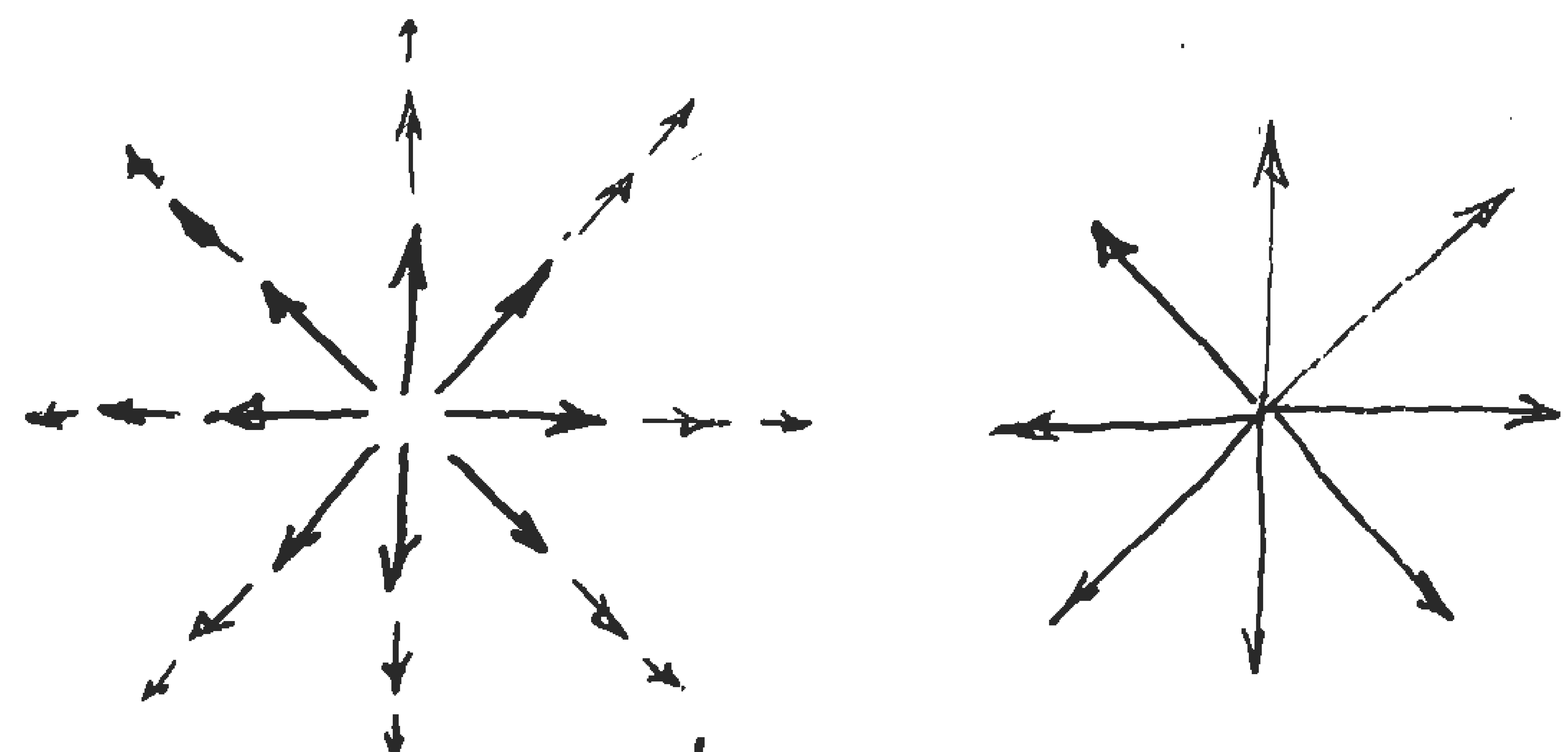
a) $\psi = \arctan\left(\frac{y}{x}\right)$

