Topic #12

16.30/31 Feedback Control Systems

State-Space Systems

- **Full-state Feedback Control**
- How do we change location of state-space eigenvalues/poles?
- Or, if we can change the pole locations **where do we put the poles?**
  - Heuristics
  - Linear Quadratic Regulator
- How well does this approach work?

- Reading: FPE 7.4

Pole Placement Approach

- So far we have looked at how to pick K to get the dynamics to have some nice properties (*i.e.* stabilize A)

$$\lambda_i(A) \rightsquigarrow \lambda_i(A - BK)$$

- **Question:** where should we put the closed-loop poles?

- **Approach #1:** use time-domain specifications to locate dominant poles – roots of:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

  - Then place rest of the poles so they are "much faster" than the dominant 2nd order behavior.
  - Example: could keep the same damped frequency w_d and then move the real part to be 2–3 times faster than the real part of dominant poles $\zeta\omega_n$
    - Just be careful moving the poles too far to the left because it takes a lot of control effort

- Recall ROT for 2nd order response (4–??):

10-90% rise time	$t_r = \dfrac{1 + 1.1\zeta + 1.4\zeta^2}{\omega_n}$
Settling time (5%)	$t_s = \dfrac{3}{\zeta\omega_n}$
Time to peak amplitude	$t_p = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}}$
Peak overshoot	$M_p = e^{-\zeta\omega_n t_p}$

- **Key difference** in this case: since all poles are being placed, the assumption of dominant 2nd order behavior is pretty much guaranteed to be valid.

Linear Quadratic Regulator

- **Approach #2:** is to place the pole locations so that the closed-loop system optimizes the cost function

$$J_{LQR} = \int_0^\infty \left[\mathbf{x}(t)^T Q \mathbf{x}(t) + \mathbf{u}(t)^T R \mathbf{u}(t)\right] dt$$

  where:

  - $\mathbf{x}^T Q \mathbf{x}$ is the **State Cost** with weight Q
  - $\mathbf{u}^T R \mathbf{u}$ is called the **Control Cost** with weight R
  - Basic form of **Linear Quadratic Regulator** problem.

- MIMO optimal control is a time invariant linear state feedback

$$\mathbf{u}(t) = -K_{\text{lqr}} \mathbf{x}(t)$$

  and K_{lqr} found by solving **Algebraic Riccati Equation** (ARE)

$$\begin{aligned} 0 &= A^T P + PA + Q - PBR^{-1}B^T P \\ K_{\text{lqr}} &= R^{-1}B^T P \end{aligned}$$

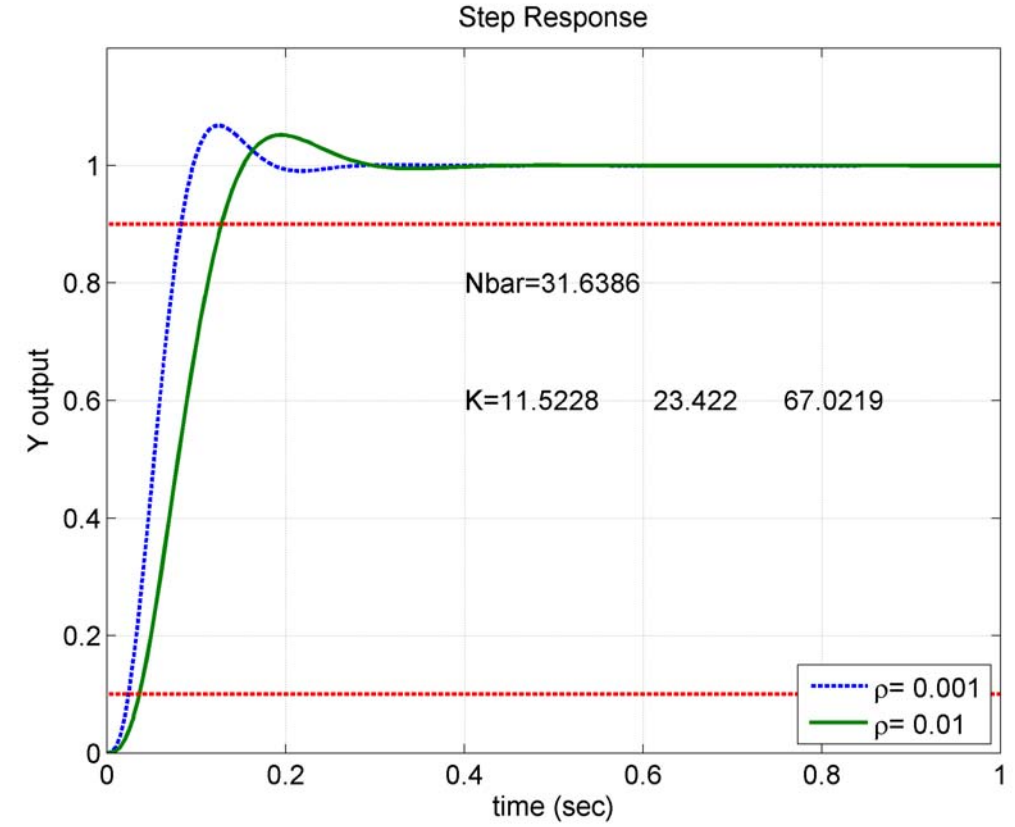
  - Some details to follow, but discussed at length in 16.323

- **Note:** state cost written using output $\mathbf{x}^T Q \mathbf{x}$, but could define a system output of interest $\mathbf{z} = C_z \mathbf{x}$ that is not based on a physical sensor measurement and use cost ftn:

$$\Rightarrow \quad J_{LQR} = \int_0^\infty \left[\mathbf{x}^T(t) C_z^T \tilde{Q} C_z \mathbf{x}(t) + \rho\ \mathbf{u}(t)^T \mathbf{u}(t)\right] dt$$

  - Then effectively have state penalty $Q = (C_z^T \tilde{Q} C_z)$
  - Selection of $\mathbf{z}$ used to isolate system states of particular interest that you would like to be regulated to "zero".
  - $R = \rho I$ effectively sets the controller bandwidth

Fig. 1: Example #1: $G(s) = \frac{8 \cdot 14 \cdot 20}{(s+8)(s+14)(s+20)}$ with control penalty ρ and 10ρ



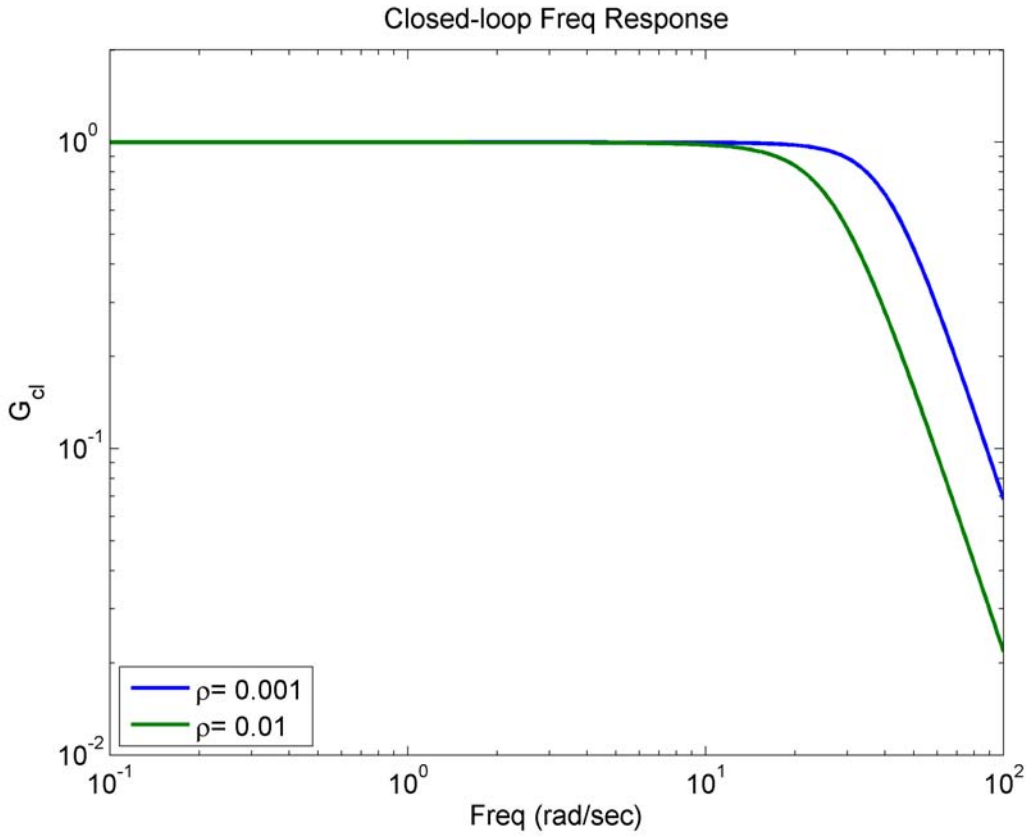


Fig. 2: Example #2: $G(s) = \frac{0.94}{s^2 - 0.0297}$ with control penalty ρ and 10ρ

