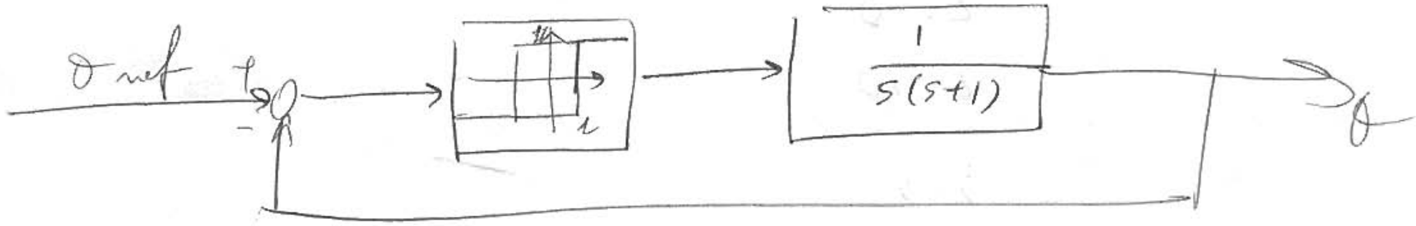
(2) (1)



a) Look to formula for toggle switch DF:

$$n_p = \frac{4}{\pi A}\sqrt{1-\left(\frac{1}{A}\right)^2} \quad \Leftarrow \text{real part}$$

$$n_q = \frac{-4}{\pi A^2} \quad \Leftarrow \text{imaginary part}$$

We need to solve:

$$1 + \frac{\frac{4}{\pi A}\left(\sqrt{1-\left(\frac{1}{A}\right)^2} - j\frac{1}{A}\right)}{j\omega(j\omega+1)} = 0$$

or:

$$-\omega^2 + j\omega + \frac{4}{\pi A}\sqrt{1-(1/A)^2} - j\frac{4}{\pi A^2} = 0$$

or:

$$-\omega^2 + \frac{4}{\pi A}\sqrt{1-(1/A)^2} = 0 \quad (1)$$

$$\omega - \frac{4}{\pi A^2} = 0 \quad (2)$$

so: from 2, $\omega = \frac{4}{\pi A^2}$

substituting in 1:

$$-\frac{16}{\pi^2 A^4} + \frac{4}{\pi A}\sqrt{1-(1/A)^2} = 0$$

or: $$-\frac{4}{\pi A^3} + \sqrt{1-(1/A)^2} = 0$$

or: $$\frac{16}{\pi^2 A^6} = 1 - \left(\frac{1}{A}\right)^2$$

At this point, solve iteratively ....

$$\begin{cases} A \simeq 1.28 \\ \omega = 0.7771 \text{ rad/sec} \end{cases}$$

b) See pic - We get A line ~ 1.3 and ω line $\simeq 0.7854$ rad/sec.

not bad, eh?

c) see figure -

The limit cycle remains, same frequency, same amplitude -

2.) G & W. 2.8

This is a memoryless nonlinearity - Thus no imaginary part!

we compute:

$$\frac{1}{\pi A}\int_0^{2\pi} \cancel{sfn}\, f(A\sin\theta)\sin\theta\, d\theta.$$

$$= \frac{1}{\pi A}\int_0^{\pi} f(A\sin\theta)\sin\theta\, d\theta.$$

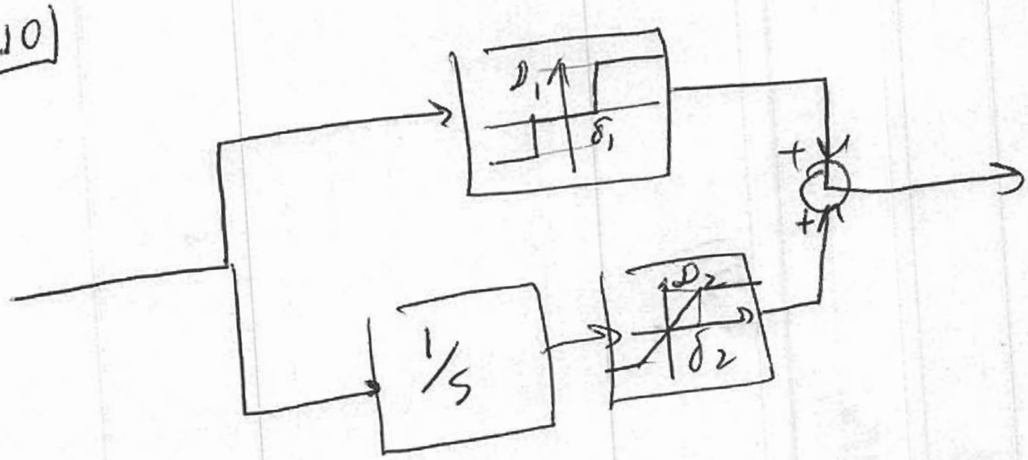
$f(u) = \sin u \quad -\pi \leq u \leq \pi.$