$$u = \frac{\partial \psi}{\partial y} = \frac{x}{x^2+y^2}$$

$$v = -\frac{\partial \psi}{\partial x} = \frac{y}{x^2+y^2}$$

Streamlines: $\arctan\left(\frac{y}{x}\right) = \theta = \text{const}$

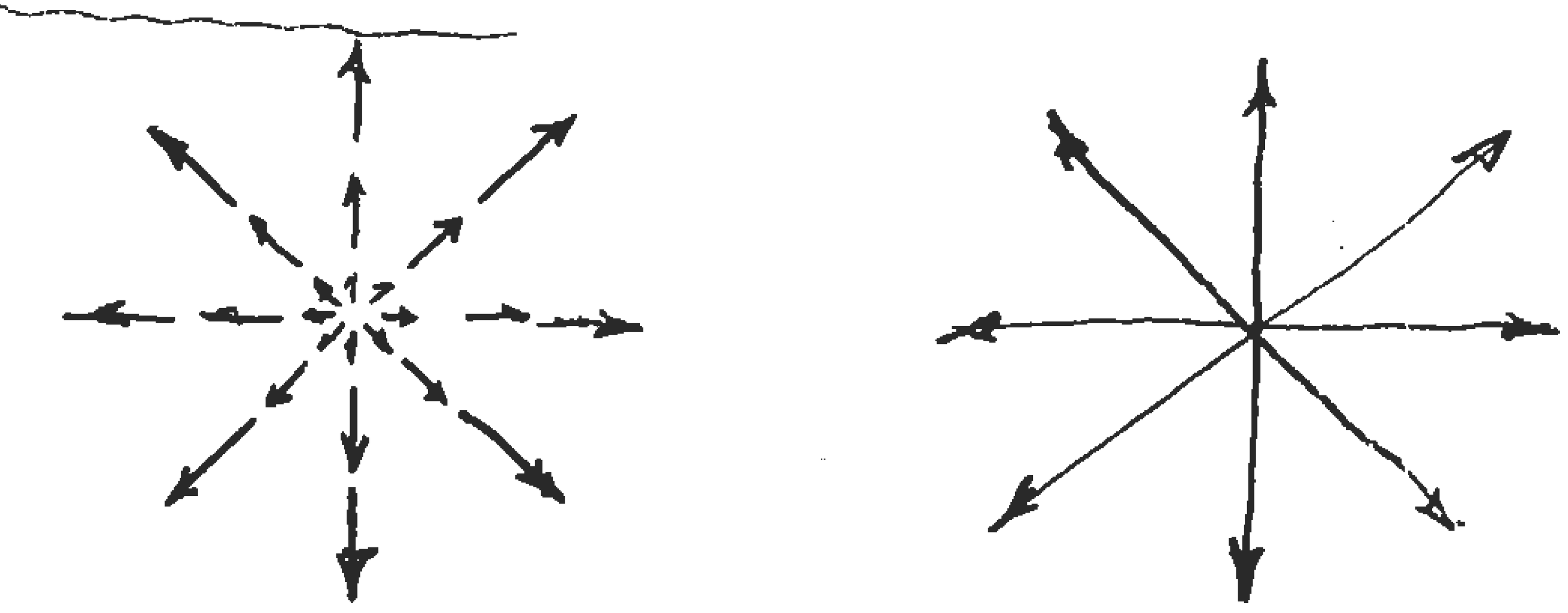


$|\vec{V}|$ varies as $\frac{1}{r}$

$$\phi = x^2 + y^2$$

$$u = \frac{\partial \phi}{\partial x} = 2x$$

$$v = \frac{\partial \phi}{\partial y} = 2y$$

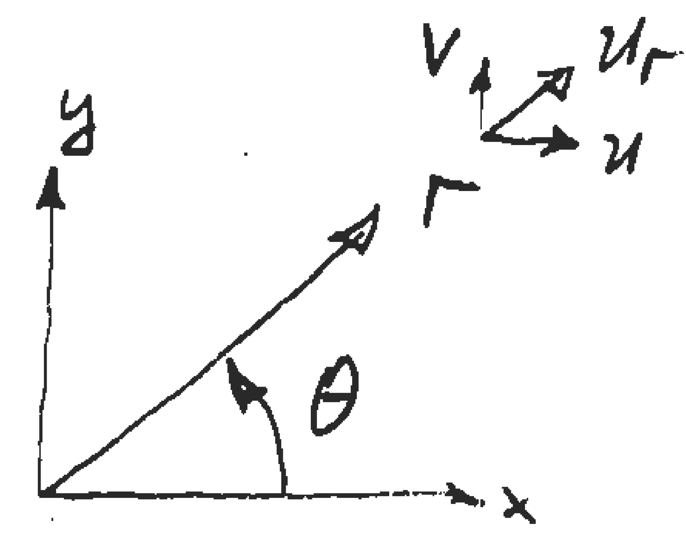


Streamlines: $\frac{dy}{dx} = \frac{v}{u} = \frac{y}{x}$

$$\frac{dy}{y} = \frac{dx}{x}$$

$$y = Cx$$

$|\vec{V}|$ varies as r

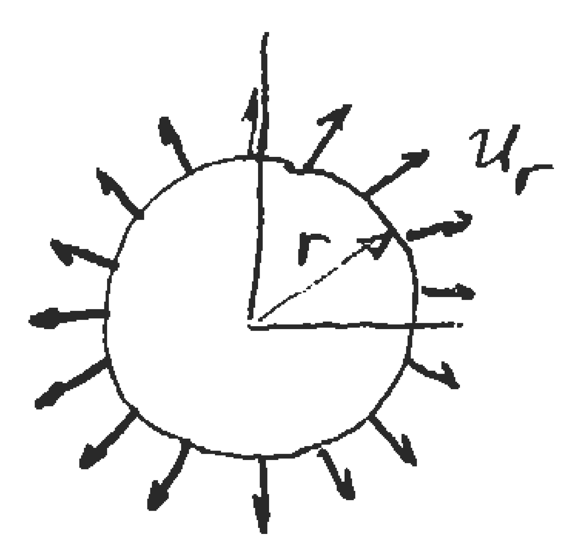


b) For $\psi = \arctan\frac{y}{x}$: $u = \frac{x}{r^2} = \frac{\cos\theta}{r}$

$$v = \frac{y}{r^2} = \frac{\sin\theta}{r}$$

Radial velocity: $u_r = u\cos\theta + v\sin\theta = \frac{1}{r}$

Volume flow rate: $\dot{V} = 2\pi r u_r = 2\pi$ (constant)



For $\phi = x^2+y^2$: $u = 2x = 2r\cos\theta$

$$v = 2y = 2r\sin\theta$$

Radial velocity: $u_r = u\cos\theta + v\sin\theta = 2r$

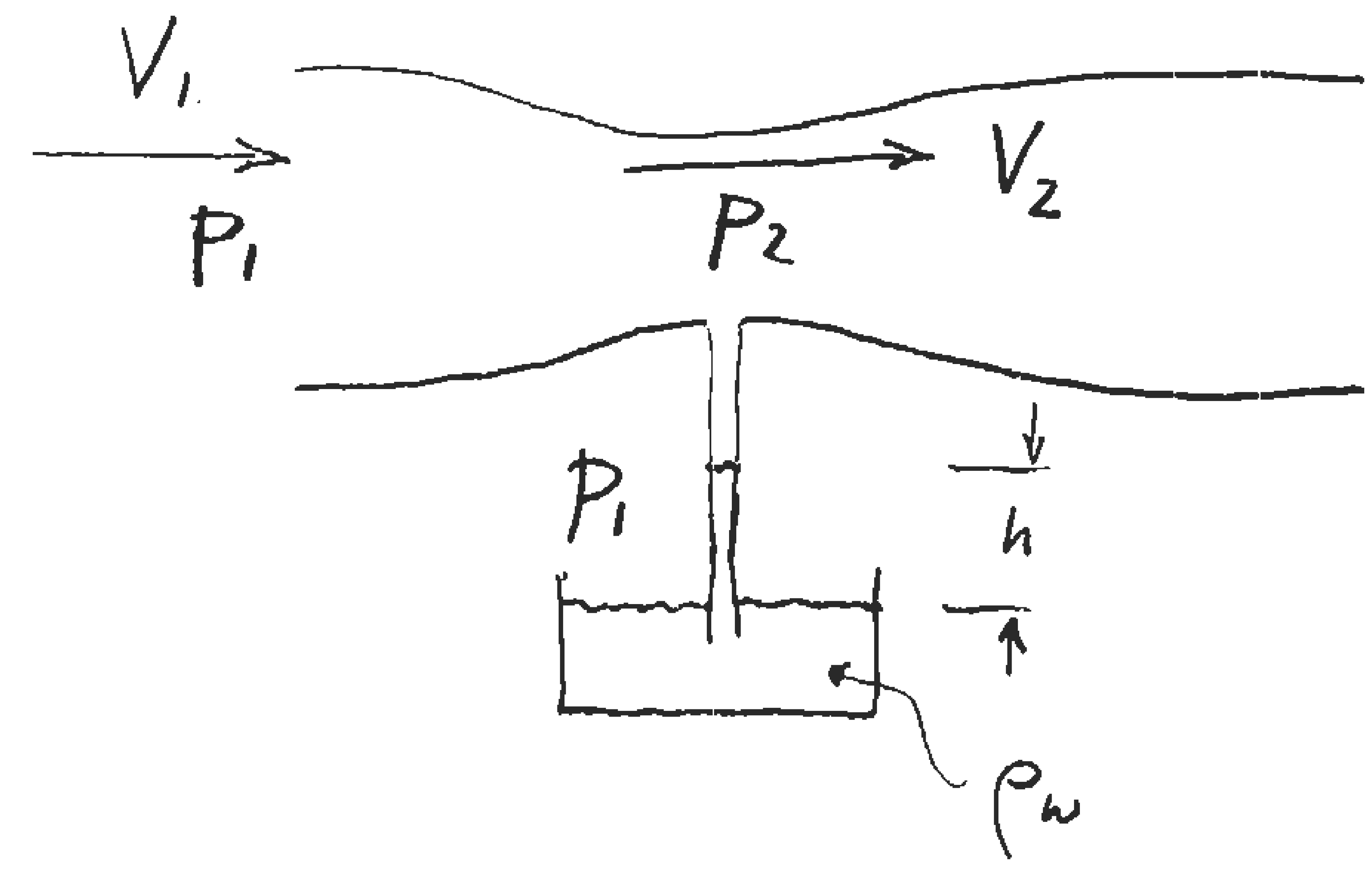
Volume flow rate: $\dot{V} = 2\pi r u_r = 4\pi r^2$ (increases as r^2)