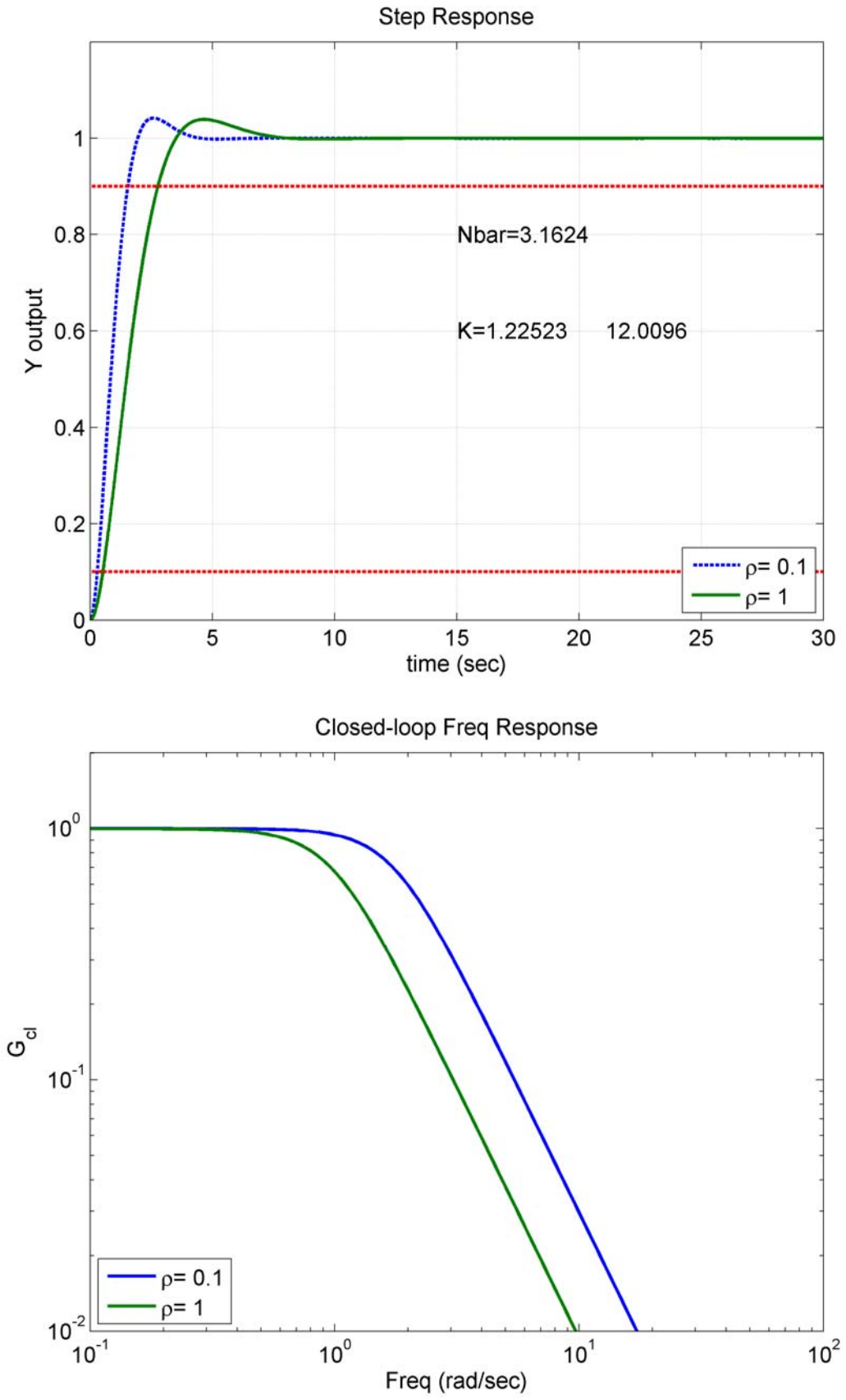


Fig. 3: Example #3: $G(s) = \frac{8 \cdot 14 \cdot 20}{(s-8)(s-14)(s-20)}$ with control penalty ρ and 10ρ

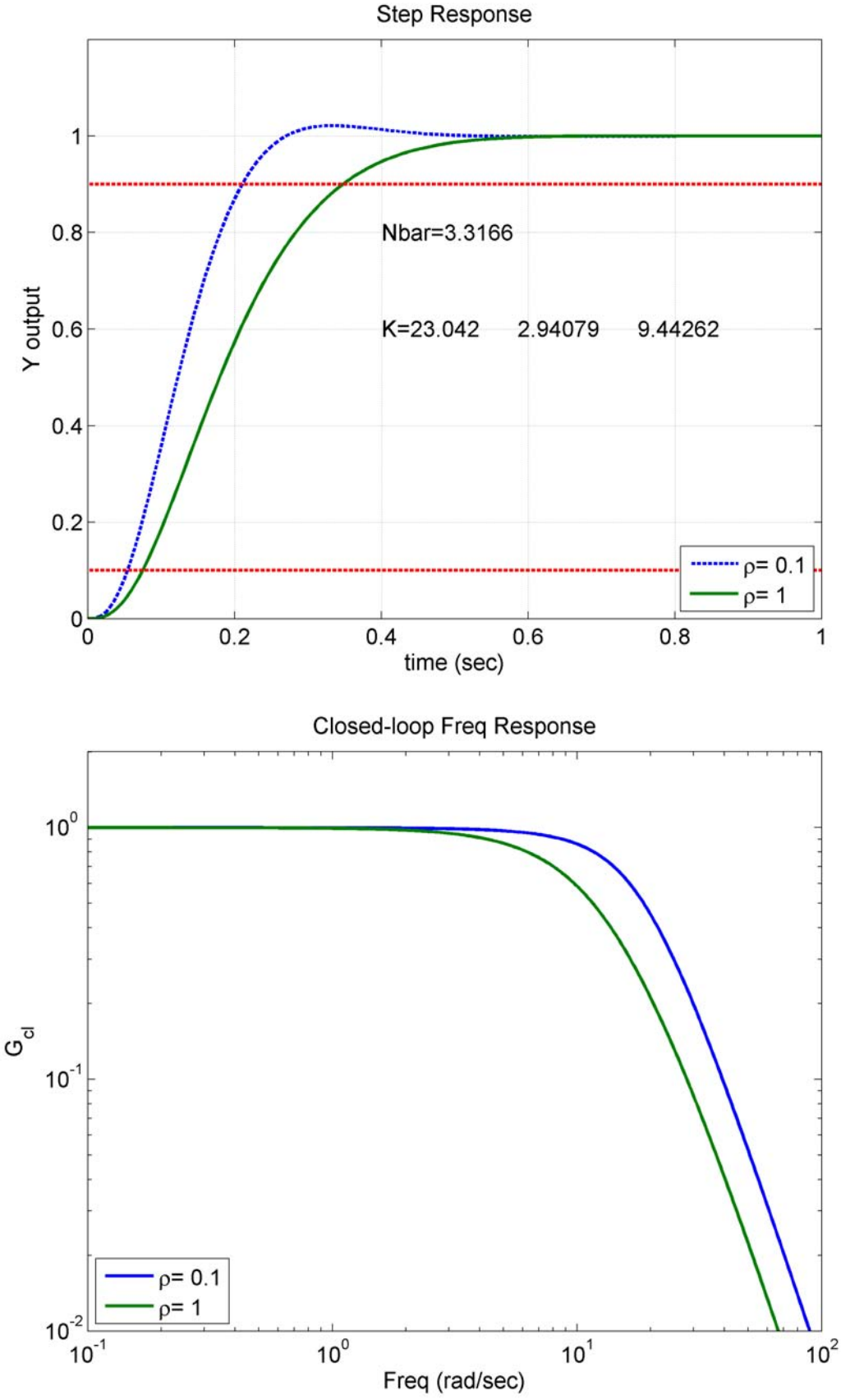


Fig. 4: Example #4: $G(s) = \frac{(s-1)}{(s+1)(s-3)}$ with control penalty ρ and 10ρ

