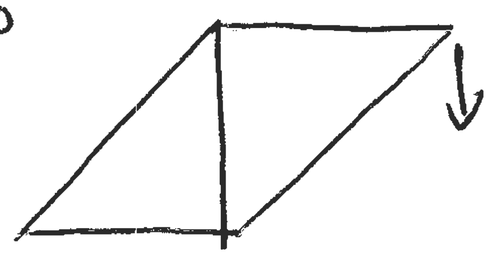
... CFRP works best for bars in truss.

... Now decide on truss configuration

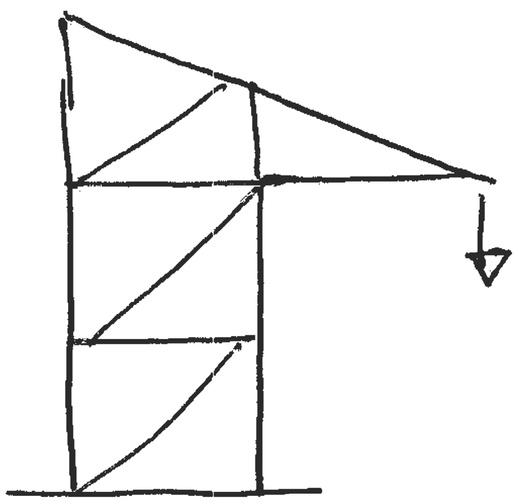
... Design considerations:
... Minimize number of bars (simplicity ... is good.

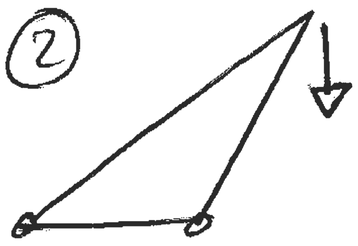
... Aim to have all bars carrying ... similar loads

