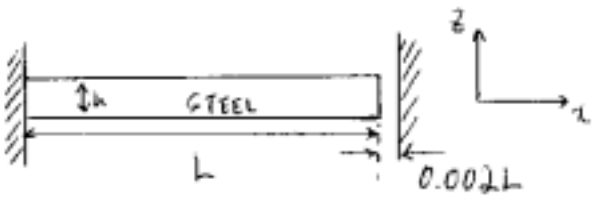
Home Assignment #5 : Solutions

Warm-Up Exercises



Given data:

steel material properties:
- $E = 30\ Msi$
- $\nu = 0.3$
- $\alpha = 6\ \mu\varepsilon/^\circ F$

$T_{Room} = 70^\circ F$.

1. If the beam is subjected to uniform temperature change and does not touch the wall, it is undergoing free expansion. Therefore, there is <u>no stress</u> in the beam (except at the cantilevered end of the beam, which we ignore by invoking St. Venant's principle). The strain in the beam is entirely due to thermal effects.

(ε: total strain, ε^T: thermal strain)

$$\varepsilon = \varepsilon^T = \alpha \Delta T$$

$$\Rightarrow \boxed{\varepsilon = 6(T - 70)\ \mu strain.}$$

2. For the bar to come in contact with the wall, it must deform by ΔL. This corresponds to a strain of

$$\varepsilon = \frac{\Delta L}{L} = \frac{0.002L}{L} = 0.002$$

Since $\varepsilon = \alpha \Delta T$, we can find the temperature required to elongate the bar by a strain of 0.002.

$$\alpha \Delta T = 0.002$$

$$\Rightarrow 6 \times 10^{-4}(T - 70) = 0.002$$

$$\Rightarrow \boxed{T = 403^\circ F}$$

As stated above, the strain is

$$\boxed{\varepsilon = 0.002}$$

and the corresponding <u>stress is zero</u> because the bar is still unconstrained.

3. If the bar does contact the wall, the stress in the bar will no longer be zero because the expansion of the bar is constrained by the wall. Thus, there will be thermal stresses. In additional, the constraints at the wall will produce a mechanical component of strain, which was not present in the uncontrained cases, problems #1 & #2. The mechanical strain can be found by considering the total strain in the bar after it comes in contact with the wall. The total strain stays constant at 0.002, i.e.,

$$\varepsilon = \varepsilon^T + \varepsilon^M = 0.002$$

$$\Rightarrow \quad \varepsilon^M = 0.002 - \varepsilon^T$$

$$\Rightarrow \quad \varepsilon^M = 0.002 - \alpha \Delta T$$

$$\therefore \quad \boxed{\varepsilon^M = 0.002 - 6\times10^{-6}(T-70)} \qquad (T > 403^\circ F)$$

The thermal strain is

$$\boxed{\varepsilon^T = \alpha \Delta T = 6\times10^{-6}(T-70)}$$

* Note that ε^M can also be written as:

$$\varepsilon^M = 0.002 - 6\times10^{-6}(T-70) = 6\times10^{-6}(403-70) - 6\times10^{-6}(T-70)$$

$$\Rightarrow \quad \varepsilon^M = -6\times10^{-6}(T-403) \qquad (T > 403^\circ F)$$

The stress is equal to the mechanical strain multiplied by the modulus, E. Thus,

$$\sigma = E\varepsilon^M = (30\ \text{Msi})(-6\times10^{-6})(T-403)$$

$$\Rightarrow \boxed{\sigma = -180(T-403)\ \text{psi}} \qquad (T > 403^\circ F)$$