Introduction to Computers and Programming

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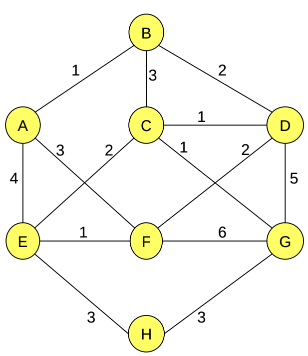
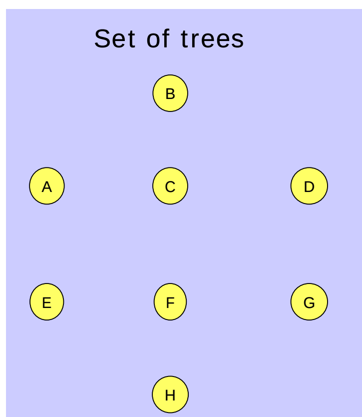
Recitation 2
April 6 2004

Minimum Spanning Tree

- Kruskal's Algorithm
  - Finds a minimum spanning tree for a connected weighted graph
    - Create a set of trees, where each vertex in the graph is a separate tree
    - Create set S containing all edges in the graph
    - While S not empty
      - Remove edge with minimum weight from S
      - if that edge connects two different trees, then add it to the forest, combining two trees into a single tree
      - Otherwise discard that edge

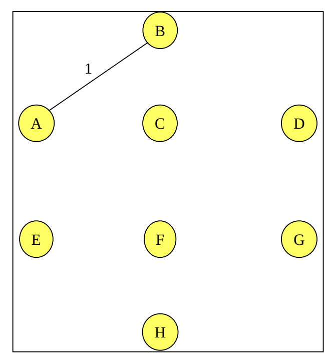


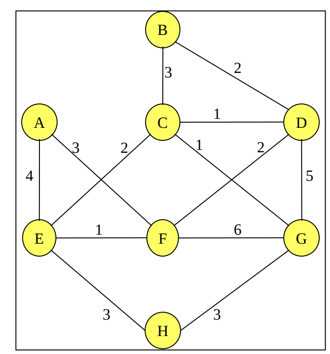
S	A	B	C	D	E	F	G	H
A	0							
B	1	0						
C		3	0					
D		2	2	0				
E	4		2		0			
F	3			2	1	0		
G			1	5		6	0	
H					3		3	0