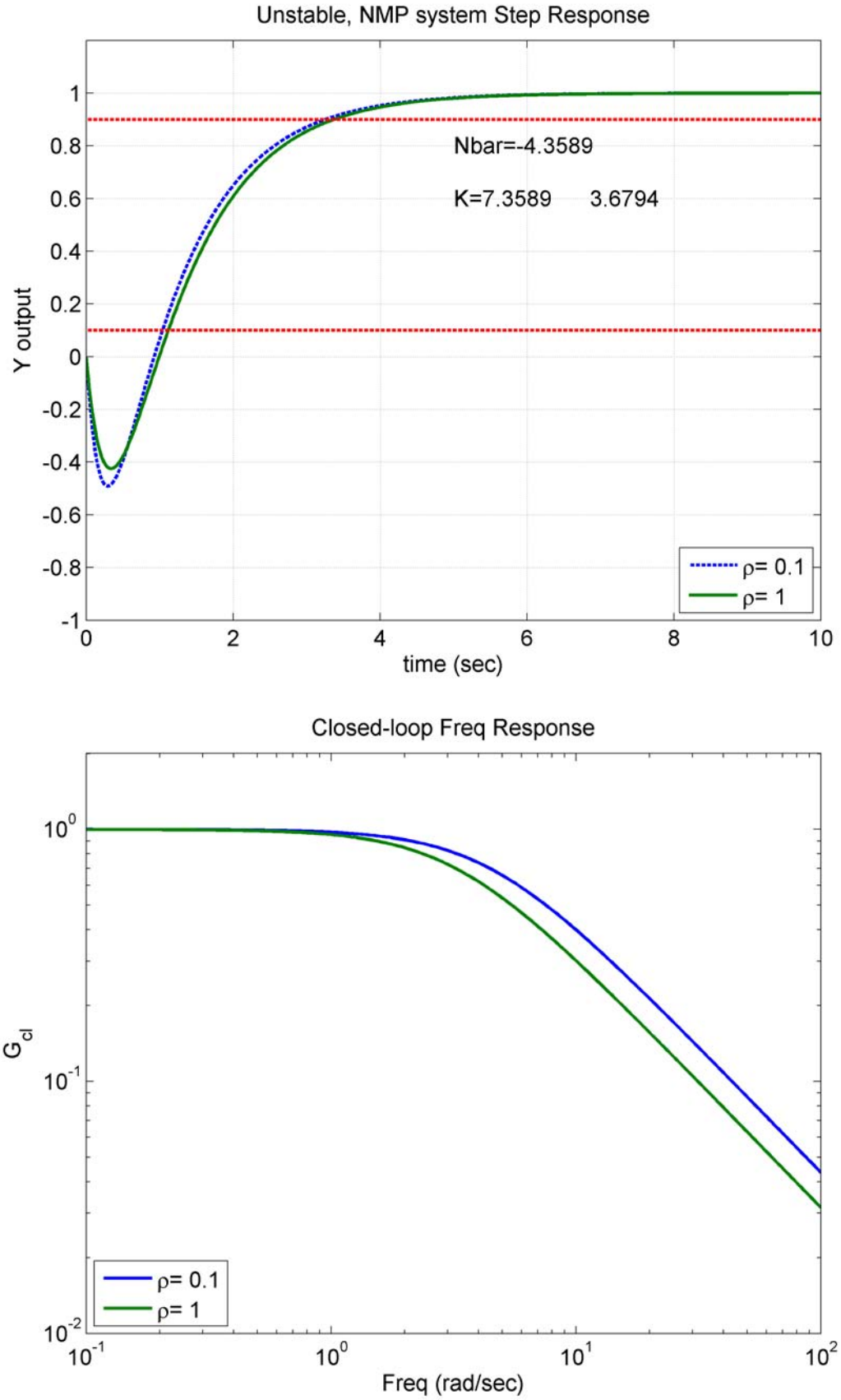
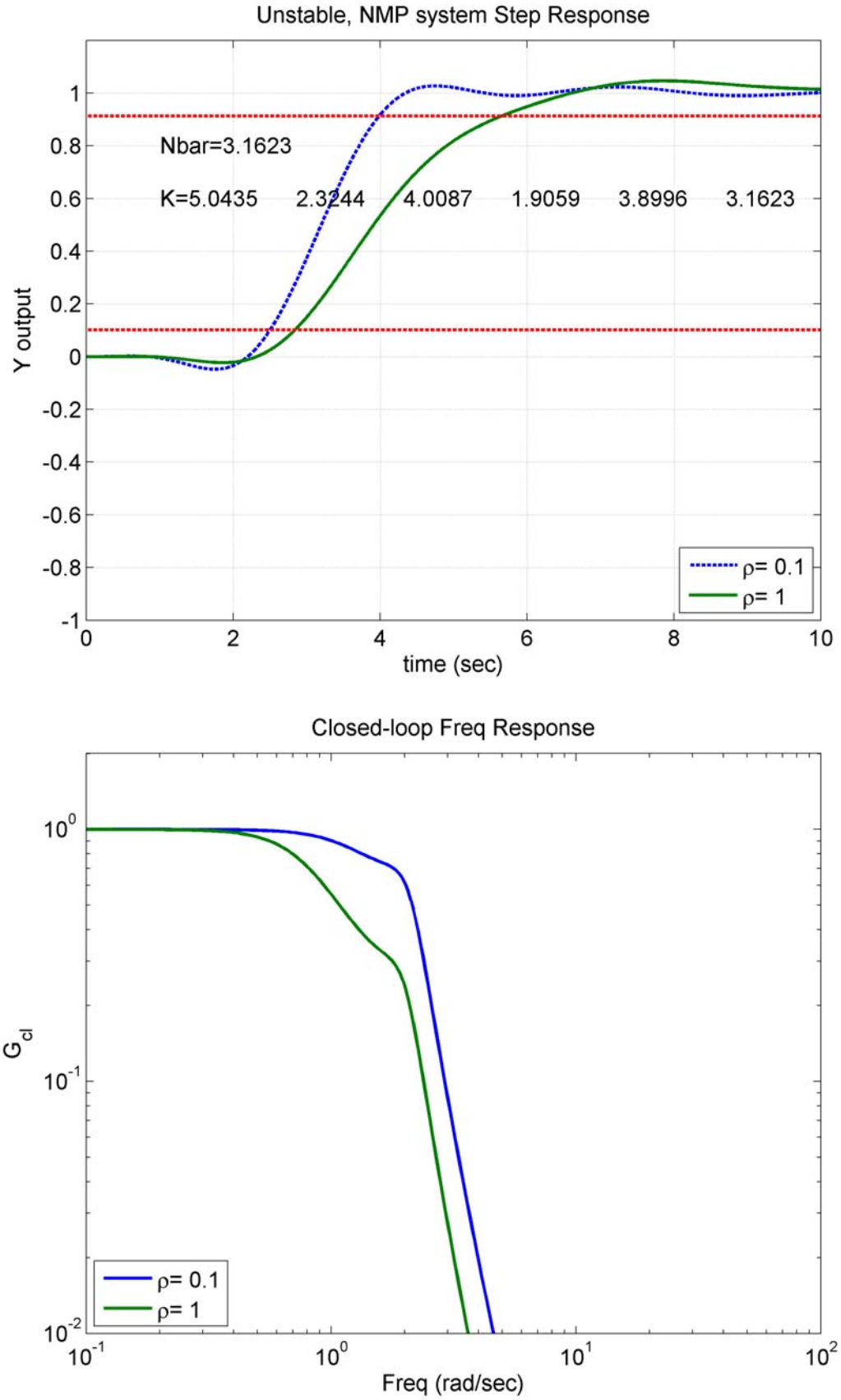


Fig. 5: Example #5: $G(s) = \frac{(s-2)(s-4)}{(s-1)(s-3)(s^2+0.8s+4)s^2}$ with control penalty ρ and 10ρ



LQR Weight Matrix Selection

- Good ROT (typically called Bryson's Rules) when selecting the weighting matrices Q and R is to normalize the signals:

$$Q = \begin{bmatrix} \frac{\alpha_1^2}{(x_1)_{\max}^2} & & & \\ & \frac{\alpha_2^2}{(x_2)_{\max}^2} & & \\ & & \ddots & \\ & & & \frac{\alpha_n^2}{(x_n)_{\max}^2} \end{bmatrix}$$

$$R = \rho \begin{bmatrix} \frac{\beta_1^2}{(u_1)_{\max}^2} & & & \\ & \frac{\beta_2^2}{(u_2)_{\max}^2} & & \\ & & \ddots & \\ & & & \frac{\beta_m^2}{(u_m)_{\max}^2} \end{bmatrix}$$

- The $(x_i)_{\max}$ and $(u_i)_{\max}$ represent the largest desired response or control input for that component of the state/actuator signal.