c) $\phi = x^2+y^2$ is not feasible to set up, since $\nabla\cdot\vec{V} \neq 0$ for this flow, so it doesn't obey mass conservation in a low speed flow situation. Lack of mass conservation is further evidenced by $\dot{V}$ increasing with r. Mass is being created "out of thin air" (pun intended)

UE Fluids F13 Solution Fall '03

To get required column height, we must have

$$p_1 - p_2 = \rho_w g h$$



$$\left.\begin{aligned} \rho_w &= 1000 \text{ kg/m}^3 \\ g &= 9.8 \text{ m/s}^2 \\ h &= 0.1 \text{ m} \end{aligned}\right] \rightarrow p_1 - p_2 = 980 \text{ Pa}$$

Using Bernoulli: $\quad p_1 + \frac{1}{2}\rho V_1^2 = p_0 = p_2 + \frac{1}{2}\rho V_2^2$

$$\frac{1}{2}\rho V_2^2 - \frac{1}{2}\rho V_1^2 = p_1 - p_2 = 980 \text{ Pa}$$

Using Continuity: $\quad V_2 = V_1 \frac{A_1}{A_2} = V_1 \frac{1}{0.7}$

$$\frac{1}{2}\rho V_1^2 \left[\frac{1}{0.7^2} - 1\right] = p_1 - p_2$$

$$V_1 = \left(\frac{2(p_1 - p_2)}{\rho\left[\frac{1}{0.7^2} - 1\right]}\right)^{1/2} = \frac{2 \cdot 980 \text{ Pa}}{1.226 \text{ kg/m}^3 \left[\frac{1}{0.7^2} - 1\right]}$$

$$V_1 = 39 \text{ m/s} = 87.5 \text{ mph}$$

M13

Need $\frac{\partial \sigma_{mn}}{\partial x_m} + f_n = 0$ $\qquad$ $f_n = 0$

a)

① $\underbrace{\frac{\partial \sigma_{11}}{\partial x_1}}^{=0} + \overbrace{\frac{\partial \sigma_{21}}{\partial x_2}}^{=0} + \frac{\partial \sigma_{31}}{\partial x_3} = 0$

Since $\frac{\partial \sigma_{11}}{\partial x_1} = 0$, $\sigma_{21} = 0 \Rightarrow \frac{\partial \sigma_{31}}{\partial x_3} = 0$

② $\overbrace{\frac{\partial \sigma_{12}}{\partial x_1}}^{=0} + \overbrace{\frac{\partial \sigma_{22}}{\partial x_2}}^{=0} + \overbrace{\frac{\partial \sigma_{32}}{\partial x_3}}^{=0} = 0$

Since $\sigma_{12} = \sigma_{22} = \sigma_{32} = 0$ $\qquad$ no additional information

③ $\frac{\partial \sigma_{13}}{\partial x_1} + \frac{\partial \sigma_{23}}{\partial x_2} + \frac{\partial \sigma_{33}}{\partial x_3}$

Since $\sigma_{23} = \sigma_{33} = 0 \Rightarrow \frac{\partial \sigma_{13}}{\partial x_1} = 0$

$\frac{\partial \sigma_{31}}{\partial x_3} = \frac{\partial \sigma_{13}}{\partial x_1} = 0$ $\qquad$ & Since $\sigma_{13} = 0$ @ $\pm h$

$\therefore \sigma_{13} = 0$ ~~everywhere~~. in x_3

σ_{13} = Constant in x_1 $\Leftarrow$

from (1)