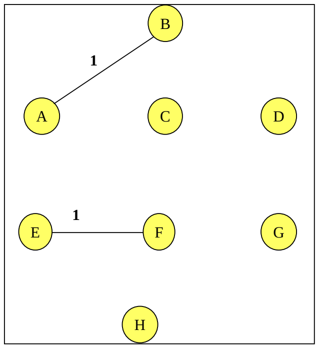
Kruskal's Algorithm

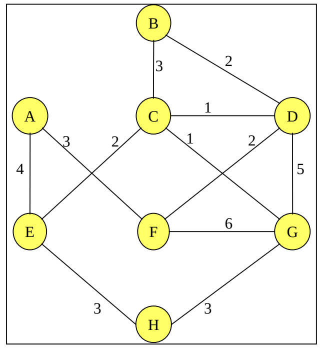
Step 1



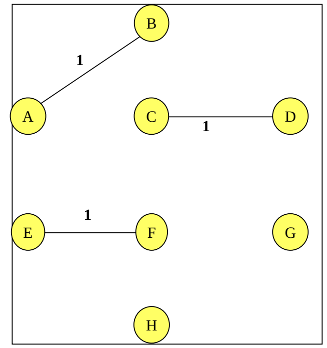


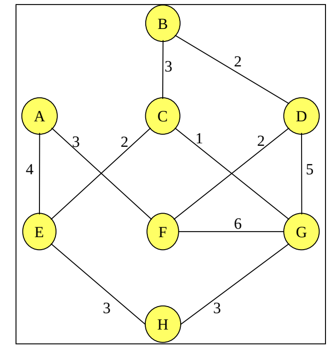
Step 2



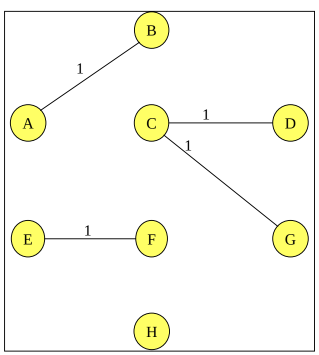


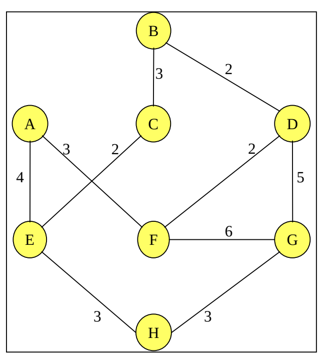
Step 3



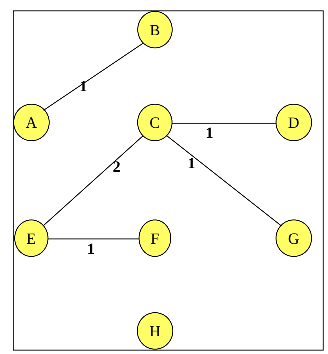


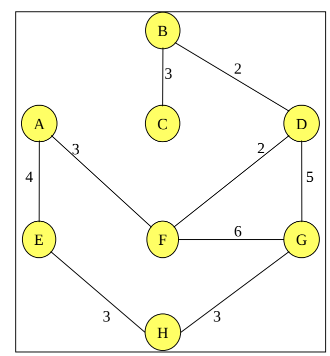
Step 4



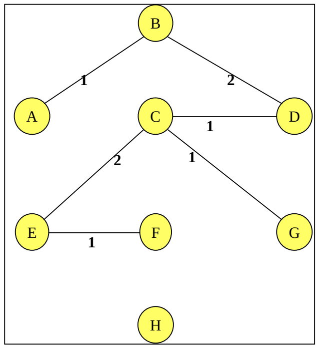


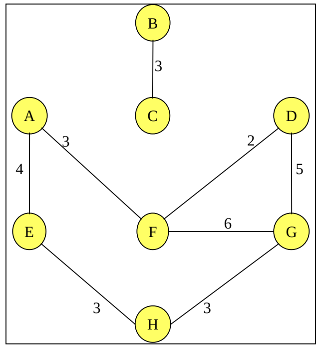
Step 5





Slide 6





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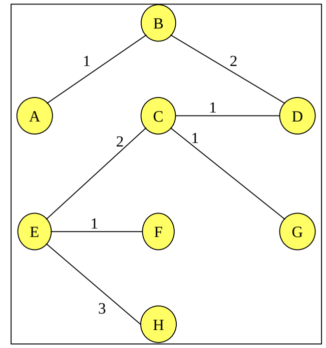
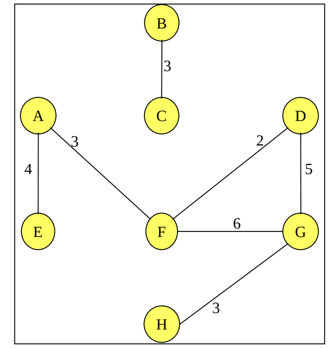




Minimum Spanning Tree

- Prim's Algorithm
  - Finds a subset of the edges (that form a tree) including every vertex and the total weight of all the edges in tree is minimized
    - Choose starting vertex
    - Create the Fringe Set

    } **Initialization**

    - Loop until the MST contains all the vertices in the graph
      - Remove edge with minimum weight from Fringe Set
      - Add the edge to MST
      - Update the Fringe Set

    Body {

Prim – Initialization

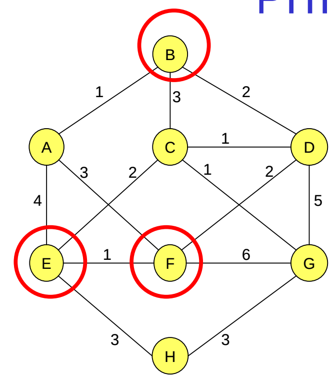
- Pick any vertex *x* as the starting vertex
- Place *x* in the Minimum Spanning Tree (MST)
- For each vertex *y* in the graph that is adjacent to *x*
  - Add *y* to the Fringe Set
- For each vertex *y* in the Fringe Set
  - Set weight of *y* to weight of the edge connecting *y* to *x*
  - Set *x* to be parent of *y*

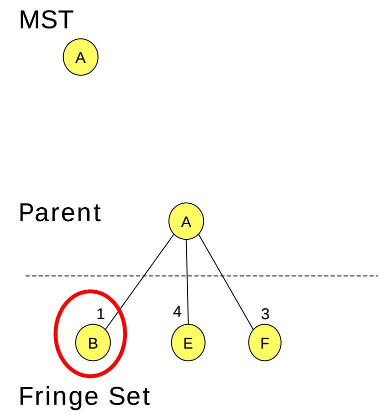
Prim – Body

While number of vertices in MST < vertices in the graph

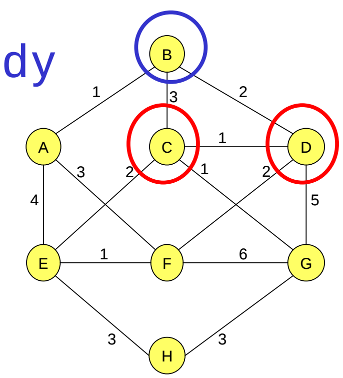
- Find vertex *y* with minimum weight in the Fringe Set
- Add vertex and the edge *x*,*y* to the MST
- Remove *y* from the Fringe Set
- For all vertices *z* adjacent to *y* that are not in MST
  - If *z* is not in the Fringe Set
    - Add *z* to the Fringe Set
    - Set parent to *y*
    - Set weight of *z* to weight of the edge connecting *z* to *y*
  - Else
    - If Weight(*y*,*z*) < Weight(*z*) then
      - Set parent to *y*
      - Set weight of *z* to weight of the edge connecting *z* to *y*