- $\sum_i \alpha_i^2 = 1$ and $\sum_i \beta_i^2 = 1$ are used to add an additional relative weighting on the various components of the state/control

- ρ is used as the last relative weighting between the control and state penalties $\Rightarrow$ gives a relatively concrete way to discuss the relative size of Q and R and their ratio Q/R

Regulator Summary

- Dominant second order approach places the closed-loop pole locations **with no regard to the amount of control effort required.**
  - Designer must iterate on the selected bandwidth (ω_n) to ensure that the control effort is reasonable.

- LQR selects closed-loop poles that **balance** between state errors and control effort.
  - Easy design iteration using R
  - Sometimes difficult to relate the desired transient response to the LQR cost function.
  - Key thing is that the designer is focused on system performance issues rather than the mechanics of the design process

Code: LQR Examples - (step4.m)

```
% LQR examples for 16.31
% Fall 2010
% Jonathan How, MIT
%
close all;clear all
set(0,'DefaultLineLineWidth',2);
set(0,'DefaultlineMarkerSize',8);set(0,'DefaultlineMarkerFace','b')
set(0, 'DefaultAxesFontSize', 12);set(0, 'DefaultTextFontSize', 12)
set(0,'DefaultAxesFontName','arial');set(0,'DefaultTextFontName','arial')
set(0,'DefaultFigureColor','w','DefaultAxesColor','w',...
      'DefaultAxesXColor','k','DefaultAxesYColor','k',...
      'DefaultAxesZColor','k','DefaultTextColor','k')

if 1
clear all;fig=1;
% system
G=tf(8*14*20,conv([1 8],conv([1 14],[1 20])));
[a,b,c,d]=ssdata(G);
R=1e-3;
k=lqr(a,b,c'*c,R);  % nominal controller
k2=lqr(a,b,c'*c,10*R);  % slower control bec of higher control penalty

% find the feedforward gains
Nbar=inv(-c*inv(a-b*k)*b);
Nbar2=inv(-c*inv(a-b*k2)*b);

sys1=ss(a-b*k,b*Nbar,c,d);
sys2=ss(a-b*k2,b*Nbar2,c,d);

t=[0:.005:1];
[y,t,x]=step(sys1,t);
[y2,t2,x2]=step(sys2,t);

figure(fig);clf;fig=fig+1;
plot(t,y,'--',t2,y2,'LineWidth',2);axis([0 1 0 1.2])
grid;
legend(['\rho  = ',num2str(R)],['\rho  = ',num2str(10*R)],'Location','SouthEast')
xlabel('time (sec)');ylabel('Y output');
title('Step Response')
hold on
plot(t2([1 end]),[.1 .1]*y2(end),'r--');
plot(t2([1 end]),[.1 .1]*9*y2(end),'r--');
hold off

text(.4,.6,['k= [ ',num2str(round(k*1000)/1000),' ]'])
text(.4,.8,['Nbar= ',num2str(Nbar)])
%!rm srl12.pdf
export_fig srl12 -pdf

if 1
  figure(fig);clf;
   f=logspace(-1,2,400);
   gcl1=freqresp(sys1,f);
   gcl2=freqresp(sys2,f);
   loglog(f,abs(squeeze(gcl1)),f,abs(squeeze(gcl2)),'LineWidth',2);%grid
   axis([.1 100 .01 2])
   xlabel('Freq (rad/sec)');ylabel('G_{cl}')
    title('Closed-loop Freq Response')
    legend(['\rho  = ',num2str(R)],['\rho  = ',num2str(10*R)],'Location','SouthWest')
    %!rm srl13.pdf
    export_fig srl13 -pdf
end

end % if 0

%%%%%%%%%
if 1
clear all
fig=4;

G=tf(8*14*20,conv([1 -8],conv([1 -14],[1 -20])));
[a,b,c,d]=ssdata(G);
R=.1
k=lqr(a,b,c'*c,R);
```

```
k2=lqr(a,b,c'*c,10*R);

% find the feedforward gains
Nbar=inv(-c*inv(a-b*k)*b);
Nbar2=inv(-c*inv(a-b*k2)*b);

sys1=ss(a-b*k,b*Nbar,c,d);
sys2=ss(a-b*k2,b*Nbar2,c,d);

t=[0:.005:1];
[y,t,x]=step(sys1,t);
[y2,t2,x2]=step(sys2,t);

figure(fig);clf;fig=fig+1;
plot(t,y,'--',t2,y2,'LineWidth',2);axis([0 1 0 1.2])
grid;
legend(['\rho  = ',num2str(R)],['\rho  = ',num2str(10*R)],'Location','SouthEast')
xlabel('time (sec)');ylabel('Y output');
title('Step Response')
hold on
plot(t2([1 end]),[.1 .1]*y2(end),'r--');
plot(t2([1 end]),[.1 .1]*9*y2(end),'r--');
hold off

text(.4,.6,['k= [ ',num2str(round(k*1000)/1000),' ]'])
text(.4,.8,['Nbar= ',num2str(Nbar)])
%!rm srl22.pdf;
export_fig srl22 -pdf

if 1
figure(fig);clf;fig=fig+1;
   f=logspace(-1,2,400);
   gcl1=freqresp(sys1,f);
   gcl2=freqresp(sys2,f);
   loglog(f,abs(squeeze(gcl1)),f,abs(squeeze(gcl2)),'LineWidth',2);%grid
   axis([.1 100 .01 2])
    xlabel('Freq (rad/sec)');ylabel('G_{cl}')
    title('Closed-loop Freq Response')
    legend(['\rho  = ',num2str(R)],['\rho  = ',num2str(10*R)],'Location','SouthWest')
    %!rm srl23.pdf;
    export_fig srl23 -pdf
end

end % if 0
%%%%%%%%%%%%%%

if 1
clear all
fig=7;

G=tf(.94,[1 0 -0.0297]);
[a,b,c,d]=ssdata(G);
R=.1
k=lqr(a,b,c'*c,R);
k2=lqr(a,b,c'*c,10*R);

% find the feedforward gains
Nbar=inv(-c*inv(a-b*k)*b);
Nbar2=inv(-c*inv(a-b*k2)*b);

sys1=ss(a-b*k,b*Nbar,c,d);
sys2=ss(a-b*k2,b*Nbar2,c,d);

t=[0:.01:30];
[y,t,x]=step(sys1,t);
[y2,t2,x2]=step(sys2,t);

figure(fig);clf;fig=fig+1;
plot(t,y,'--',t2,y2,'LineWidth',2);axis([0 30 0 1.2])
grid;
legend(['\rho  = ',num2str(R)],['\rho  = ',num2str(10*R)],'Location','SouthEast')
xlabel('time (sec)');ylabel('Y output');
title('Step Response')
hold on
plot(t2([1 end]),[.1 .1]*y2(end),'r--');
plot(t2([1 end]),[.1 .1]*9*y2(end),'r--');
hold off
```

```
152
153 text(15,.6,['k= [ ',num2str(round(k*1000)/1000),' ]'])
154 text(15,.8,['Nbar= ',num2str(Nbar)])
155 %!rm srl32.pdf;
156 export_fig srl32 -pdf
157
158 if 1
159 figure(fig);clf;fig=fig+1;
160    f=logspace(-3,3,400);
161    gcl1=freqresp(sys1,f);
162    gcl2=freqresp(sys2,f);
163    loglog(f,abs(squeeze(gcl1)),f,abs(squeeze(gcl2)),'LineWidth',2);%grid
164    axis([.1 100 .01 2])
165     xlabel('Freq (rad/sec)');ylabel('G_{cl}')
166     title('Closed-loop Freq Response')
167     legend(['\rho  = ',num2str(R)],['\rho  = ',num2str(10*R)],'Location','SouthWest')
168     %!rm srl33.pdf;
169     export_fig srl33 -pdf
170 end
171
172 end % if 0
173
174 %%%%%%%%%%%%
175 if 1
176 clear all
177 fig=10;
178
179 G=tf([1 -1],conv([1 1],[1 -3]));
180 [a,b,c,d]=ssdata(G);
181 R=.1
182 k=lqr(a,b,c'*c,R);
183 k2=lqr(a,b,c'*c,10*R);
184
185 % find the feedforward gains
186 Nbar=inv(-c*inv(a-b*k)*b);
187 Nbar2=inv(-c*inv(a-b*k2)*b);
188
189 sys1=ss(a-b*k,b*Nbar,c,d);
190 sys2=ss(a-b*k2,b*Nbar2,c,d);
191
192 t=[0:.01:10];
193 [y,t,x]=step(sys1,t);
194 [y2,t2,x2]=step(sys2,t);
195
196 figure(fig);clf;fig=fig+1;
197 plot(t,y,'--',t2,y2,'LineWidth',2);axis([0 10 -1 1.2])
198 grid;
199 legend(['\rho  = ',num2str(R)],['\rho  = ',num2str(10*R)],'Location','SouthEast')
200 xlabel('time (sec)');ylabel('Y output');
201 title('Unstable, NMP system Step Response')
202 hold on
203 plot(t2([1 end]),[.1 .1]*y2(end),'r--');
204 plot(t2([1 end]),[.1 .1]*9*y2(end),'r--');
205 hold off
206
207 text(5,.6,['k= [ ',num2str(round(k*1000)/1000),' ]'])
208 text(5,.8,['Nbar= ',num2str(Nbar)])
209 %!rm srl42.pdf;
210 export_fig srl42 -pdf
211
212 if 1
213 figure(fig);clf;fig=fig+1;
214    f=logspace(-2,2,400);
215    gcl1=freqresp(sys1,f);
216    gcl2=freqresp(sys2,f);
217    loglog(f,abs(squeeze(gcl1)),f,abs(squeeze(gcl2)),'LineWidth',2);%grid
218    axis([.1 100 .01 2])
219     xlabel('Freq (rad/sec)');ylabel('G_{cl}')
220     title('Closed-loop Freq Response')
221     legend(['\rho  = ',num2str(R)],['\rho  = ',num2str(10*R)],'Location','SouthWest')
222     %!rm srl43.pdf;
223     export_fig srl43 -pdf
224 end
225
226 end % if 0
227
228 %%%%%%%%%%%%
```

```
clear all
fig=13;

G=tf(conv([1 -2],[1 -4]),conv(conv([1 -1],[1 -3]),[1 2*.2*2 2^2 0 0]));
[a,b,c,d]=ssdata(G);
R=.1
k=lqr(a,b,c'*c,R);
k2=lqr(a,b,c'*c,10*R);

% find the feedforward gains
Nbar=inv(-c*inv(a-b*k)*b);
Nbar2=inv(-c*inv(a-b*k2)*b);

sys1=ss(a-b*k,b*Nbar,c,d);
sys2=ss(a-b*k2,b*Nbar2,c,d);

t=[0:.01:10];
[y,t,x]=step(sys1,t);
[y2,t2,x2]=step(sys2,t);

figure(fig);clf;fig=fig+1;
plot(t,y,'--',t2,y2,'LineWidth',2);axis([0 10 -1 1.2])
grid;
legend(['\rho  = ',num2str(R)],['\rho  = ',num2str(10*R)],'Location','SouthEast')
xlabel('time (sec)');ylabel('Y output');
title('Unstable, NMP system Step Response')
hold on
plot(t2([1 end]),[.1 .1]*y2(end),'r--');
plot(t2([1 end]),[.1 .1]*9*y2(end),'r--');
hold off

text(1,.6,['k= [ ',num2str(round(k*1000)/1000),' ]'])
text(1,.8,['Nbar= ',num2str(Nbar)])
%!rm srl52.pdf;
export_fig srl52 -pdf

if 1
    figure(fig);clf;fig=fig+1;
    f=logspace(-2,2,400);
    gcl1=freqresp(sys1,f);
    gcl2=freqresp(sys2,f);
    loglog(f,abs(squeeze(gcl1)),f,abs(squeeze(gcl2)),'LineWidth',2);%grid
    axis([.1 100 .01 2])
    xlabel('Freq (rad/sec)')
    ylabel('G_{cl}')
    title('Closed-loop Freq Response')
    legend(['\rho  = ',num2str(R)],['\rho  = ',num2str(10*R)],'Location','SouthWest')
    %!rm srl53.pdf;
    export_fig srl53 -pdf
end
```

