NAME : ....

Massachusetts Institute of Technology

16.07 Dynamics

Problem Set 9

Out date: Oct 31, 2007

Due date: Nov. 7, 2007

	Time Spent [minutes]
Problem 1	
Problem 2	
Problem 3	
Study Time	

Turn in **each problem** *on* **separate sheets** *so that grading can be done in parallel*

Problem 1 (10 points)

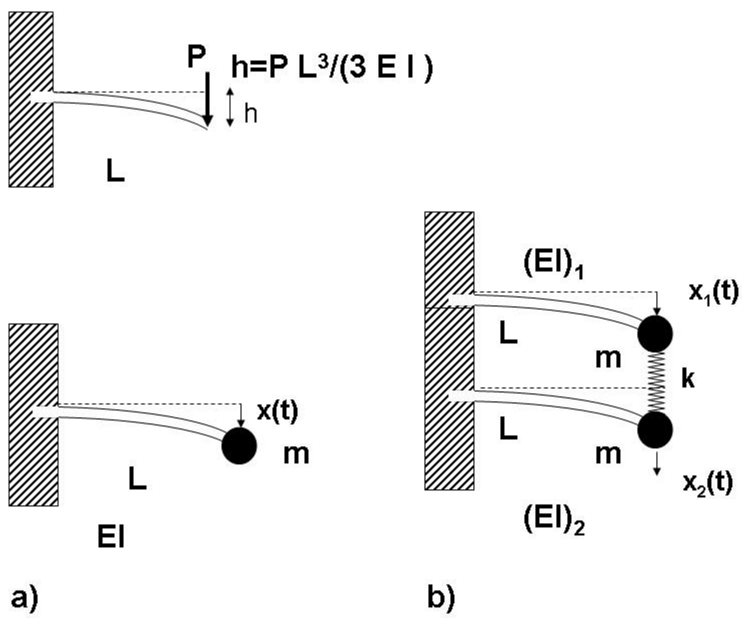
The displacement h of an elastic beam under the action of a point load P is given by

$$h = \frac{PL^3}{3EI} \tag{1}$$

where L is the length; EI is the product of Young's modulus and the cross-sectional area moment of the beam I. Consider case a): a beam with a mass m at its tip.

a-1) Take the beam as massless; ignore gravity; write Newton's law for the motion of the mass $x(t)$.

a-2) What is the natural frequency of oscillation of the beam-mass system?



Now consider case b): two beams of different stiffnesses EI_1 and EI_2 are joined by a spring of stiffness k which exerts a force proportional to its change in length. Put an equal mass m at the end of each beam. Ignore gravity.

b-1) Write Newton's law for the coupled motion of both $x_1(t)$ and $x_2(t)$.

b-2) Carry through the procedure outlined in Lecture 19 to determine the 2 natural frequencies of oscillation and the normal modes for each natural frequency.

b-3) What would the 2 frequencies have been if the beams were not joined? i.e. for k=0. How are they modified by the coupling?

b-4) What happens when $EI_1 = EI_2$? Discuss the geometry of the normal modes in this case.

Problem 2 (10 points)

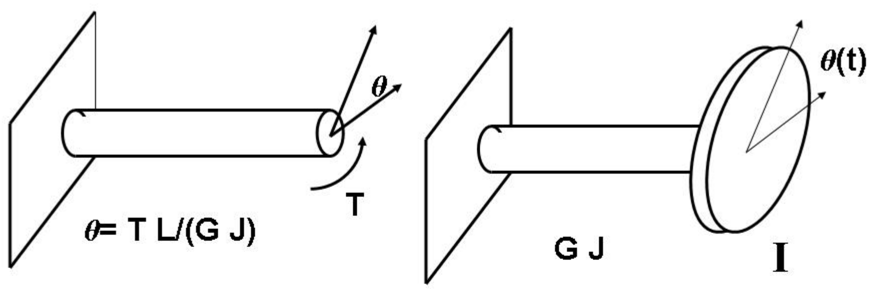
The angular displacement θ of an elastic beam, fixed at one end, under the action of a torque T applied at the other end, is given by

$$\theta = \frac{TL}{GJ} \tag{2}$$

where L is the length; GJ is the product of the shear modulus and the cross-sectional area moment of the beam J. Just take these as given quantities. Now consider the beam with a circular disk of mass moment of inertia I at its tip.