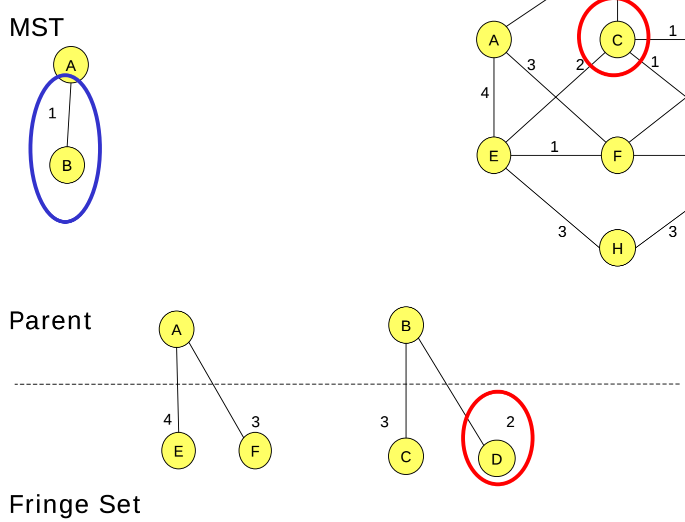
Prim's Algorithm



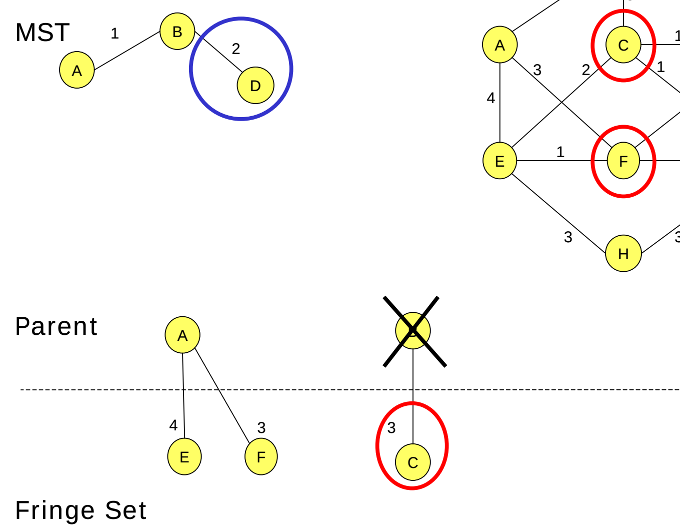


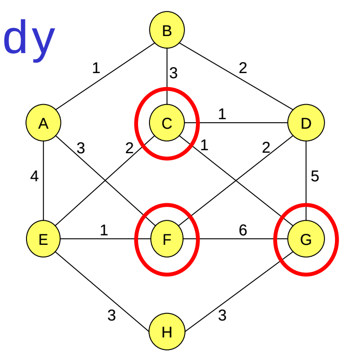
Prim's Body





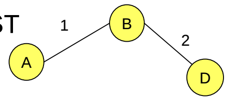
Prim's Body

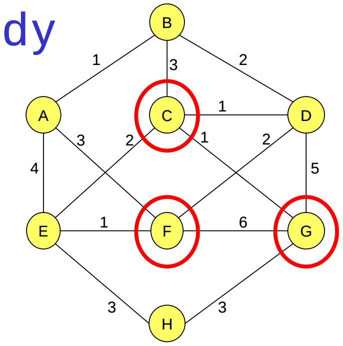




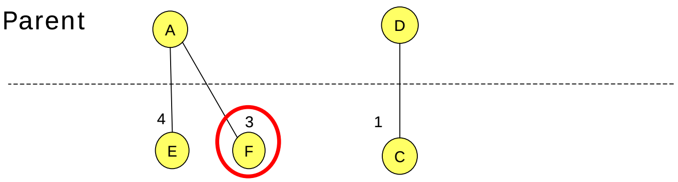
Prim’s Body

MST





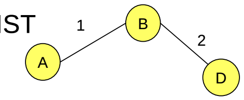
Parent

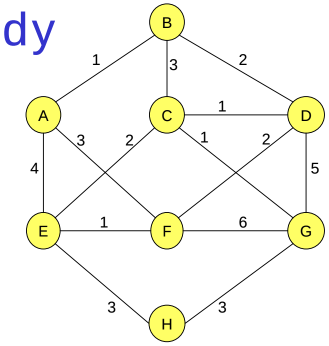


Fringe Set

Prim’s Body

MST





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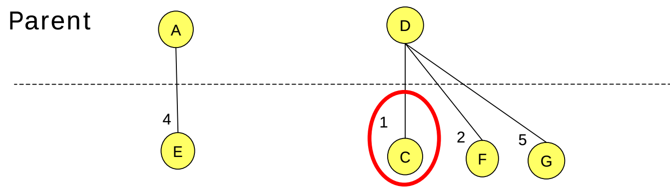
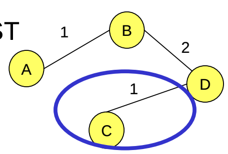
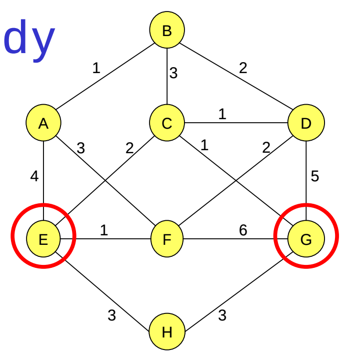