Example: B747

- Consider the lateral dynamics of a B747 at cruise (40,000ft, M=0.8)
  - Variables of interest now include lateral velocity (side slip, β), yaw ψ and yaw rate r, roll ϕ and roll rate p.
  - Actuators are aileron δ_a and rudder δ_r (Figure 10.30 from FPE)

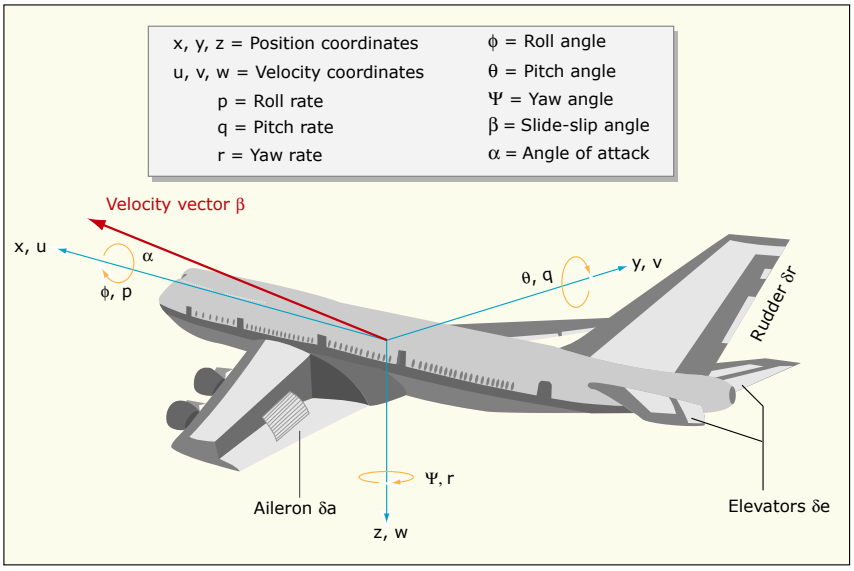


Image by MIT OpenCourseWare.

- Form nonlinear dynamics as before and linearize about forward cruise flight condition to get equations of motion for **lateral dynamics**

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \ , \ \mathbf{x} = \begin{bmatrix} \beta \\ r \\ p \\ \phi \end{bmatrix} \ , \ \mathbf{u} = \begin{bmatrix} \delta_r \\ \delta_a \end{bmatrix} \qquad \dot{\psi} = r$$

$$A = \begin{bmatrix} -0.0558 & -0.9968 & 0.0802 & 0.0415 \\ 0.598 & -0.115 & -0.0318 & 0 \\ -3.05 & 0.388 & -0.4650 & 0 \\ 0 & 0.0805 & 1 & 0 \end{bmatrix} \tag{1}$$

$$B = \begin{bmatrix} 0.00729 & 0 \\ -0.475 & 0.00775 \\ 0.153 & 0.143 \\ 0 & 0 \end{bmatrix} \tag{2}$$

and we can sense the yaw rate r and bank angle ϕ.

- Goal: Investigate OL behavior and correct parts that we don't like.

Open-Loop Dynamics

- Code gives the numerical values for all of the stability derivatives.
  - Solve for the eigenvalues of the matrix A to find the **lateral modes**.

$$-0.0329 \pm 0.9467\mathbf{i}, -0.5627, -0.0073$$

  - Stable, but there is one very slow pole.

- There are 3 modes, but they are a **lot more complicated** than the longitudinal case.

