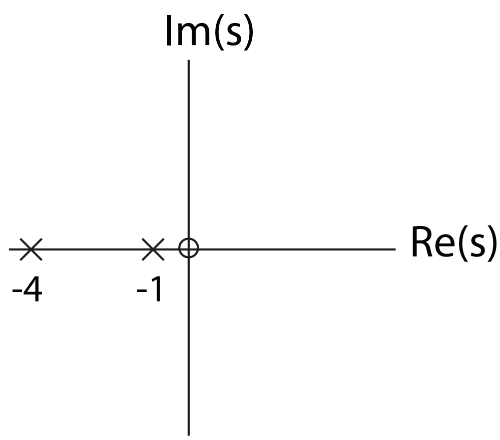
16.06 Principles of Automatic Control
Recitation 4

Problem 1.

Sketch the root locus for $L(s) = \frac{s}{(s+1)(s+4)}$.

$\phi_R = \frac{180^\circ + 360^\circ}{2-1} = 180^\circ$, open loop pole at $s = -1$, $s = -4$. Zero at $s = 0$.



Problem 2.

Sketch the root locus for $L(s) = \frac{s}{(s-1)(s-4)}$.

$\phi_R = \frac{180^\circ + 360^\circ}{2-1} = 180^\circ$, open loop pole at $s = 1$, $s = 4$. Zero at $s = 0$.

To find departure/arrival point from real axis, use characteristic equation:

$$1 + kL(s) = 0 \rightarrow 1 + \frac{ks}{s^2 - 5s + 4} = 0$$

$$\Rightarrow s^2 + (k-5)s + 4 = 0$$

Use quadratic formula

$$\frac{-(k-5)}{2} \pm \frac{\sqrt{(k-5)^2 - 16}}{2}$$

The $\frac{\sqrt{(k-5)^2-16}}{2}$ term may be real or imaginary. If we sent it equal to zero and solve for k, that is the gain at which the transition from real to imaginary occurs.

$$\frac{\sqrt{(k-5)^2 - 16}}{2} = 0$$

$$\begin{aligned} (k-5)^2 =& 15 \\ |k-5| =& 4 \\ \rightarrow k =& 1, 9 \end{aligned}$$

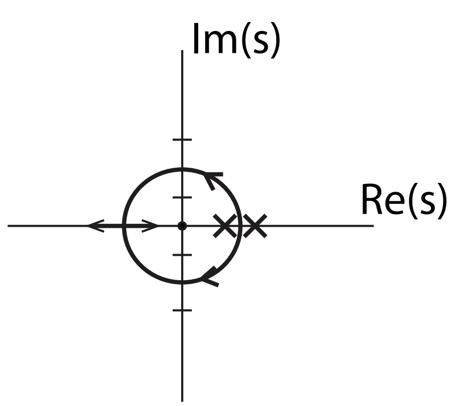
Now need to put k values back into characteristic equation, and solve for s. This will tell us the location of the roots.

$k = 1 \rightarrow s^2 - 4s + 4 = 0$, two roots at $s = 2$, $k = 9 \rightarrow s^2 + 4s + 4 = 0$.

Two roots at $s = -2$.

When $k = 5$, the real part of the quadratic equation is zero, so this is the value of k for when the locus intersects the imaginary axis. Plugging $k = 5$ into characteristic equation:

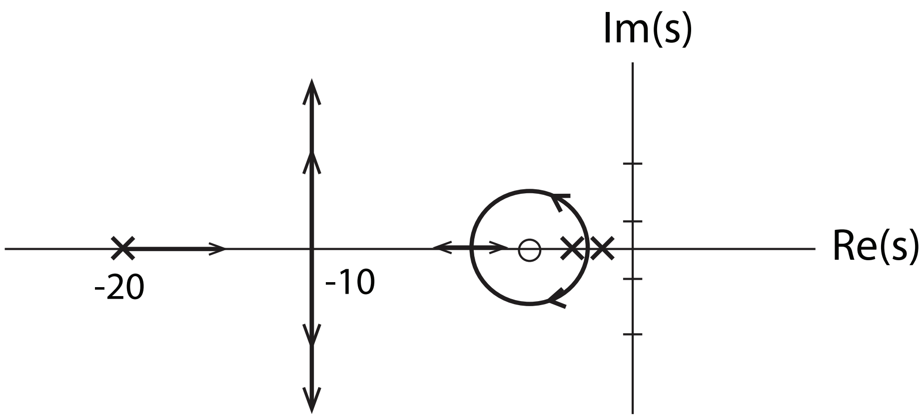
$s^2 + 4 = 0 \rightarrow$Intersects imaginary axis at $s = \pm 2j$.