A-1 1. Take the beam as massless; fix a circular disk at the end of the beam with a mass moment of inertia I; write Newton's law for the angular motion of the mass $\theta(t)$.

A-2 What is the natural frequency of oscillation of the beam-disk system?

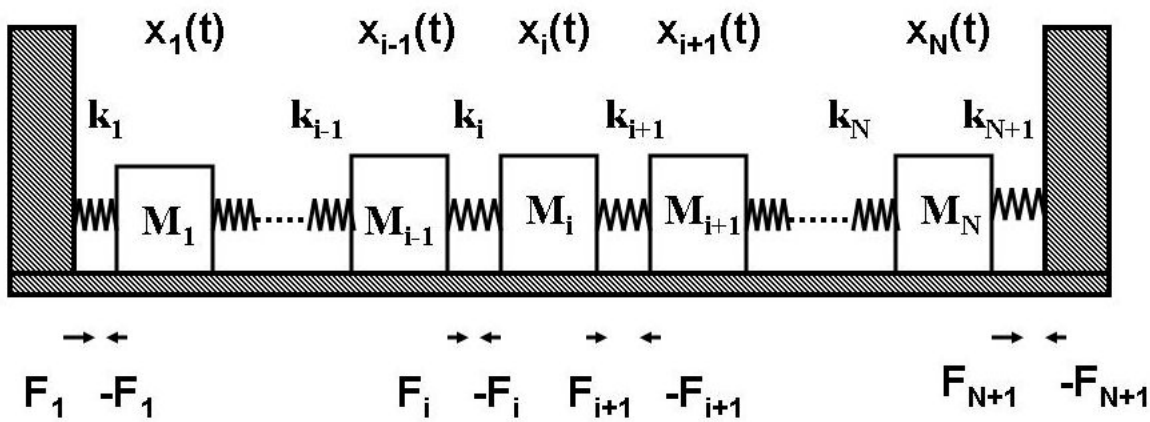


Problem 3 MATLAB (20 points)

3-1) We are going to use the power of MATLAB to solve an N degree of freedom system. We will end up taking N=10. Consider the general system shown in the figure. N masses M_i connected by $N+1$ springs of stiffness k_i. At equilibrium, the masses are an equal distance apart.
The forces acting on the masses due to the extensions of the springs are sketched.

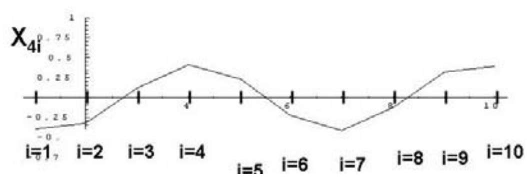
- Write Newton's law for the behavior of mass i.
- Write Newton's law for the behavior of mass 1.
- Write Newton's law for the behavior of mass N.
- For N=10, and for all the masses equal to m, and all the spring constants equal to k, write the matrix [A] that appears in the characteristic value problem as shown below. In this case instead of a 2×2 matrix we will have a $? \times ?$ matrix.

$$\left(\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right) \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \tag{3}$$