Slow mode	-0.0073	$\Rightarrow$ **Spiral Mode**
Fast real	-0.5627	$\Rightarrow$ **Roll Damping**
Oscillatory	$-0.0329 \pm 0.9467\mathbf{i}$	$\Rightarrow$ **Dutch Roll**

- Can look at normalized eigenvectors:

	Spiral	Roll	Dutch Roll	
$\beta = v/U_0$	0.0067	-0.0197	0.3085	0°
$\hat{p} = p/(2U_0/b)$	-0.0009	-0.0712	0.1131	120°
$\hat{r} = r/(2U_0/b)$	0.0052	0.0040	0.0348	-84°
ϕ	1.0000	1.0000	0.9438	28°

  - **Not as enlightening as the longitudinal case** – the lateral dynamics tightly couple all of the states in a complex motion

- Note that the Dutch roll is too lightly damped, and we would typically like to increase that damping using closed-loop control

Washout Filter

- A hidden complexity in the design of an autopilot for the lateral dynamics is that there are various flight modes that require a steady yaw rate ($r_{ss} \neq 0$). For example, steady turning flight.
  - So the control that we implement must not be concerned if the steady state value r is non-zero

- Typically achieved using a washout filter $H_w(s)$ – a high pass version of the r signal.
  - High pass cuts out the low frequency content in the signal

- For now we will penalize that filtered state in the LQR cost function
  - When we get to output feedback, it is only the filtered r that is available to the controller

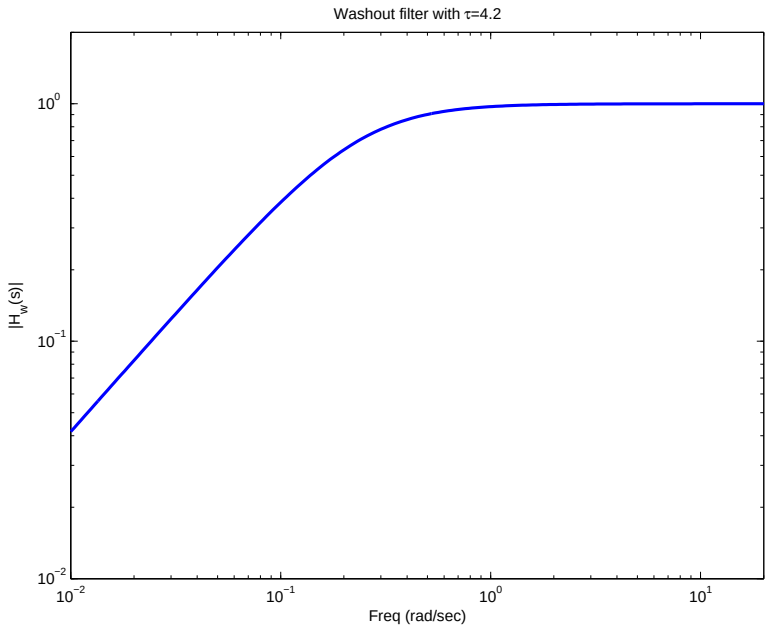


Fig. 6: Washout filter with $\tau = 4.2$

- Add the rudder dynamics $H_r(s) = 10/(s+10)$, i.e. effectively a lag
- New control picture (shows a nominal yaw damper – we will replace that with a state feedback controller)

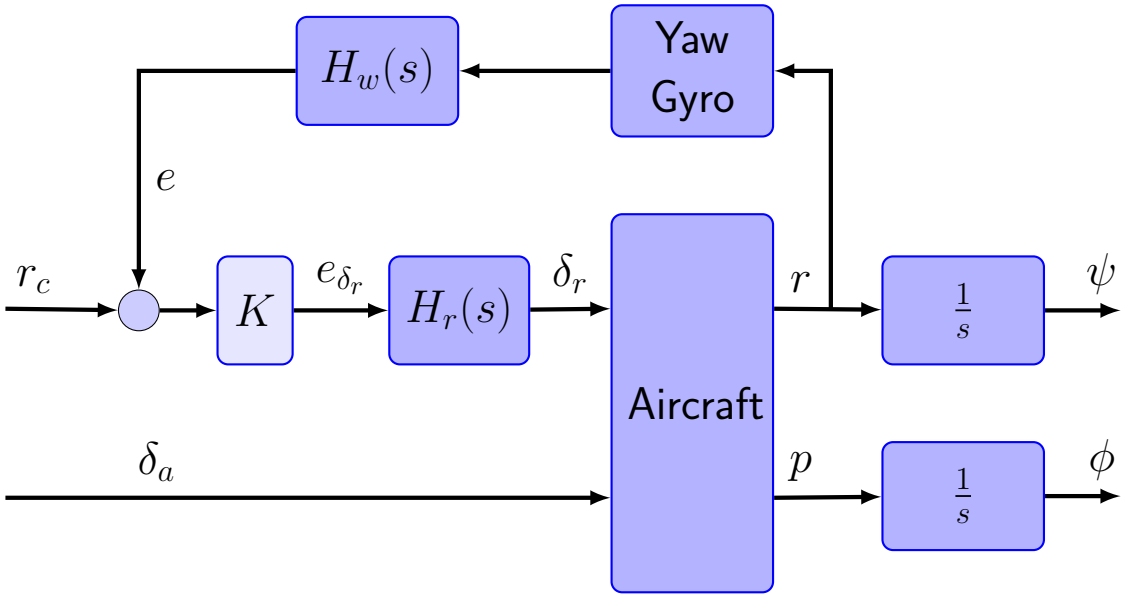


Fig. 7: Typical implementation of the LQ servo

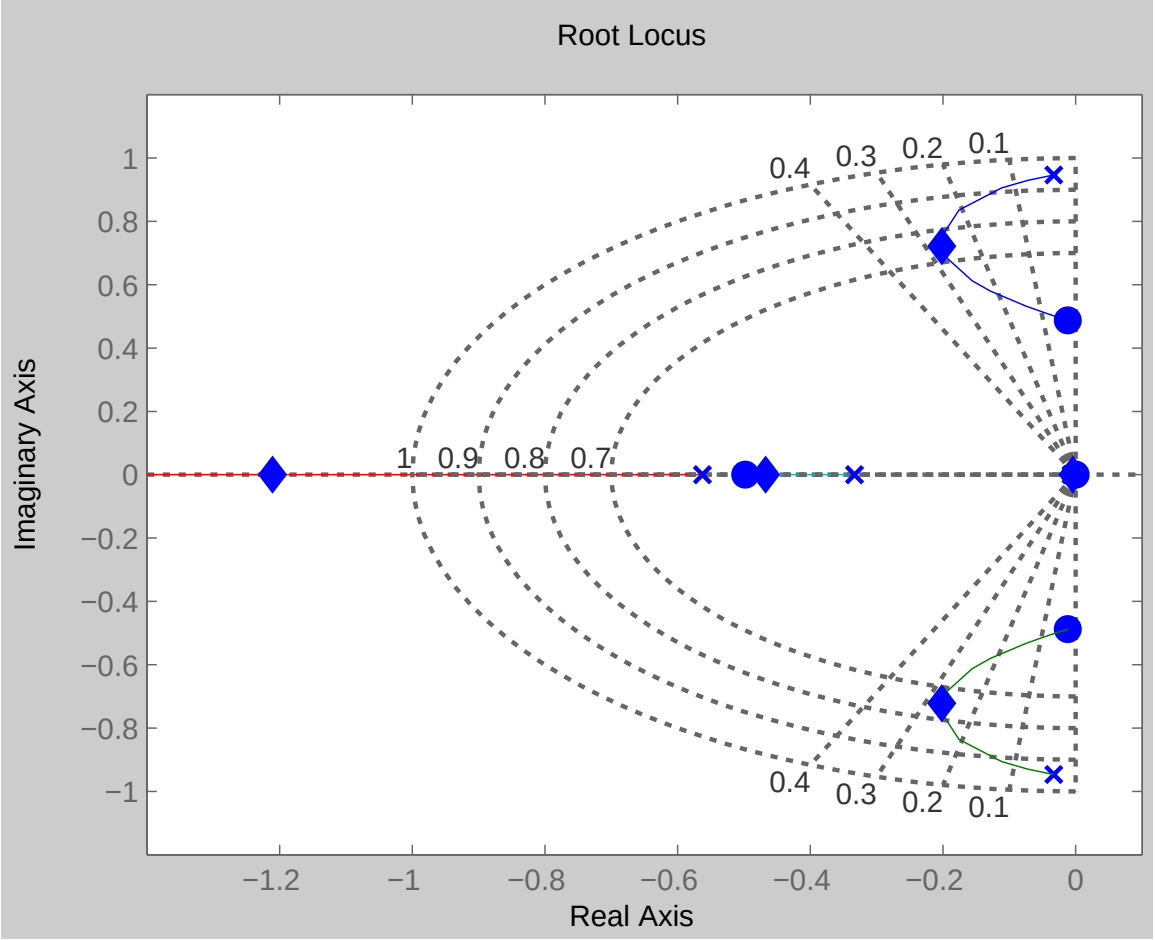
- To proceed, must augment the filter and actuator dynamics to the state model
  - Algebra of this will be discussed later, but Matlab has easy ways of doing this augmentation because we can multiply systems together (note code using a I/O order different than picture above)

```
sys = ss(A,B,C,D);
set(sys, 'inputname', {'rudder' 'aileron'},...
         'outputname', {'yaw rate' 'bank angle'});
set(sys, 'statename', {'beta' 'yaw rate' 'roll rate' 'phi'});
% actuator dynamics are a lag at 10
actn=10;actd=[1 10]; % H(s) in notes
H=tf({actn 0;0 1},{actd 1;1 1});
%
% Washout filter
tau=3;
washn=[1 0];washd=[1 1/tau]; % washout filter on yaw rate
WashFilt=tf({washn 0;0 1},{washd 1;1 1});
%
Gp=WashFilt*sys*H;
```