6) $\sigma_{11} = c\left(\frac{M}{I}\right) x_3 x_1$

$$\frac{\partial \sigma_{11}}{\partial x_1} = \left(\frac{cM}{I}\right) x_3$$

$$\Rightarrow \frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{31}}{\partial x_3} = 0 \Rightarrow \frac{\partial \sigma_{31}}{\partial x_3} = -\left(\frac{cM}{I}\right) x_3$$

$$\sigma_{31} = -\left(\frac{cM}{I}\right) \frac{x_3^2}{2} + D$$

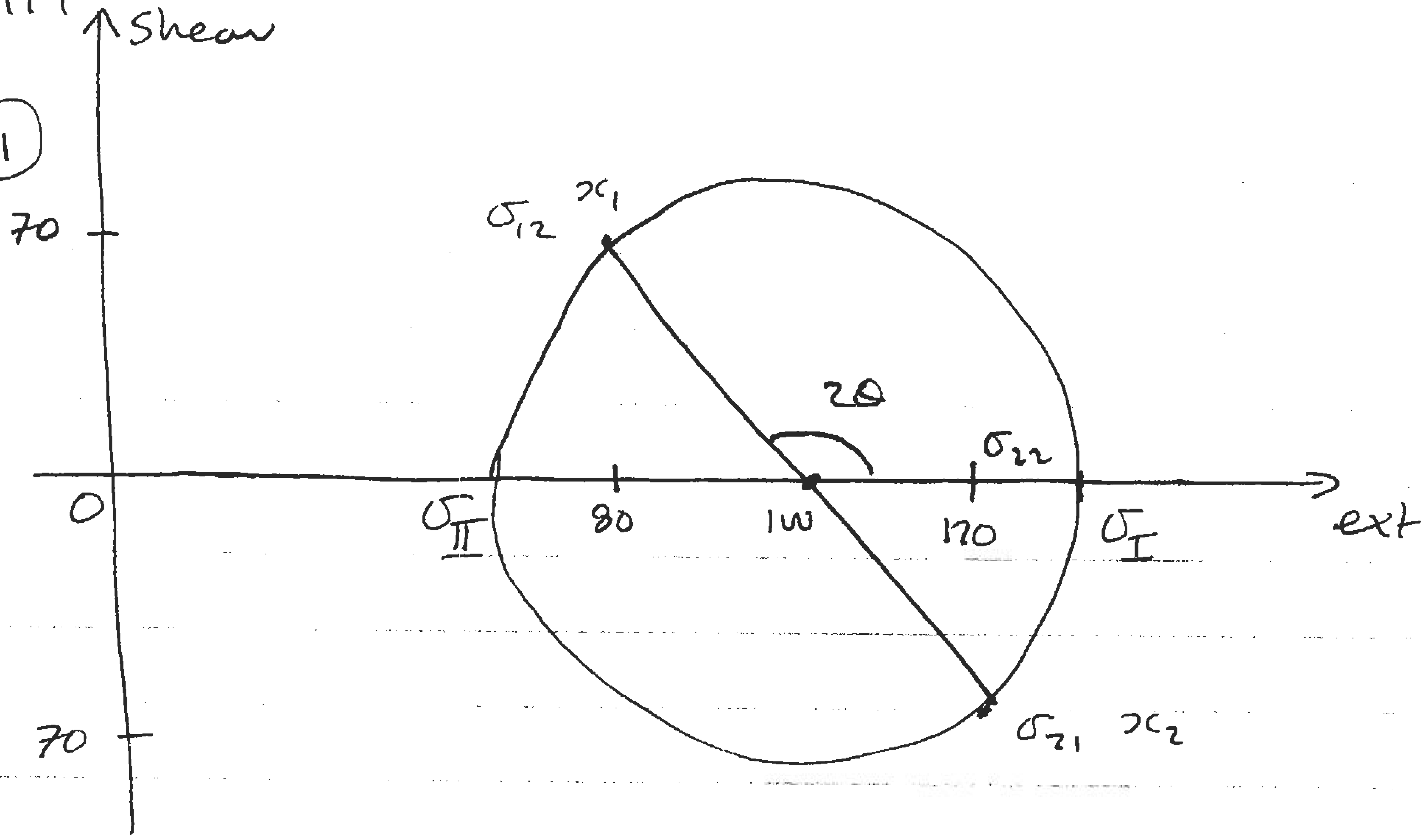
but $\sigma_{31} = 0$ for $\pm h$.

$$\therefore D = \frac{cM}{I}\frac{h^2}{2} \Rightarrow \sigma_{31} = \frac{cM}{2I}\left(h^2 - x_3^2\right) \Leftarrow$$

from (3) $\frac{\partial \sigma_{13}}{\partial x_1} = 0$ i.e. shear stress constant along length.

M14

(1)



Radius = $\sqrt{20^2 + 70^2}$ = 72.8 MPa = Max Shear.

2) Max

Max Principal Stress = 100 + 72.8 = 172.8 MPa ⇐

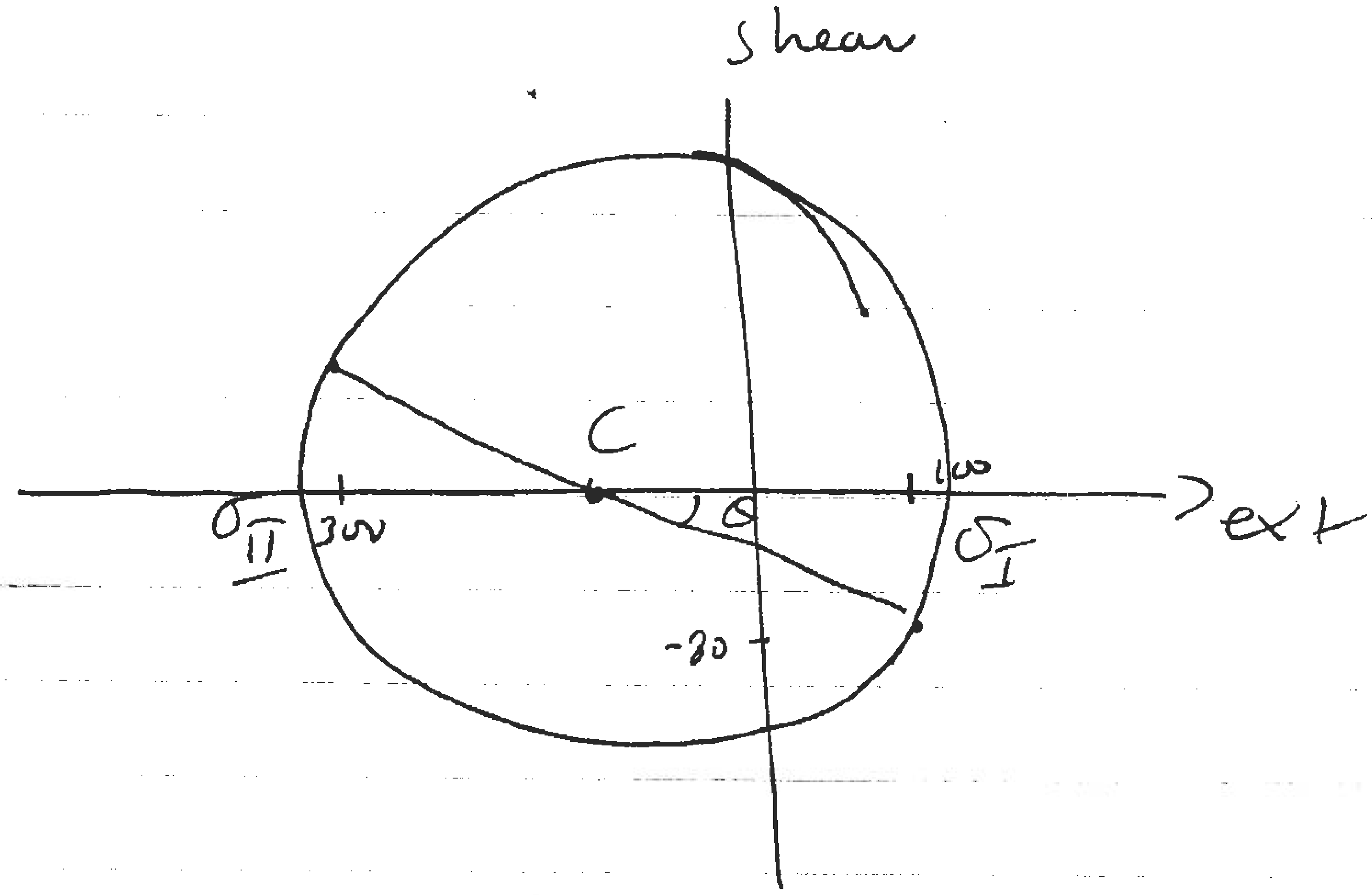
Min Principal Stress = 100 − 72.8 = 27.2 MPa ⇐

