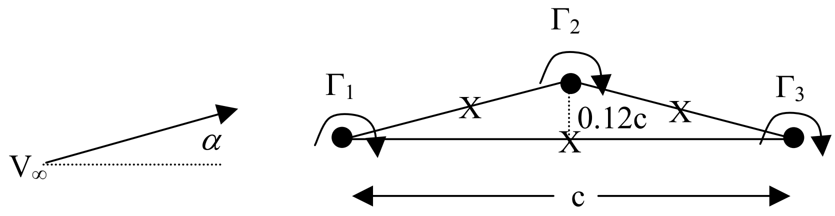
16.100 Homework Assignment # 3

Due: Monday, September 26th, 9am

Problem 1 (70%)
Useful reading: Anderson: Section 3.14, 3.16, 4.5, 4.10



● nodes where vortices are located

X control points where $\vec{u} \cdot \vec{n} = 0$ is enforced

Consider the isosceles triangle airfoil with 12% thickness as shown above. A simple potential flow model for the flow over the triangle is to place point vortices at each of the three nodes and then determine the strengths (i.e. the circulations Γ_i) which satisfy flow tangency at three control points. For this model, let the control points be located at the midpoints of the three edges of the triangle.

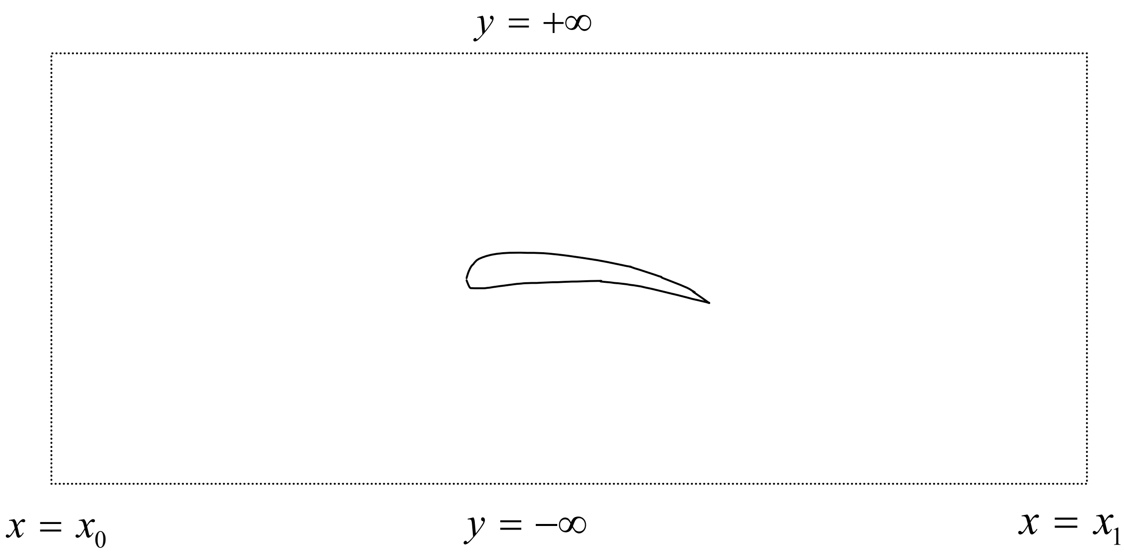
a) Set-up the 3x3 system of equations that expresses $\vec{u} \cdot \vec{n} = 0$ at the control points.

b) Show that the 3x3 matrix for this system is singular (i.e. it has at least one zero eigenvalue).

c) The singularity of the matrix can be removed by applying another condition in addition to flow tangency. Since the solution that is often of interest is attached flow in which the flow leaves the trailing edge smoothly and does not wrap around the trailing edge (i.e. the flow that satisfies the Kutta condition). Any non-zero value of the circulation at node 3 would imply the flow wraps around the trailing edge (note: the velocity becomes infinite at a point vortex and therefore cannot be counteracted by the other point vortices). Thus, in this simple flow model, the Kutta condition would require that $\Gamma_3 = 0$. Replace the flow tangency condition at the control point between nodes 2 and 3 with the Kutta condition. What is the solution for Γ_1 and Γ_2 that satisfies the remaining flow tangency conditions and the Kutta condition as a function of the freestream angle of attack?

d) Calculate and plot the lift coefficient as a function of angle of attack from –5 to +5 degrees. What is the lift slope and what is the angle of zero-lift? Note: use the Kutta-Joukowski theorem to find the lift.

Problem 2 (30%)
Useful reading: Anderson: Section 2.6, 3.16



a) Using the integral form of the momentum equation, show that the lift (per unit span) is related to the amount of the airfoil turns the flow and is given by the following formula:

$$L' = \left. \int_{-\infty}^{+\infty} \rho u v \, dy \right|_{x=x_0} - \left. \int_{-\infty}^{+\infty} \rho u v \, dy \right|_{x=x_1}$$

b) The velocity a few chords away from a lifting airfoil is well approximated by the freestream and a point-vortex,

$$\vec{V} = V_\infty \vec{i} - \frac{\Gamma_\infty}{2\pi r} \vec{e}_\theta$$

where Γ_∞ is a constant. Using this assumed velocity field and the result from part a), determine the lift (per unit span) for this flow (you may assume incompressible flow). Does this result look familiar?