Fringe Set

Prim’s Body

MST





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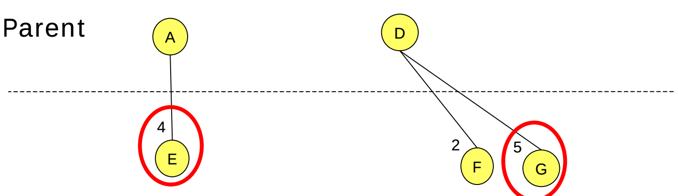
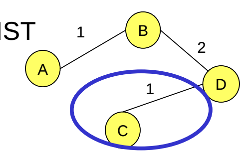
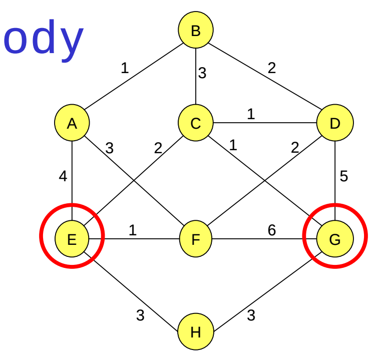


Fringe Set

Prim’s Body

MST





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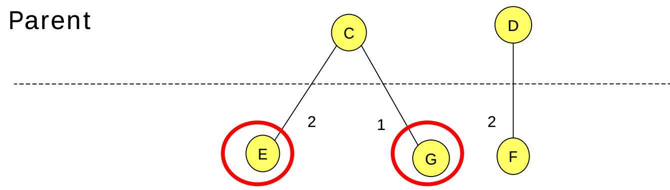
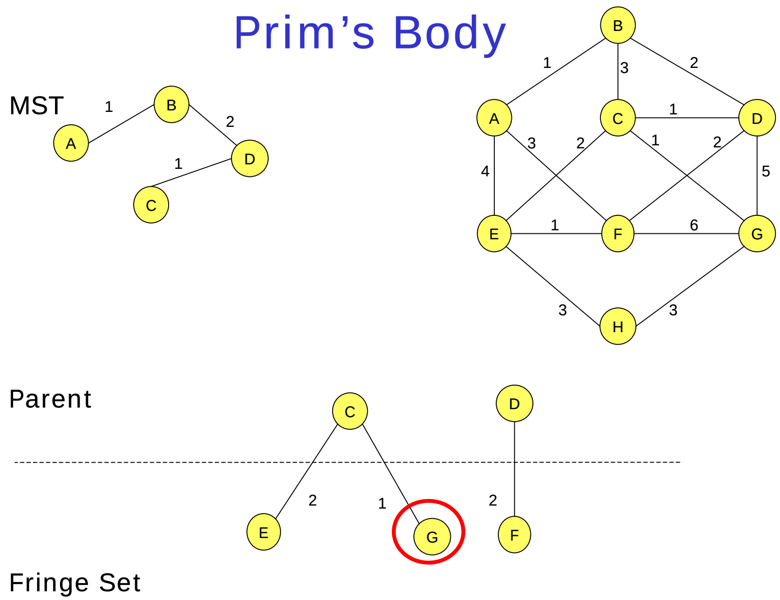


Fringe Set

Prim’s Body



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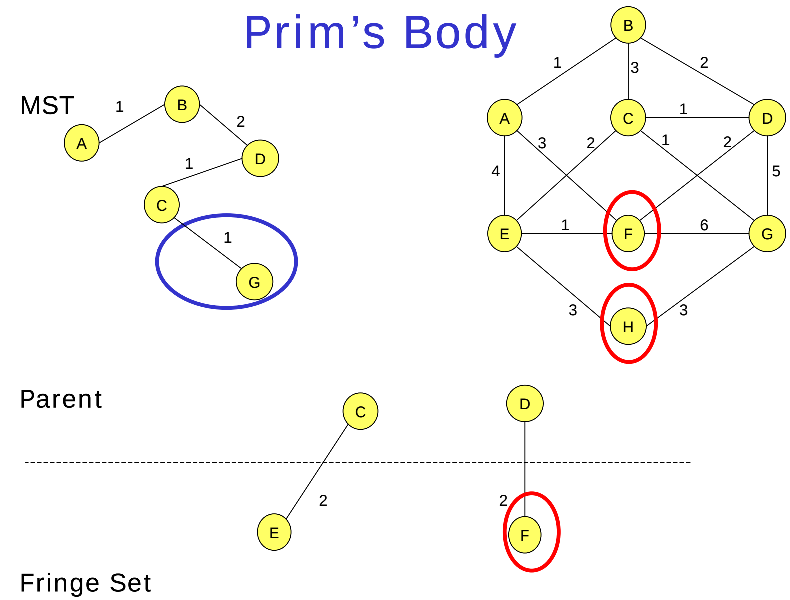