- Inputs are now e_{δ_r} and δ_a, and outputs are e and ϕ – from this MIMO system, pull out the SISO system from $e_{\delta_r} \rightarrow e$
  - Sixth order because of the augmented states
- Easiest control is of course to try gain feedback from e to e_{δ_r} – will compare that with LQR

B747 Lateral Gain Feedback

- Try gain feedback - see root locus

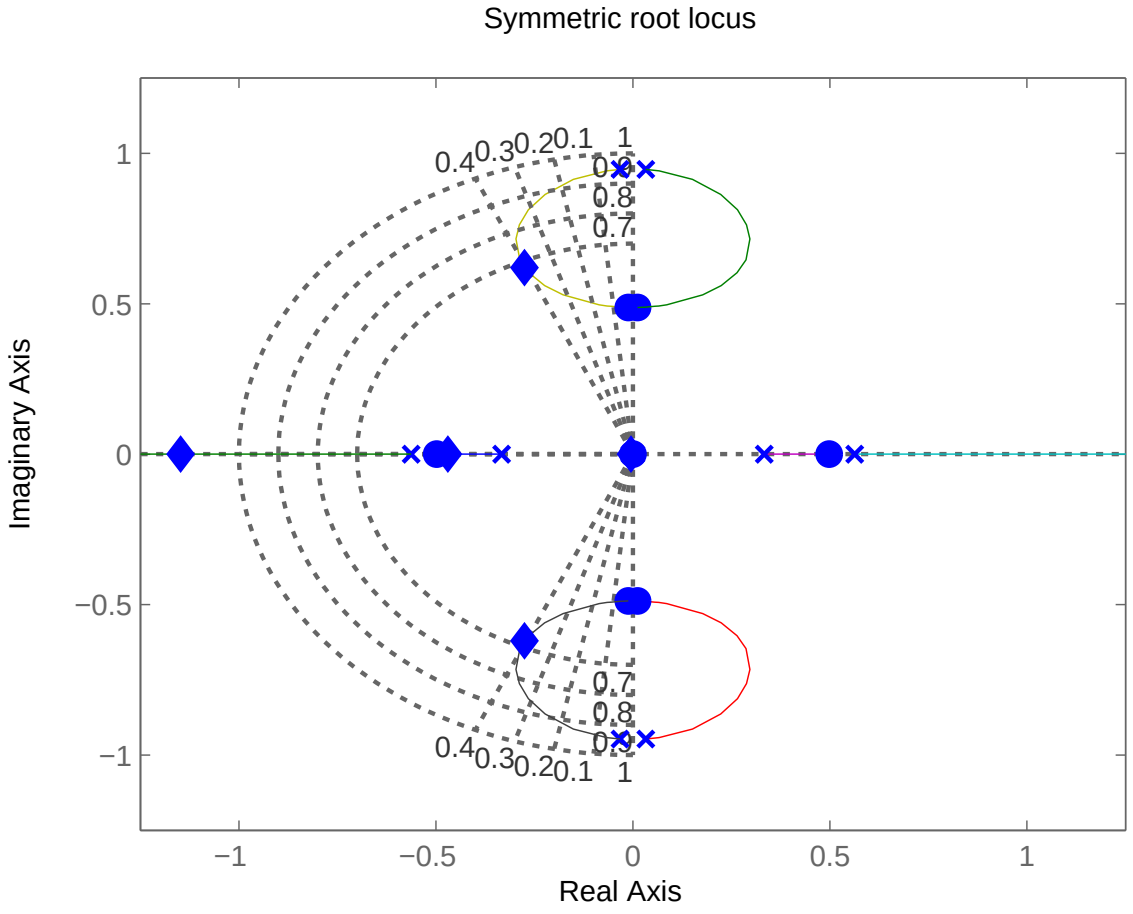


- Presence of zero on imaginary axis "traps" the Dutch roll pole so that it cannot be moved far into the LHP

- Net result is that we are limited to about ≈30% damping in that pole

B747 Lateral LQR

- Using an LQR state cost based on e^2, can design an LQR controller for various $R = \rho$.
  - But where do the poles go as a ftn of ρ?
  - Follow a pattern defined by a **symmetric root locus** (SRL) – form pole/zero map from input e_{δ_r} to output e, put these on the s-plane, introduce mirror image of these dynamics about the imaginary axis, plot root locus using standard root-locus tools.



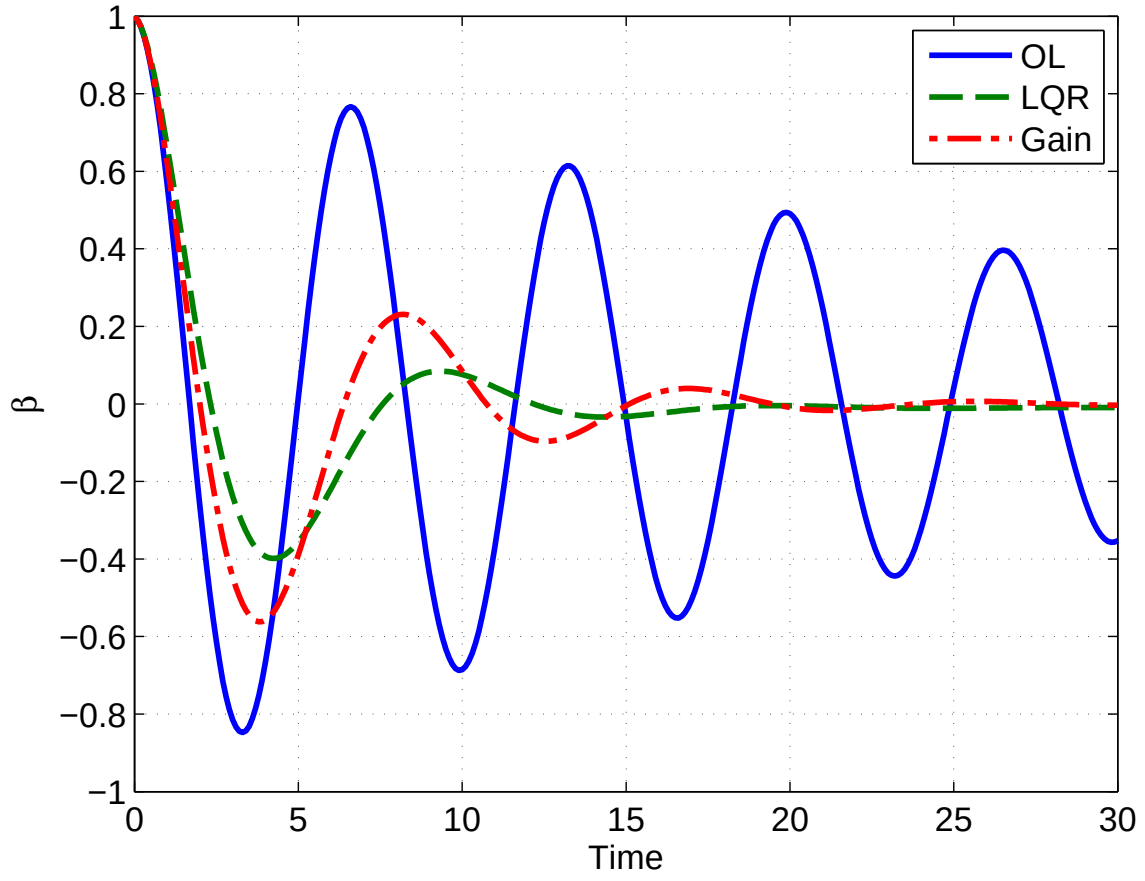
- Dutch roll mode is strongly influenced by this control approach, but effect of zero still apparent
  - Suggests $\rho \approx 0.1$, which gives about 40% damping in that mode

```
Gp=WashFilt*sys*H;
set(Gp, 'statename', {'xwo' 'beta' 'yaw rate' 'roll' 'phi' 'xa'});
set(Gp, 'inputname', {'rudder inputs' 'aileron'},...
         'outputname', {'filtered yaw rate' 'bank angle'});
[Ap,Bp,Cp,Dp]=ssdata(Gp);
[Klqr,S,Elqr]=lqr(Ap,Bp(:,1),Cp(1,:)'*Cp(1,:),0.1)
```