(3) From x_1 d^n

$\theta = \frac{1}{2}\left(180 - \tan^{-1}\left(\frac{70}{20}\right)\right) = 53^\circ$ clockwise from x_1

6)

1)



2) Center @ $\dfrac{100 + (-300)}{2} = -100$ MPa

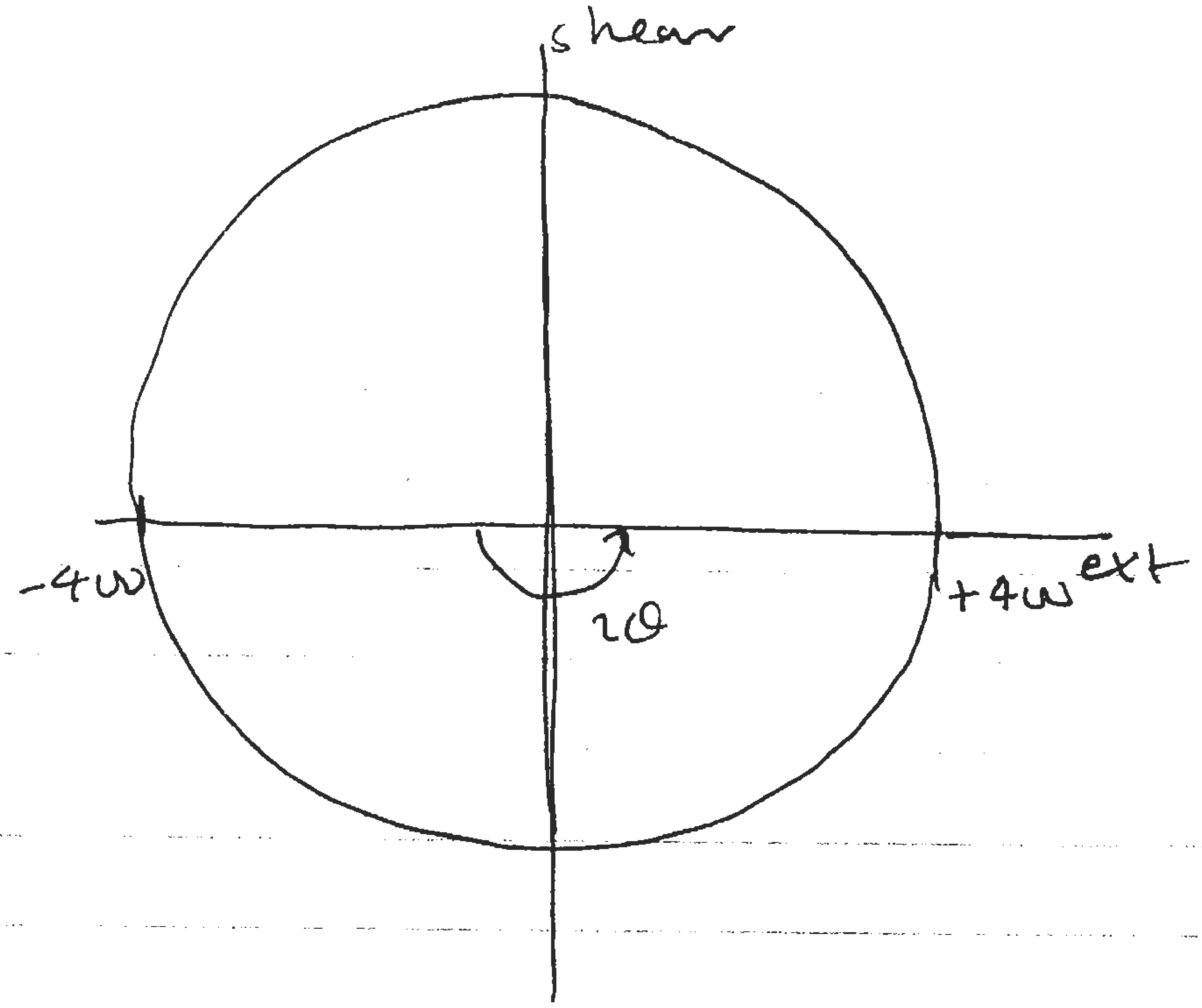
Radius $= \sqrt{200^2 + 80^2} = 215.4$ MPa = Max shear.

$\sigma_I = -100 + 215.4 = 115.4$ MPa $\Leftarrow$

$\sigma_{II} = -100 - 215.4 = -315.4$ MPa $\Leftarrow$

3) ~~tan~~ $\theta = \frac{1}{2}\tan^{-1}\left(\frac{80}{200}\right) = 10.9°$ counterclockwise.

c)



center at origin

Max shear = 400 MPa

σ_I = 400 MPa

σ_{II} = -400 MPa

Max σ_I in x_2 direction $\therefore$ 90° CCW from x_1.