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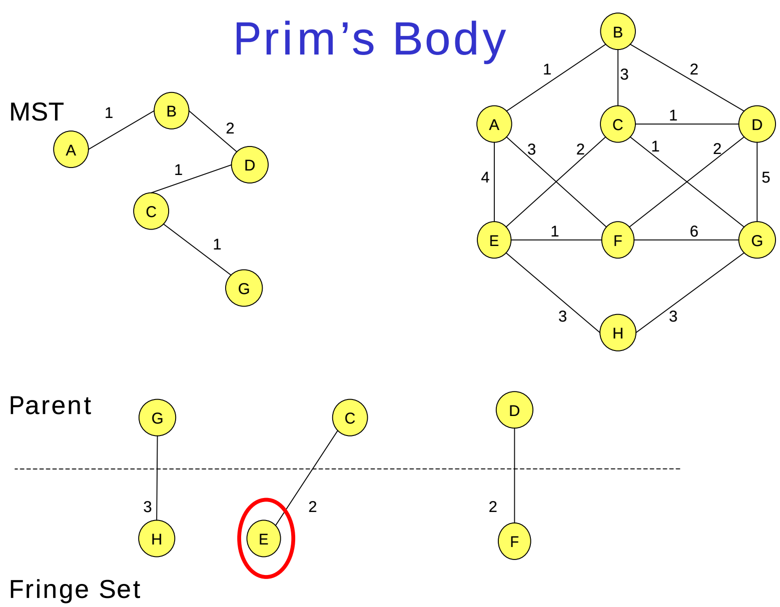
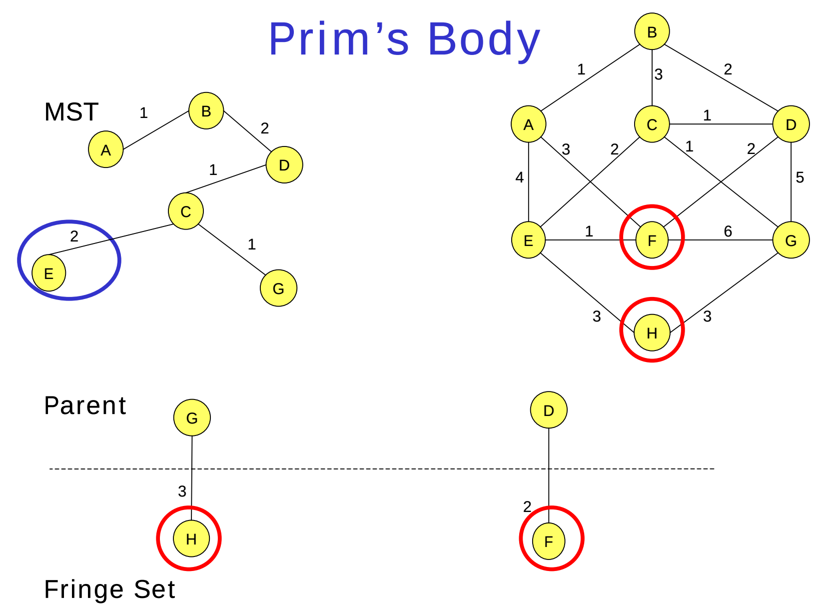
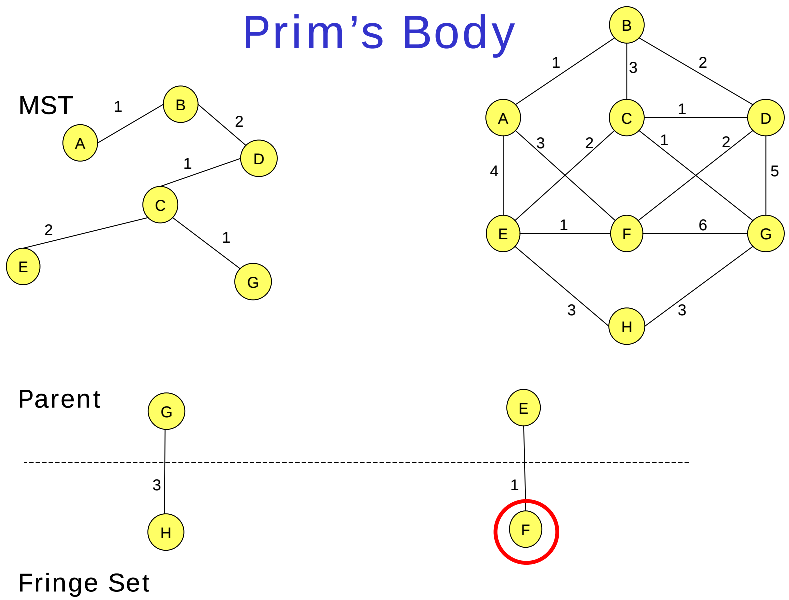