$f(u) = 0$ otherwise.

from there, use numerical approximations -

the DF is shown in printout -

2.10)



a) DF? These are two (easy) nonlinearities mounted in parallel. The DF is the sum of the individual DFs:

$$N(A) = \frac{4D_1}{\pi A}\sqrt{1-\left(\frac{\delta_1}{A}\right)^2} - \frac{j}{\omega}\,\frac{D_2}{\delta_2}\, f\left(\frac{\delta_2}{(A/\omega)}\right)$$

with $f\left(\frac{\delta_2}{(A/\omega)}\right) = 1$ if $A/\omega \leq \delta_2$.

$$= \frac{2}{\pi}\left(\sin^{-1}\left(\frac{\delta_2}{A/\omega}\right) + \frac{\delta_2}{A/\omega}\sqrt{1-\left(\frac{\delta_2}{A/\omega}\right)^2}\right)$$

if $\frac{A}{\omega} \geq \delta_2$.

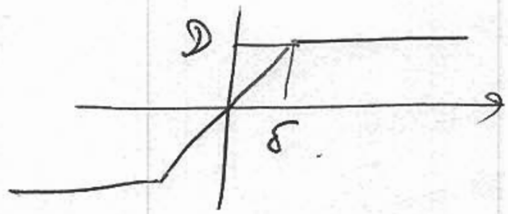
b) Identify $D_1, \delta_1, D_2, \delta_2$.

First identify D_2, δ_2 by running sinusoidal or other signals of amplitude less than δ_1 into system.

Then get δ_1 and D_1 (by subtracting output of other nonlinearity).

3-3]

$$L(s) = \frac{K(\tau s + 1)^3}{s^3}.$$



<u>determining existence of limit cycles</u>

<u>use analytical method;</u>

$$1 + L(s)\, N(A) = 0$$

<u>or</u> $$1 + K\frac{(\tau j\omega + 1)^3}{(j\omega)^3}\frac{D}{\delta} f\left(\frac{\delta}{A}\right) = 0$$

(f has been defined in problem 3)

<u>or</u> $$-j\omega^3 + K(-j\tau^3\omega^3 - 3\tau^2\omega^2 + 3\tau j\omega + 1)\frac{D}{\delta} f\left(\frac{\delta}{A}\right) = 0$$

using real part of equation, we get:

$$-3\tau^2\omega^2 + 1 = 0$$

or: $$\omega = +\frac{1}{\tau\sqrt{3}}$$

imaginary part of equation is:

$$-\omega^3 - \left(K\tau^3\omega^3 - K\,3\tau\omega\right)\frac{D}{\delta} f\left(\frac{\delta}{A}\right) = 0$$

or: $$-\frac{1}{\tau^3 3\sqrt{3}} - \left(\frac{K}{3\sqrt{3}} - K\sqrt{3}\right)\frac{D}{\delta} f\left(\frac{\delta}{A}\right) = 0$$

or: $$\frac{1}{\tau^3 \cancel{3\sqrt{3}}} + K\left(\frac{1-9}{\cancel{3\sqrt{3}}}\right)\frac{D}{\delta} f\left(\frac{\delta}{A}\right) = 0$$

or: $$\boxed{f\left(\frac{\delta}{A}\right) = \frac{\delta}{8DK\tau^3}}$$

so: we have a limit cycle if

$$\boxed{\frac{\delta}{8DK\tau^3} < 1}$$

(otherwise the above equation has no solution since $f(u) < 1$ $u \geq 0$)