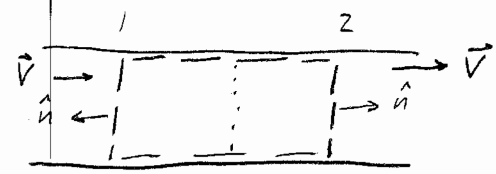
Fluids Quiz 2 | Solution | Spring '04

1.



a) $\oint \rho \hat{V}\cdot n \, h_o \, dA = \iiint \rho \dot{q} \, d\mathcal{V}$

$$\boxed{\begin{aligned} &\rho_2 V_2 A h_{o_2} - \rho_1 V_1 A h_{o_1} = \dot{Q} \\ &\text{or} \quad h_{o_2} - h_{o_1} = \frac{\dot{Q}}{\dot{m}} \\ &\text{or} \quad h_2 + \tfrac{1}{2}V_2^2 - h_1 - \tfrac{1}{2}V_1^2 = \frac{\dot{Q}}{\dot{m}} \end{aligned}}$$

b) $h_2 - h_1 \approx \frac{\dot{Q}}{\dot{m}}$ neglect V^2, $h = c_p T$

$T_2 - T_1 = \frac{1}{c_p}\frac{\dot{Q}}{\dot{m}}$, $\dot{m} = \rho_1 V_1 A = 1 \cdot 1 \cdot 0.1 = 0.1 \text{ kg/s}$

$$\boxed{T_2 = T_1 + \frac{1}{c_p}\frac{\dot{Q}}{\dot{m}} = 250\,K + \frac{1}{1000}\,\frac{5000}{0.1} = 300\,K}$$

c) $$\boxed{\frac{\rho_2}{\rho_1} = \frac{p_2}{p_1}\cdot\frac{T_1}{T_2} \simeq \frac{T_1}{T_2} = \frac{250}{300} = \frac{5}{6} = 0.8333}$$

$$\boxed{\frac{V_2}{V_1} = \frac{\rho_1}{\rho_2} = \frac{6}{5} = 1.2}$$

d) Momentum eqn: $p_2 + \rho_2 V_2^2 - p_1 - \rho_1 V_1^2 = 0$

$p_2 - p_1 = \rho_1 V_1^2 - \rho_2 V_2^2 = \rho_1 V_1 (V_1 - V_2)$ since $\rho_1 V_1 = \rho_2 V_2$

$$\boxed{p_2 - p_1 = 1 \cdot 1 \cdot (1 - 1.2) = -0.2 \text{ Pa}}$$

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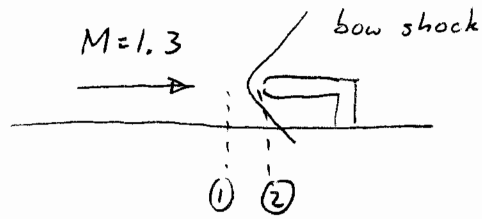
2.

a)



Pitot senses total pressure p_{o_2} behind bow shock.

From normal-shock table: for $M=1.3$, $p_{o2}/p_1 = 2.714$

since $p_1 = p_\infty$, $\boxed{p_{o_2} = 2.714 p_\infty}$

Can also use table's $p_{o2}/p_{o1} = 0.9794$

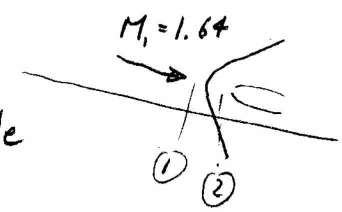
since $p_{o_1} = p_\infty \left[1 + \frac{\gamma-1}{2} M_\infty^2\right]^{\frac{\gamma}{\gamma-1}} = 2.771\, p_\infty$

$\rightarrow \quad p_{o_2} = 2.771\, p_\infty \cdot 0.9794 = 2.714$ same result

b) Now p_1 is behind expansion fan, with $\theta = 10°$

$\nu(M_1) - \nu(M_\infty) = 10°$, $\quad \nu(M_\infty) = \nu(1.3) = 6.17°$

$\nu(M_1) = 16.17° \rightarrow M_1 \approx 1.64$



Now, $p_{o2}/p_1 = 3.97$ from normal-shock table

$$\frac{p_1}{p_\infty} = \overbrace{\left(\frac{p_{o_1}}{p_{o_\infty}}\right)}^{=1} \frac{\left[1 + \frac{\gamma-1}{2} M_\infty^2\right]^{\frac{\gamma}{\gamma-1}}}{\left[1 + \frac{\gamma-1}{2} M_1^2\right]^{\frac{\gamma}{\gamma-1}}} = \frac{2.771}{4.511} = 0.614$$

$$p_{o2} = 3.97\, p_1 = 3.97 \cdot 0.614\, p_{o2} = \underline{2.438\, p_\infty}$$

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3.

a) $A^* = A_{t_1}$ since flow is choked.

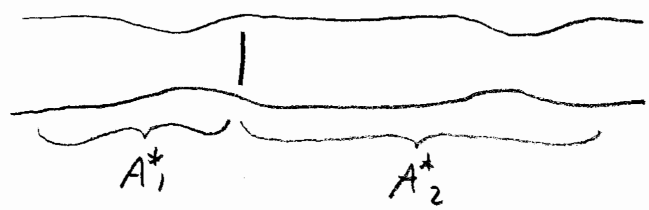
At shock location, we still have $A^* = A_{t_1}$

from Isentropic Flow table, for $M = 1.5$ ... $A/A^* = 1.176$

$$A = 1.176\, A^* = 1.176\, A_{t_1} = 1.176 \cdot 0.8 = 0.941$$

b) Behind the shock, $h_{o_2} = h_{o_1}$, $p_{o_2} = 0.9298\, p_{o_1}$ (from Table)

Need to find A_2^*



In general, $\rho^* a^* A^* = \dot{m} = \text{constant}$

so $\rho_1^* a_1^* A_1^* = \rho_2^* a_2^* A_2^* \rightarrow \dfrac{A_2^*}{A_1^*} = \dfrac{\rho_1^* a_1^*}{\rho_2^* a_2^*}$

$a^* = \sqrt{(\gamma-1) h_0} \left[1 + \frac{\gamma-1}{2}\right]^{-\frac{1}{2}}$ same for 1 and 2 since $h_{o_1} = h_{o_2}$

$\rho^* = \rho_0 \left(1 + \frac{\gamma-1}{2}\right)^{-\frac{1}{\gamma-1}}$

so $\dfrac{\rho_1^*}{\rho_2^*} = \dfrac{p_1^*}{p_2^*}\dfrac{h_2^*}{h_1^*} = \dfrac{p_1^*}{p_2^*} = \dfrac{p_{o_1}}{p_{o_2}} = \dfrac{1}{0.9298}$

$$\frac{A_2^*}{A_1^*} = \frac{A_{t_2}}{A_{t_1}} = \frac{\rho_1^*}{\rho_2^*} = \frac{1}{0.9298} = 1.07$$

$$\boxed{A_{t_2} = 1.07\, A_{t_1} = 1.07 \cdot 0.8 = 0.856\ m^2}$$