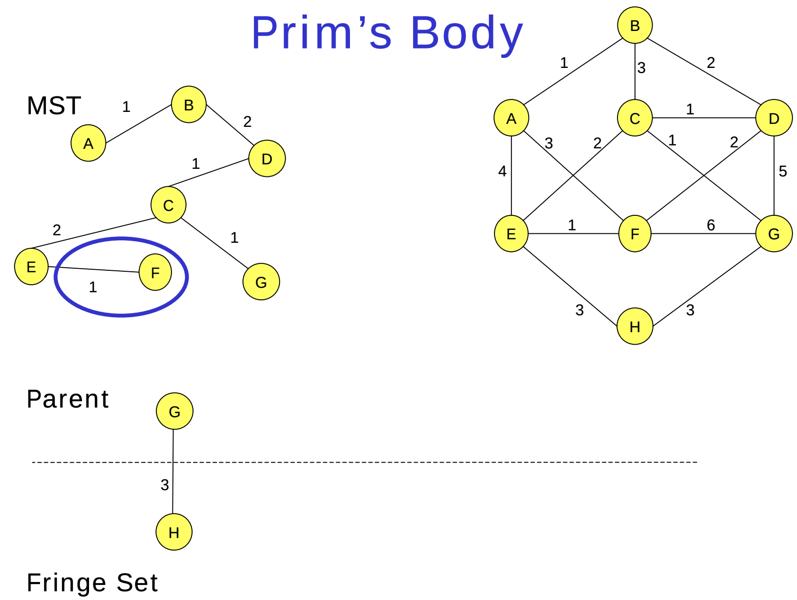
Prim’s Body



Prim's Body



Prim's Body



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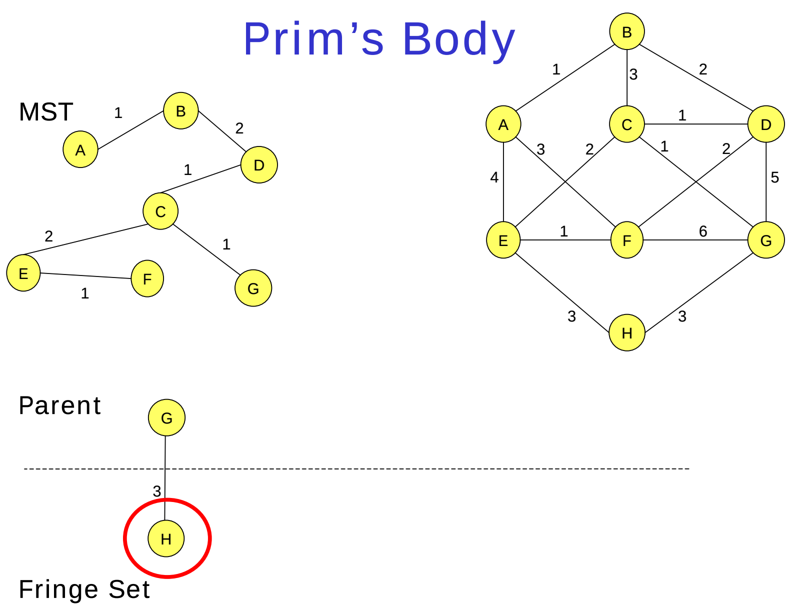

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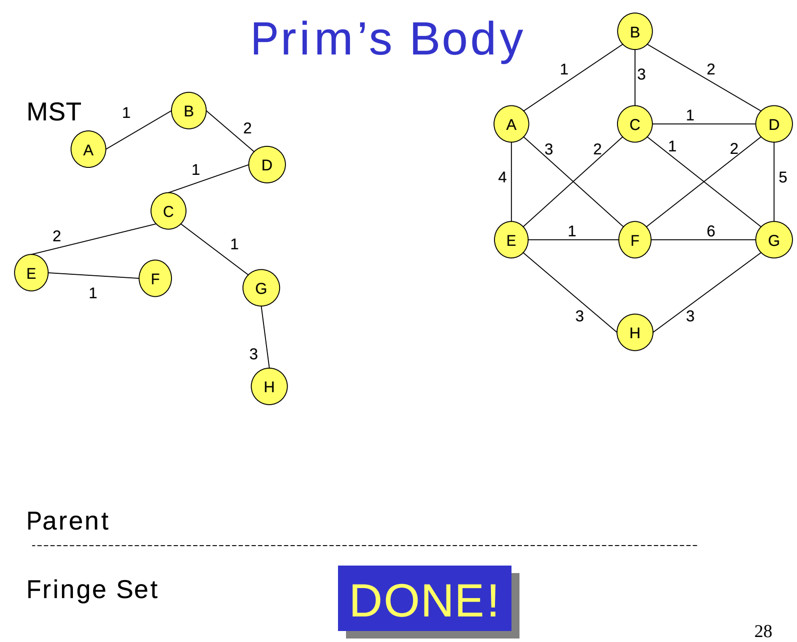
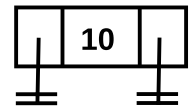


Inserting into Ordered Binary Tree

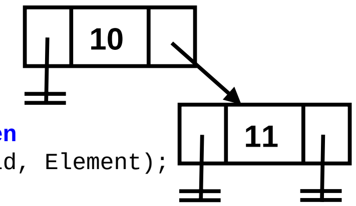
```
-- insert procedure
procedure Insert (Root    : in out Nodeptr;
                  Element : in     Integer ) is
      New_Node : Nodeptr;
   begin
      if Root = null then
         New_Node:= new Node;
         New_Node.Element := Element;
         Root := New_Node;
      else
         if Root.Element < Element then
            Insert(Root.Right_Child, Element);
         else
            Insert(Root.Left_Child, Element);
         end if;
      end if;
   end Insert;
```