Candidates

① 

or

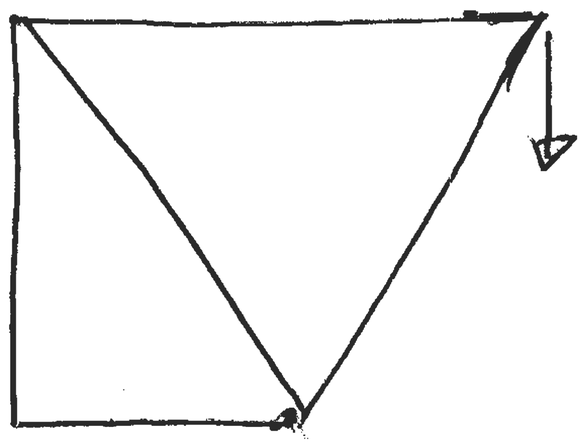


② 

or

③

or

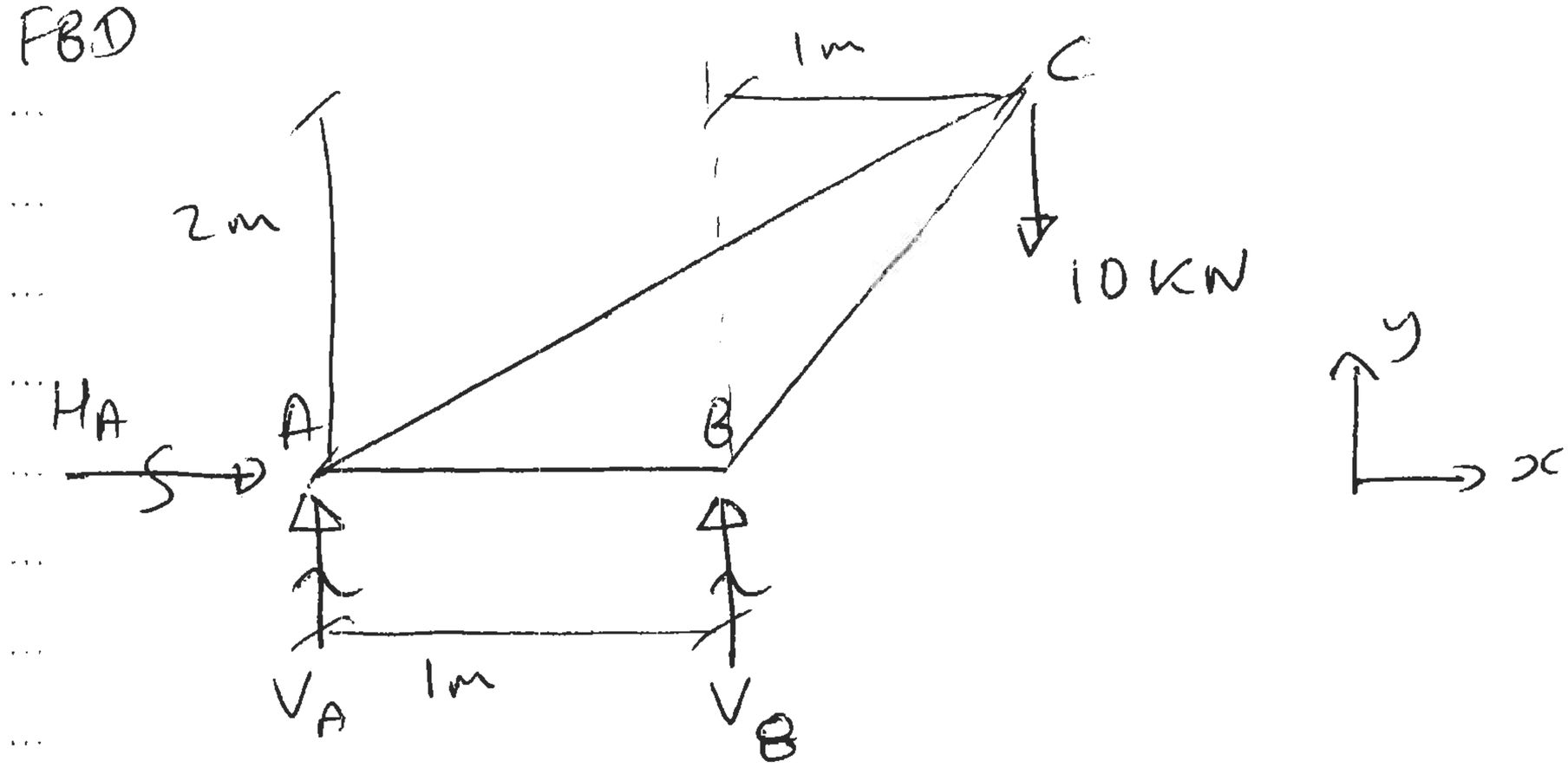


② looks simplest - smallest number of bars
∴ probably lowest mass

suspect ① may have more bars at some others - ∴ more efficient

In any case go with ②

FBD



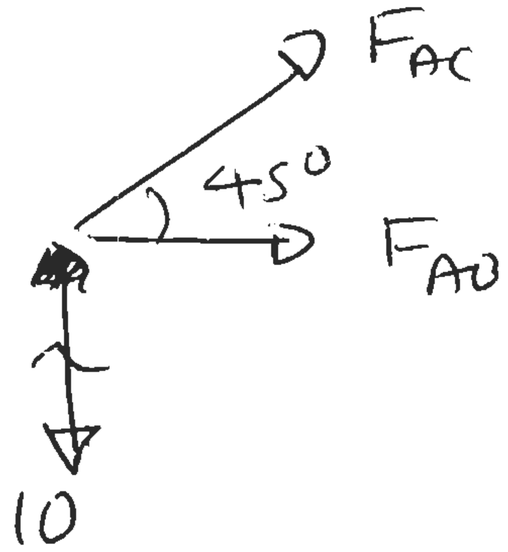
$\rightarrow \Sigma F_x = 0 : H_A + 0 = 0 \Rightarrow H_A = 0$

$\uparrow \Sigma F_y = 0 : V_A + V_B - 10 = 0$

$\circlearrowleft \Sigma M_A = 0 : 1 . V_B - 10 . 2 = 0 : V_B = +20 \text{ kN}$

$\therefore V_A = -10 \text{ kN} \Leftarrow$

M.O.J @ A



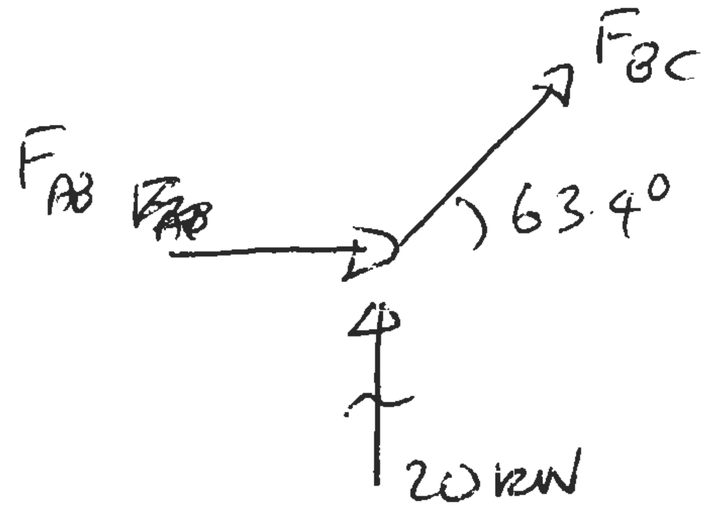
$\Sigma F_y \uparrow = 0 : F_{AC} \sin 45^\circ - 10 = 0$

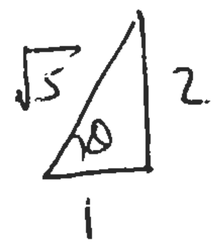
$F_{AC} = +10\sqrt{2} \text{ kN} \Leftarrow$

$\Sigma \vec{F}_x = 0 : F_A \cos 45^\circ + F_{AB} = 0$

$F_{AB} = -10 \text{ kN} \Leftarrow$

MoJ @ B





$\sin\theta = \frac{2}{\sqrt{5}}$

$\Sigma F_y \uparrow = 0 \qquad F_{BC} \sin\theta + 20 = 0$

$$F_{BC} = \frac{-20.\sqrt{5}}{2} = -22.4 \text{ kN}$$

Critical bar is BC - carries highest load
∴ This determines cross-section

$$\frac{22.4 \times 10^3}{\sigma_f} = A_{crit}$$

$$\frac{22.4 \times 10^3}{300 \times 10^6} = 31.9 \times 10^{-6} \text{ m}^2 = 31.9 \text{ mm}^2 \text{ (=)}$$

Total length of bars

$= L_{AC} + L_{AO} + L_{OC}$

$2\sqrt{2} + 1 + \sqrt{5} = 6.06 \text{ m}$

∴ Total mass $= 1500 \times 6.06 \times 31.9 \times 10^{-6} = 0.29 \text{ kg}$

i.e. $\frac{1}{2}$ a 16! seems right!