M15) i) consider element of initial sides 1 unit in length

deformed element has sides $1+\varepsilon_1$, $1+\varepsilon_2$, $1+\varepsilon_3$

$\therefore$ Volume (deformed) = $(1+\varepsilon_1)(1+\varepsilon_2)(1+\varepsilon_3)$

neglecting high order terms $(\varepsilon_1\varepsilon_2, \varepsilon_1\varepsilon_3, \varepsilon_2\varepsilon_3, \varepsilon_1\varepsilon_2\varepsilon_3)$

deformed volume = $1+\varepsilon_1+\varepsilon_2+\varepsilon_3$

$$\therefore \frac{V_{def} - V_{undef}}{V_{undef}} = \varepsilon_1+\varepsilon_2+\varepsilon_3$$

ii)

ii a) $\varepsilon_{11} = \dfrac{\partial u_1}{\partial x_1} = (x_1 + 0.5x_2) \times 10^{-3}$

$$\varepsilon_{22} = \frac{\partial u_2}{\partial x_2} = (0.5x_2 - x_1) \times 10^{-3}$$

$$\varepsilon_{33} = \frac{\partial u_3}{\partial x_3} = 0$$

$$\varepsilon_{12} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right) = \frac{1}{2}\left(-x_2 + 0.5x_1 + 0.5x_1 - x_2\right)$$

$$= \frac{1}{2}(x_1 - 2x_2) \times 10^{-3} \Leftarrow$$

$$\varepsilon_{23} = \frac{1}{2}\left(\frac{\partial u_2}{\partial x_3} + \frac{\partial u_3}{\partial x_2}\right) = 0$$

$$\varepsilon_{13} = \frac{1}{2}\left(\frac{\partial u_3}{\partial x_1} + \frac{\partial u_1}{\partial x_3}\right) = 0$$

b) Rigid body rotation

$$\theta = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} - \frac{\partial u_2}{\partial x_1}\right)$$

$$= \frac{1}{2}\left(-x_2 + \cancel{0.5x_1} - \cancel{0.5x_1} - x_2\right) \times 10^{-3}$$

$$= \frac{1}{2}(-2x_2) = x_2 \times 10^{-3} \Leftarrow$$

c) at $x_1 = 5\text{mm}$, $x_2 = 7\text{mm}$.

$$\epsilon_{11} = \left(5 + \frac{7}{2}\right) \times 10^{-3} = 8.5 \times 10^{-3}$$

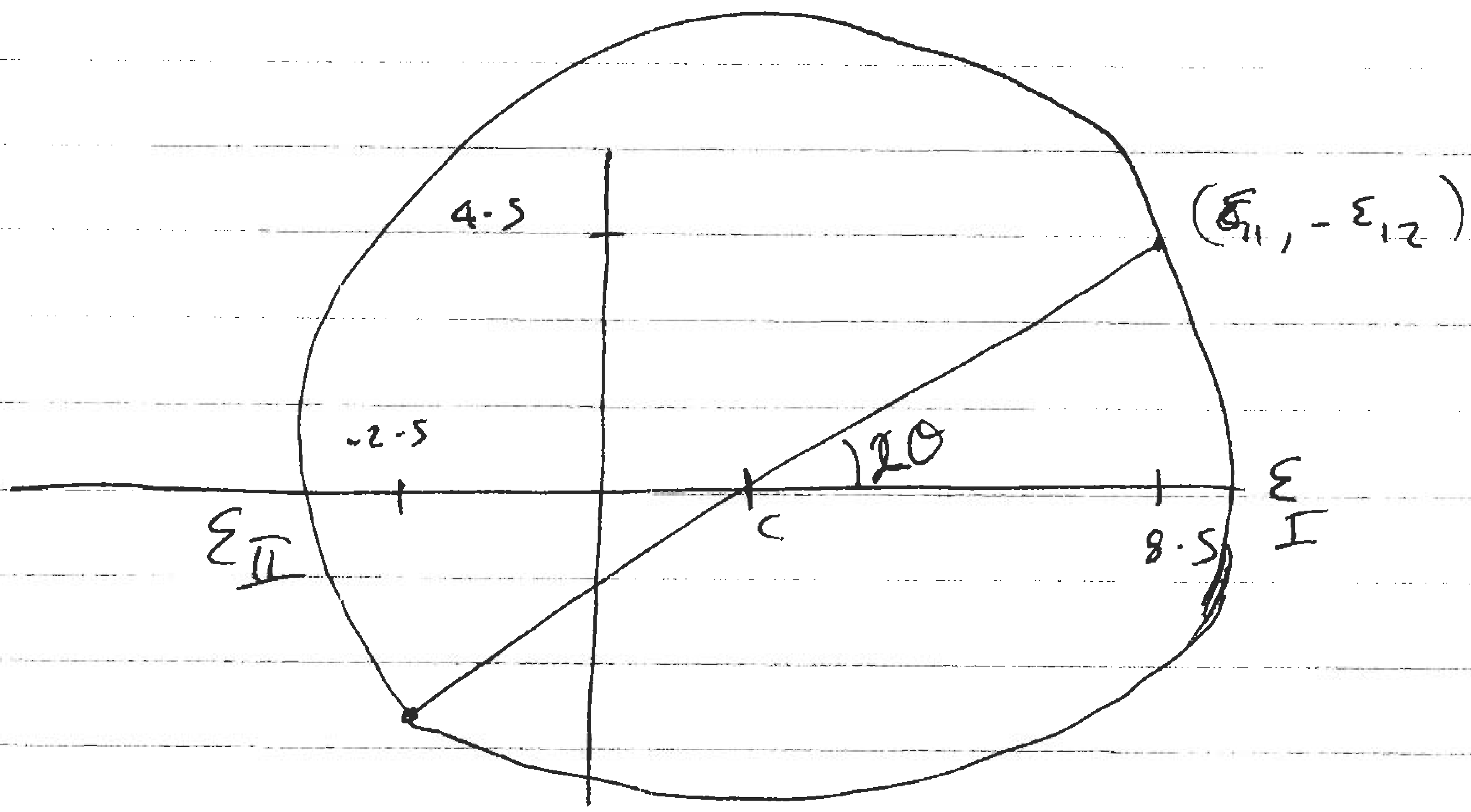
$$\varepsilon_{22} = \left(\frac{7}{2} - 5\right) \times 10^{-3} = -2.5 \times 10^{-3}$$