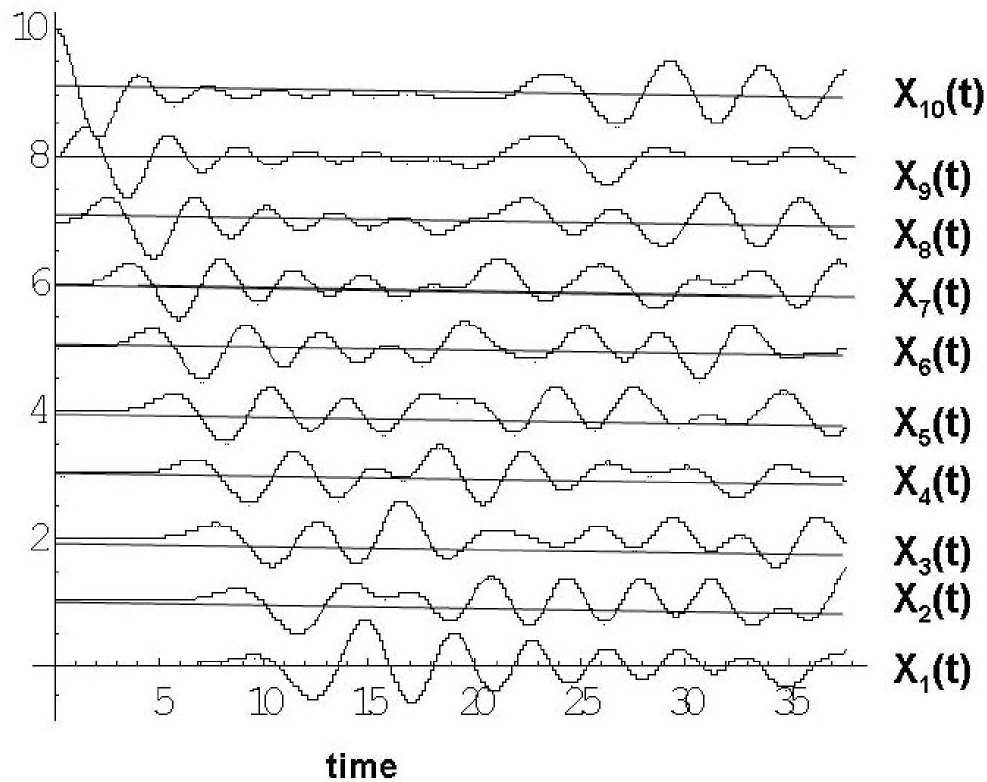
3-2) Now feed your matrix [A] to MATLAB, and tell MATLAB you want the eigenvalues and the eigenvectors vectors of [A]. Present your results as a table of modal displacements for each mode, giving its natural frequency as well.

Sketch the "shape" of the first 3 modes (first is defined as the mode with the lowest frequency, etc.) ; i.e. plot X_{1i} vs i–that would be the first mode corresponding to ω_1. Repeat for X_{2i} (corresponding to ω_2) and X_{3i} (corresponding to ω_3. Indicate the ω's for each mode. The figure shows a plot of the 4th mode. That is, the X_{4i} are plotted "vs" i. **The points are joined to aid visualization, but of course they just represent the sidewise displacement of the x_i's due to the 4th mode.**



3-3) Now consider initial conditions. Take all masses to be at rest, and all masses to be at their "zero" position except the 10th mass for which $x_{10}(0) = 1$, ie. the initial- condition vector is $(0, 0, 0, 0, 0, 0, 0, 0, 0, 1)$. For this case, calculate the coefficients A_i for each of the "normal modes" (or eigenvectors) $\vec{X}_i$ oscillating at its natural frequency ω_i. Sum up to find a complete solution: $x_i(t)$, $i = 1$ to $i = 10$ for the 10 degree of freedom system.

The figure shows the results: each $x_i(t)$ is plotted for $t = 0$ to $T = 8\pi$. The curves are displaced by $i - 1$ to make viewing understandable.



Interpret these result. Remember that the initial condition was that only $x_{10}(0)$ was displaced. All other $x_i(0) = 0$. Can you see the satisfaction of this initial condition at $t = 0$? What happens with time? How does the disturbance behave? How long does it take for x_1 to respond?