Inserting into a Binary node

- Insert 10, 11, 9, 7, 8, 12
- Insert 10



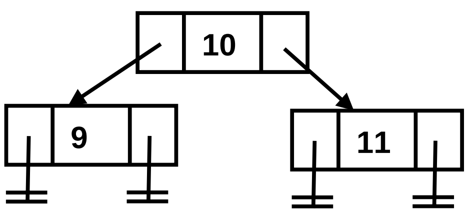
- Insert 11




```
if Root.Element < Element then
      Insert(Root.Right_Child, Element);
else
      Insert(Root.Left_Child, Element);
end if;
```


Inserting into Ordered Binary Tree

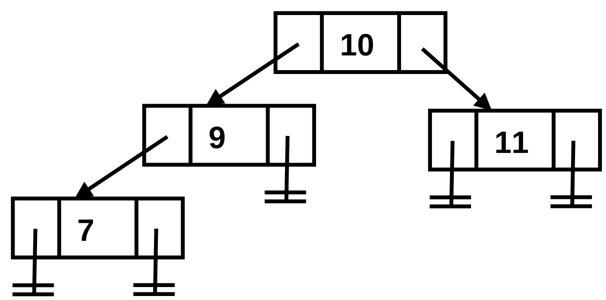
- Insert 9




```
if Root.Element < Element then
      Insert(Root.Right_Child, Element);
else
      Insert(Root.Left_Child, Element);
end if;
```

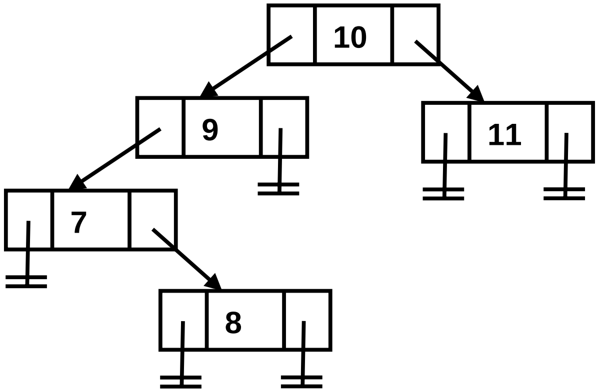

Inserting into Ordered Binary Tree

- Insert 7



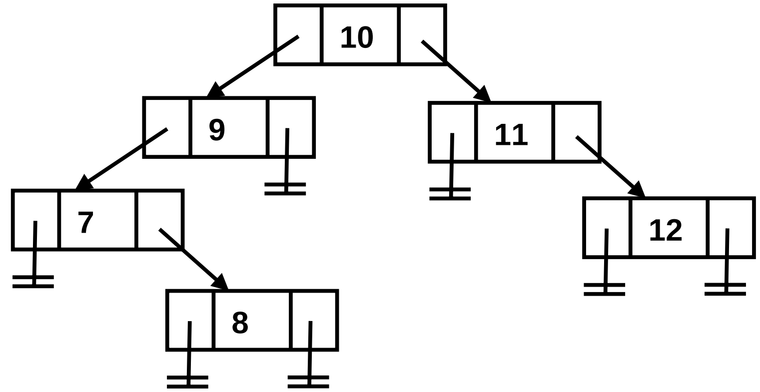
Inserting into Ordered Binary Tree

- Insert 8



Inserting into Ordered Binary Tree

- Insert 12



Shortest Path Problems

- **Dijkstra's algorithm**
  - Finds shortest path for a directed and connected graph G(V,E) which has non-negative weights.
  - Applications:
    - Internet routing
    - Road generation within a geographic region
    - ...

Dijkstra's Algorithm

- Dijkstra(G,w,s)

```
Init_Source(G,s)
S := empty set
Q := set of all vertices

while Q is not an empty set loop
    u := Extract_Min(Q)
    S := S union {u}
    for each vertex v which is a neighbor of u loop
        Relax(u,v,w)
```

Dijkstra’s Algorithm

- Init_Source(G,s)

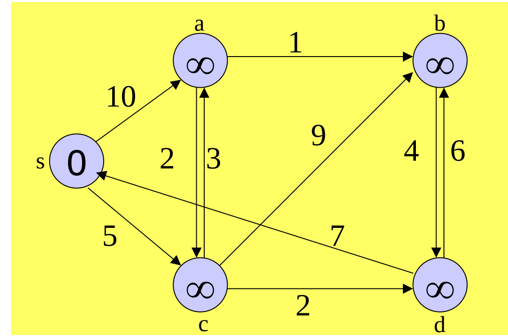
  for each vertex v in V[G] **loop**
  d[v] := infinite
  previous[v] := 0
  d[s] := 0

- v = Extract_Min(Q) searches for the vertex v in the vertex set Q that has the least d[v] value. That vertex is removed from the set Q and then returned.