$$\varepsilon_{12} = \frac{1}{2}(5 - 14) \times 10^{-3} = -4.5 \times 10^{-3}$$

$$\varepsilon_{33} = 0$$

~~d) Volumetric strain = $(8.5 + (-2.5) + (0)) \times 10^{-3} = +.5 \times 10^{-3}$~~

c)



$$\text{Center @ } \frac{1}{2}[8.5 + (-2.5)] \times 10^{-3} = +3 \times 10^{-3}$$

$$\text{Radius} = 10^{-3}\sqrt{(4.5)^2 + (5.5)^2} = 7.11 \times 10^{-3}$$

$$\varepsilon_I = 3 \times 10^{-3} + 7.11 \times 10^{-3} = 10.11 \times 10^{-3}$$

$$\varepsilon_{II} = 3 \times 10^{-3} - 7.11 \times 10^{-3} = -4.11 \times 10^{-3}$$

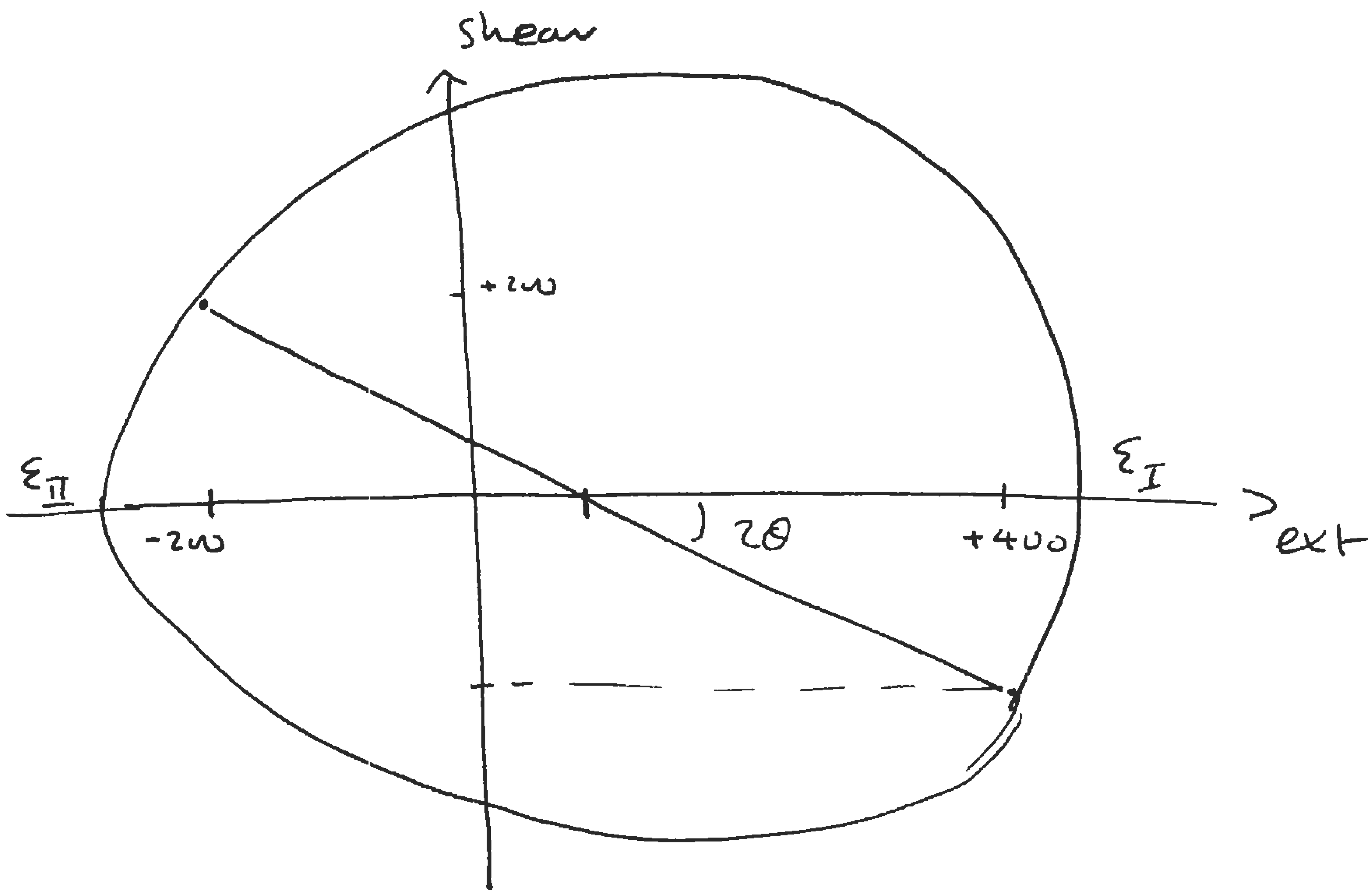
$\tan\theta = \frac{1}{2}\tan^{-1}\left(\frac{4.5}{5.5}\right) = 19.6°$ clockwise from x,

d) Volumetric strain =

$10.11 + (-4.11) + 0 = 6 \times 10^{-3}$

M16

a)



center @ $+100\,\mu\varepsilon$.

Radius $= \sqrt{300^2 + 200^2} = 360\,\mu\varepsilon$.