- $K_{lqr} = [\ 1.0105 \quad -0.1968 \quad -2.3864 \quad 0.1005 \quad 0.0374 \quad 0.2721\ ]$ giving

$$\lambda(A_p - B_p(:,1)K_{lqr}) = \begin{bmatrix} -9.8852 \\ -1.1477 \\ -0.2750 \pm 0.6200\mathbf{i} \\ -0.4696 \\ -0.0050 \end{bmatrix}$$

- Compare the initial condition response to a perturbation to β



- Closed-loop response looks pretty good
- But note that the original gain feedback design wasn't that much worse – is the additional complexity of the full state feedback justified?

Code: B747 LQR, Estimator and DOFB

```
% B747 lateral dynamics
% Jonathan How, Fall 2010
% working from Matlab Demo called jetdemo
clear all;%close all
%
A=[-.0558 -.9968 .0802 .0415;
      .598 -.115 -.0318 0;
    -3.05 .388 -.4650 0;
         0 0.0805 1 0];
B=[ .00729  0;
   -0.475   0.00775;
    0.153   0.143;
     0      0];
C=[0 1 0 0; 0 0 0 1];D=[0 0; 0 0];
sys = ss(A,B,C,D);
set(sys, 'inputname', {'rudder' 'aileron'},...
         'outputname', {'yaw rate' 'bank angle'});
set(sys, 'statename', {'beta' 'yaw rate' 'roll rate' 'phi'});
[Yol,Tol]=initial(ss(A,B,[1 0 0 0],zeros(1,2)),[1 0 0 0]',[0:.1:30]);
[V,E]=eig(A)
%
% CONTROL - actuator dynamics are a lag at 10
actn=10;actd=[1 10]; % H(s) in notes
H=tf({actn 0;0 1},{actd 1;1 1});
%
tau=3;washn=[1 0];washd=[1 1/tau]; % washout filter on yaw rate
WashFilt=tf({washn 0;0 1},{washd 1;1 1});
%
Gp=WashFilt*sys*H;
set(Gp, 'statename', {'xwo' 'beta' 'yaw rate' 'roll' 'phi' 'xa'});
set(Gp, 'inputname', {'rudder inputs' 'aileron'},...
         'outputname', {'filtered yaw rate' 'bank angle'});
[Ap,Bp,Cp,Dp]=ssdata(Gp);
[Klqr,S,Elqr]=lqr(Ap,Bp(:,1),Cp(1,:)'*Cp(1,:),0.1);rifd(Elqr)
%
% gain feedback on the filter yaw rate
figure(1);clf
rlocus(Ap,Bp(:,1),-Cp(1,:),0);
sgrid([.1 .2 .3 .4],[.7 .8 .9 1]);grid on;axis([-1.4 .1 -1.2 1.2])
Kgain=-2;
Egain=eig(Ap-Bp(:,1)*Kgain*Cp(1,:));rifd(Egain)
hold on;plot(Egain+eps*sqrt(-1),'bd');hold off
export_fig b747_lqr1 -pdf

Cpbeta=[0 1 0 0 0 0]; % performance output variable
Ggain=ss(Ap-Bp(:,1)*Kgain*Cp(1,:),Bp,Cpbeta,0);
xp0=[0 1 0 0 0 0]';
[Ygain,Tgain]=initial(Ggain,xp0,Tol); %'

figure(2);clf
srl(ss(Ap,Bp(:,1),Cp(1,:),0));
sgrid([.1 .2 .3 .4],[.7 .8 .9 1])
axis([-1.25 1.25 -1.25 1.25]);grid on
hold on;plot(Elqr+eps*sqrt(-1),'bd');hold off
export_fig b747_lqr2 -pdf

% close the LQR loop
Acl=Ap-Bp(:,1)*Klqr;
Bcl=Bp(:,1);
% choose output to just state beta and control u
Ccl=[Cpbeta;Klqr];
Dcl=[0;0];
Glqr=ss(Acl,Bcl,Ccl,Dcl);
[Y,T]=initial(Glqr,xp0,Tol); %'

figure(3);clf
plot(Tol,Yol,T,Y(:,1),Tgain,Ygain);axis([0 30 -1 1]);
setlines(2)
legend('OL','LQR','Gain')
ylabel('\beta');xlabel('Time')
grid on
export_fig b747_lqr3 -pdf

% estimator poles made faster by jfactor
```

```
jfactor=2.05;
Eest=jfactor*Elqr;
Kest=place(Ap',Cp(1,:)',Eest);Kest=Kest';

na=size(Ap,1);
% Form compensator and closed-loop
% see 15-5
ac=Ap-Bp(:,1)*Klqr-Kest*Cp(1,:);bc=Kest;cc=Klqr;dc=0;
Gc=ss(ac,bc,cc,dc);
% see 15-7
acl=[Ap Bp(:,1)*cc;-bc*Cp(1,:) ac];
bcl=[zeros(na,1);bc];
% choose output to be state beta and control u
ccl=[Cpbeta,zeros(1,na);zeros(1,na) cc];
dcl=[0;0];
Gcl=ss(acl,bcl,ccl,dcl);

figure(5);clf
[p,z]=pzmap(Gc(1,1));
plot(z+sqrt(-1)*eps,'mo','MarkerFace','m')
hold on
plot(p+sqrt(-1)*eps,'bs','MarkerFace','b')
hold off
legend('Gc zeros','Gc poles','Location','NorthWest')
grid on;axis([-5.5 .5 -3 3])
export_fig b747_lqr5 -pdf

figure(4);clf
rlocus(Gp(1,1)*Gc);axis([-5.5 .5 -3 3])
hold on
[p,z]=pzmap(Gp(1,1));
plot(p+sqrt(-1)*eps,'rx','MarkerFace','r')
plot(z+sqrt(-1)*eps,'ro','MarkerFace','r')
[p,z]=pzmap(Gc(1,1));
plot(p+sqrt(-1)*eps,'mx','MarkerFace','m')
plot(z+sqrt(-1)*eps,'mo','MarkerFace','m')
p=eig(acl);
plot(p+sqrt(-1)*eps,'bd')
hold off
export_fig b747_lqr4 -pdf

% initial comp at zero
[Ycl,Tcl]=initial(Gcl,[xp0;xp0*0],Tol); %'

figure(3);clf
plot(Tol,Yol,T,Y(:,1),Tgain,Ygain,Tcl,Ycl(:,1));axis([0 30 -1 1]);
setlines(2)
legend('OL','LQR','Gain','DOFB')
ylabel('\beta');xlabel('Time')
grid on
export_fig b747_lqr3a -pdf

figure(6);clf
plot(T,Y(:,1).^2,Tcl,Ycl(:,1).^2,T,Y(:,2).^2,Tcl,Ycl(:,2).^2);
axis([0 10 0 1]);
hold off
setlines(2)
legend('LQR x^2','DOFB x^2','LQR u^2','DOFB u^2')
ylabel('x^2 and u^2');xlabel('Time')
grid on
export_fig b747_lqr3b -pdf
```

16.30 / 16.31 Feedback Control Systems

Fall 2010