- Relax(u,v,w)

  if d[v] > d[u] + w(u,v) **then**
  d[v] := d[u] + w(u,v)
  previous[v] := u

Dijkstra’s Algorithm

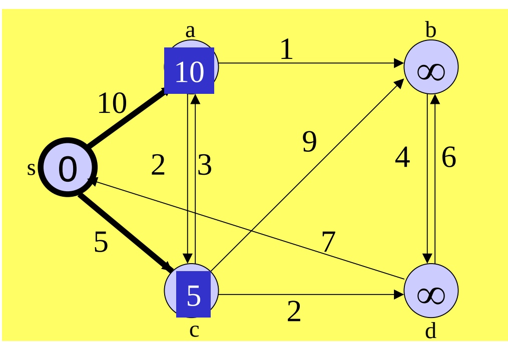


V = {a, b, c, d, s}
E = {(s,c), (c,d), (d,b), (b,d), (c,b), (a,c), (c,a), (a,b), (s,a)}

S = {∅}
Q = {s, a, b, c, d}

d = (0, ∞, ∞, ∞, ∞)   prev = (0, 0, 0, 0, 0)

Dijkstra’s Algorithm



S = {s}
Q = {a, b, c, d}

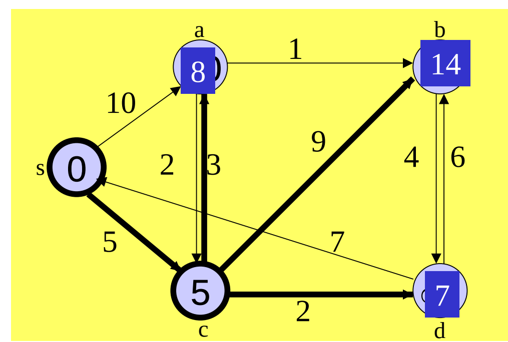
d = (0, ∞, ∞, ∞, ∞) → (0, 10, ∞, 5, ∞)

prev = (0, 0, 0, 0, 0) → (0, s, 0, s, 0)

Extract_Min (Q) → s
Neighbors of s = a, c

Relax (s,c,5)
Relax (s,a,10)

Dijkstra’s Algorithm



S = {s, c}
Q = {a, b, d}

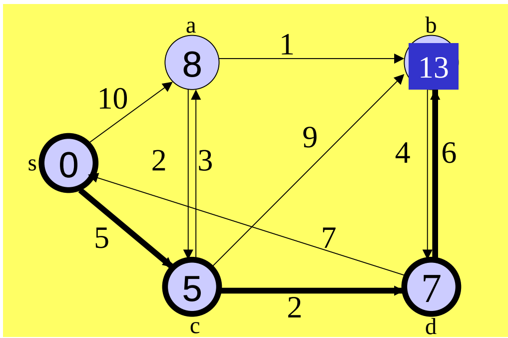
d = (0, 10, ∞, 5, ∞) → (0, 8, 14, 5, 7)

prev = (0, s, 0, s, 0) → (0, c, c, s, c)

Extract_Min (Q) → c
Neighbors of c = a, b, d

Relax (c,a,3)
Relax (c,b,9)
Relax (c,d,2)

Dijkstra's Algorithm



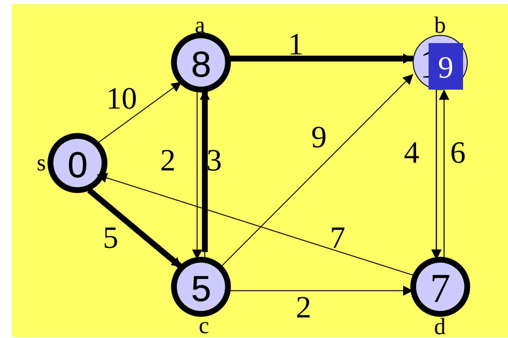
S = {s, c, d}
Q = {a, b}

d = (0, 8, 14, 5, 7) → (0, 8, 13, 5, 7)

prev = (0, c, c, s, c) → (0, c, d, s, c)

Extract_Min (Q) → d
Neighbors of d = b
Relax (d,b,6)

Dijkstra's Algorithm



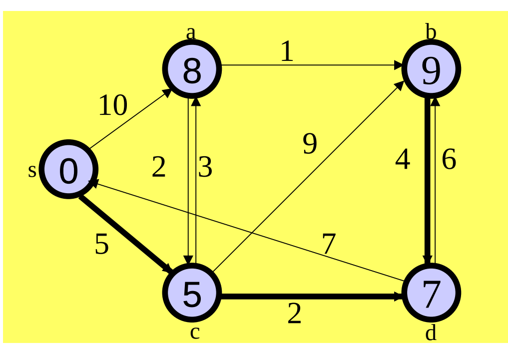
S = {s, c, d, a}
Q = {b}

d = (0, 8, 13, 5, 7) → (0, 8, 9, 5, 7)

prev = (0, c, d, s, c) → (0, c, a, s, c)

Extract_Min (Q) → a
Neighbors of a = b, c
Relax (a,b,1)
Relax (a,c,3)

Dijkstra's Algorithm



S = {s, c, d, a, b}
Q = {}

d = (0, 8, 9, 5, 7) → (0, 8, 9, 5, 7)

prev = (0, c, a, s, c) → (0, c, a, s, c)

Extract_Min (Q) → b
Neighbors of b = d
Relax (b, d, 4)