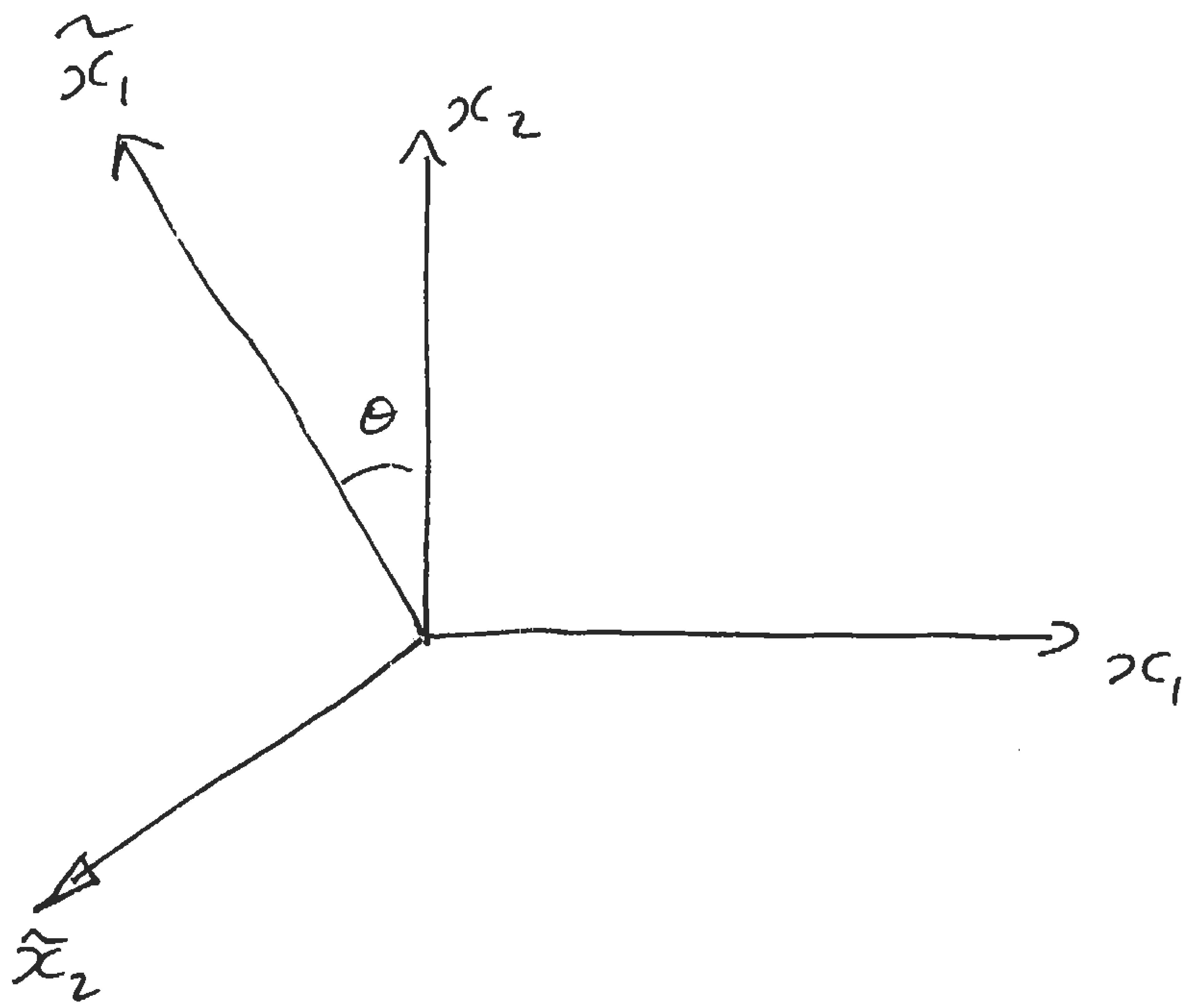
b) $\varepsilon_{I} = 100 + 361 = 401\,\mu\varepsilon$

$\varepsilon_{II} = 100 - 361 = -261\,\mu\varepsilon$.

$2\theta = \frac{1}{2}\tan^{-1}\left(\frac{200}{300}\right) = 16.85^\circ$

$\therefore\ \hat{x}_1 = 90 + \theta = 106.85^\circ$ ccw from x_1

$\hat{x}_2 = \theta = 16.85^\circ$ ccw from x_1 (106.85° ccw from x_2)



$$l_{\tilde{1}1} = \cos(106.8) = -0.290 \Leftarrow$$

$$l_{\tilde{1}2} = \cos(16.85) = 0.957 \Leftarrow$$

$$l_{\tilde{2}2} = \cos(106.8) = -0.290 \Leftarrow$$

$$l_{\tilde{2}1} = \cos(180+16.85) = -0.957 \Leftarrow$$

$$l_{\tilde{3}3} = 1, \quad l_{\tilde{1}3} = l_{\tilde{2}3} = l_{\tilde{3}1} = l_{\tilde{3}2} = 0. \Leftarrow$$

d) $\tilde{\varepsilon}_{mn} = l_{\tilde{m}p}\, l_{\tilde{n}q}\, \varepsilon_{pq}$

$$\tilde{\varepsilon}_{11} = l_{\tilde{1}1} l_{\tilde{1}1} \varepsilon_{11} + l_{\tilde{1}1} l_{\tilde{1}2} \varepsilon_{12} + l_{\tilde{1}2} l_{\tilde{1}1} \varepsilon_{21}$$

$$(-0.290)^2(-200) + (-0.290)(0.957)(-200) + (-0.290)(0.957)(-200)$$

$$+ l_{\tilde{1}2} l_{\tilde{1}2} \varepsilon_{22} + 0 + 0 + 0$$

$$+ (0.957)^2 (400)$$

$$= 460.5\, \mu\varepsilon$$

$$\tilde{\varepsilon}_{22} = l_{\tilde{2}1} l_{\tilde{2}1} \varepsilon_{11} + l_{\tilde{2}1} l_{\tilde{2}2} \varepsilon_{12} + l_{\tilde{2}2} l_{\tilde{2}1} \varepsilon_{21} + l_{\tilde{2}2} l_{\tilde{2}2} \varepsilon_{22}$$

$$(-0.957)^2(-200) + (-0.957)(-0.290)(-200) + (-0.290)(-0.957)(-200) + (-0.290)^2(+400)$$

$$= -261\, \mu\varepsilon.$$

$$\tilde{\varepsilon}_{12} = l_{\tilde{1}1} l_{\tilde{2}1} \varepsilon_{11} + l_{\tilde{1}2} l_{\tilde{2}1} \varepsilon_{21} + l_{\tilde{1}1} l_{\tilde{2}2} \varepsilon_{12} + l_{\tilde{1}2} l_{\tilde{2}2} \varepsilon_{22} + 0\text{'s}$$

$$= (-0.290)(-0.957)(-200) + (0.957)(-0.957)(-200) + (-0.290)(-0.290)(-200) +$$
$$+ (0.957)(-0.290)(400)$$

$$= -0.2 \simeq 0$$

$$\tilde{\varepsilon}_{11} = +460, \quad \tilde{\varepsilon}_{22} = -261, \quad \tilde{\varepsilon}_{12} = 0$$

$\therefore$ agrees with Mohr's circle.