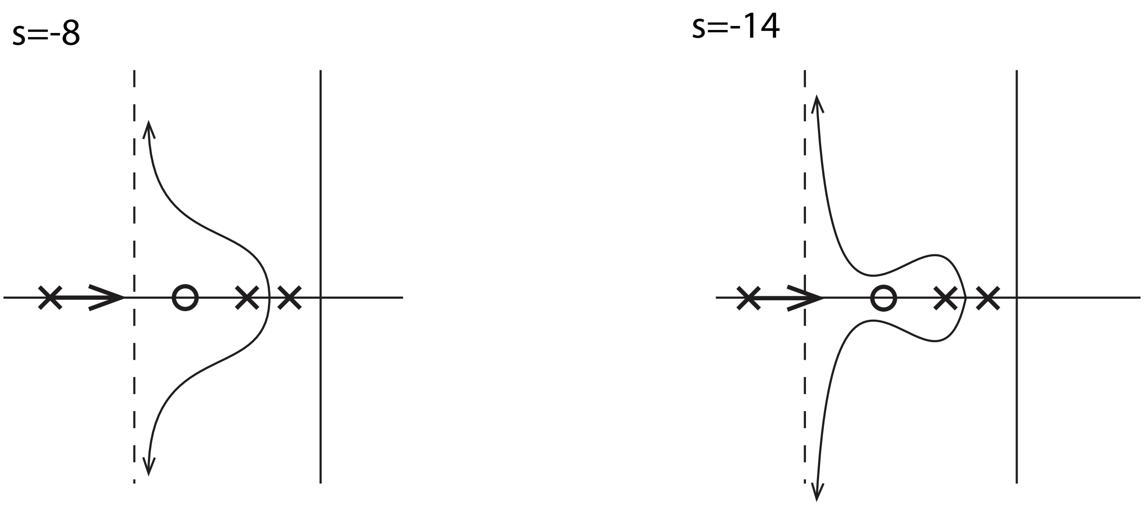
Problem 3.

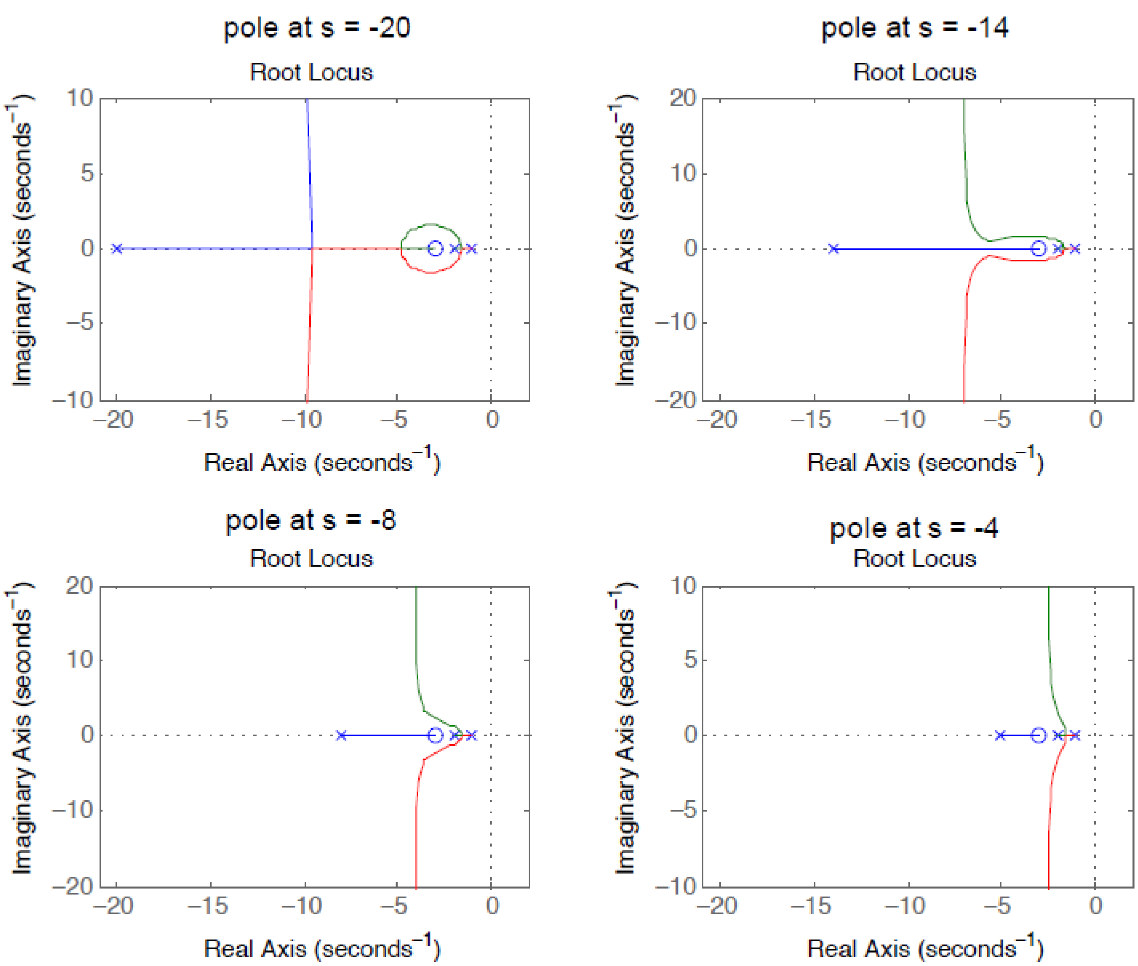
Sketch the locus of $L(s) = \frac{s+3}{(s+1)(s+2)(s+20)}$

$\alpha = \frac{-1-2+3-20}{3-1} = -10$



If the pole at $s = -20$ were closer to the zero, the locus would look more like




pole at s = -20
Root Locus
pole at s = -14
Root Locus
pole at s = -8
Root Locus
pole at s = -4
Root Locus
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
10
5
0
−5
−10
20
−20
−15
−10
−5
0