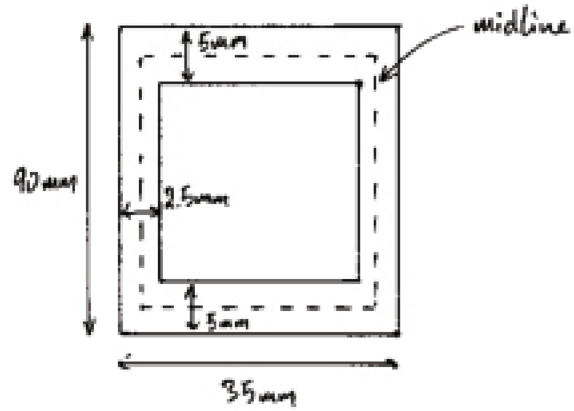
Solutions to Home Assignment #1

Warm-Up Exercises

We will use our knowledge of thin-walled closed sections to solve problems 1 through 4.

1.



For a thin-walled

$$J = \frac{4A^2}{\oint \frac{ds}{t}} \qquad \text{——— ①}$$

where

A = area enclosed by midline

s = midline perimeter

Using the known geometry of the closed section, we can calculated J.

$$A = (90-5)(35-2.5) = 85 \times 32.5$$

$$\therefore \quad A = 2760 \text{ mm}^2 \qquad \text{——— ②}$$

$$\oint \frac{ds}{t} = \sum \frac{s_i}{t_i} = \frac{85}{2.5} + \frac{32.5}{5} + \frac{85}{2.5} + \frac{32.5}{5}$$

$$\therefore \quad \oint \frac{ds}{t} = 81 \qquad \text{——— ③}$$

Plugging ② and ③ into ①, we get

$$J = \frac{4 \times (2160\,mm^2)^2}{81}$$

$$\therefore \boxed{J = 376000\,mm^4}$$

2. The angle of twist is

$$\alpha = \frac{TL}{GJ} \qquad \text{———————} \qquad ①$$

Therefore the ratio of angle of twist of the configuration with the closed section to the configuration of the open section is

closed section →

$$\frac{\alpha_c}{\alpha_o} = \frac{TL}{GJ_c} \Big/ \frac{TL}{GJ_o} = \frac{J_o}{J_c}$$

open section →

$$\therefore \frac{\alpha_c}{\alpha_o} = \frac{6300\,mm^4}{376000\,mm^4}$$

$$\Rightarrow \boxed{\frac{\alpha_c}{\alpha_o} = 0.0168}$$

3. The resultant shear stress is given by the equation

$$\tau = \frac{T}{2At} \qquad \text{——— (1)}$$

(Bredt's formula, notes unit 12, p 17)

The maximum resultant shear stress will be at the minimum thickness. For the configuration given, the thickness of the vertical walls are

$$t_{min} = 2.5mm.$$

Therefore,

$$\tau_{max} = \frac{T}{2(2760mm^2)(2.5mm)}$$

$$\Rightarrow \boxed{\tau_{max} = \frac{T}{13800mm^3}} \quad \text{at vertical walls.}$$

4. Let's compare the torsional constant, angle of twist, and the maximum resultant shear stress of the current thin-walled closed section to the previous open section beam in HA#6, problem #5. The applied torque is taken to be

$$T = 40Nm$$

	torsional constant, J	angle of twist, α	maximum resultant shear, τ_{max}
closed section	371600 mm^4	$0.454°$	2.90 MPa
open section	6300 mm^4	$27°$	32MPa

Let's consider the angle of twist, α, first. The angle of twist for the closed section beam is much smaller than the open section beam (the ratio was 0.0168 as found in problem #2). The reason for the large difference in angle of twist even though the same amount of material was used in the two cases is due to the torsional constant, J. Recall that the rate of twist, $\frac{d\alpha}{dz}$, is inversely proportional to the torsional rigidity, GJ. Thus, if the same material is used, the beam with a larger torsional rigidity will have a smaller rate of twist which translates into a smaller angle of twist when the length is equal.

Next, let's look at the maximum resultant shear stress. Recall that in closed sections, the torque creates a "shear flow" whose magnitude is τt $(=q)$ and constant. Thus, the shear across a constant thickness section is constant. In the open section case, shear is linearly distributed across the thickness. Therefore, if the thickness is small (i.e. thin-walled open section), the slope of the shear stress needs to be very steep, causing very large stresses at the outer surfaces. Thus, it is not surprising that the maximum resultant shear stress for the open section case is much greater than the closed section case.