16.36 Communication Systems Engineering
Spring 2009

Lectures 7: Modulation with 2-D signal

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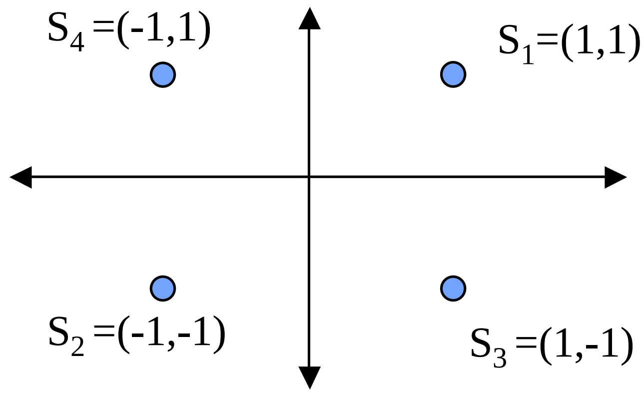
Two-dimensional signals

- $S_i = (S_{i1}, S_{i2})$

- Set of signal points is called a constellation



- 2-D constellations are commonly used

- Large constellations can be used to transmit many bits per symbol
  - More bandwidth efficient
  - More error prone
- The "shape" of the constellation can be used to minimize error probability by keeping symbols as far apart as possible

- Common constellations
  - QAM: Quadrature Amplitude Modulation
    - PAM in two dimensions
  - PSK: Phase Shift Keying
    - Special constellation where all symbols have equal power

Symmetric M-QAM

$$S_m = (A_m^x, A_m^y),\ A_m^x, A_m^y \in \{+/-1, +/-3, \ldots, +/-(\sqrt{M}-1)\}$$

M is the total number of signal points (symbols)

$\sqrt{M}$ signal levels on each axis

Constellation is symmetric

$\Rightarrow M = K^2$, for some K

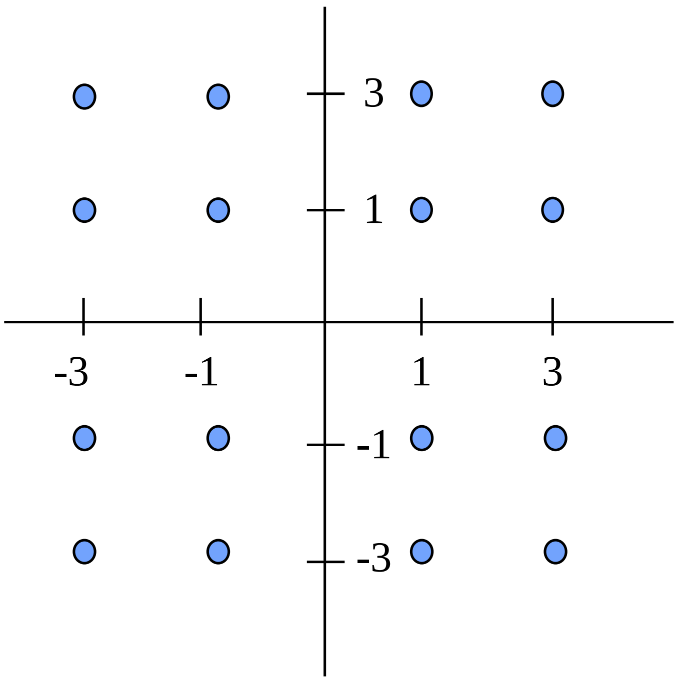
Signal levels on each axis are

the same as for PAM

$$E.g., 4-QAM \Rightarrow A_m^x, A_m^y \in \{+/-1\}$$

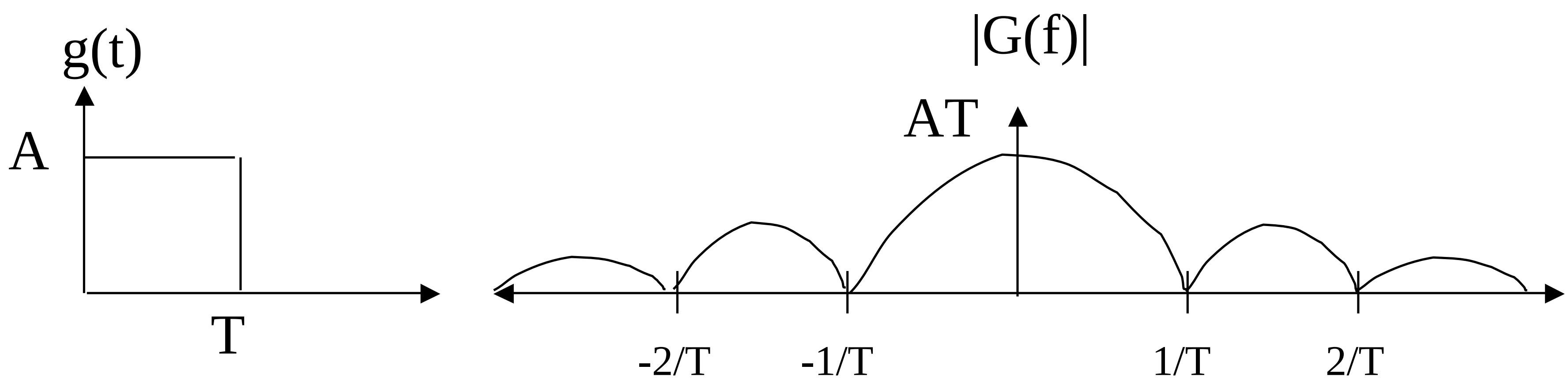
$$16-QAM \Rightarrow A_m^x, A_m^y \in \{+/-1, +/-3\}$$

16-QAM



Bandwidth occupancy of QAM

- **When using a rectangular pulse, the Fourier transform is a Sinc**



- **First null BW is still 2/T**
  - **$Log_2(M)$ bits per symbol**
  - **$Rb = Log_2(M)/T$**
  - **Bandwidth Efficiency = Rb/BW = $Log_2(M)/2$**

  $\Rightarrow$ **"Same as for PAM"**

But as we will see next, QAM is more energy efficient than PAM

Energy efficiency

$$E_{sm} = [(A_m^x)^2 + (A_m^y)^2]E_g$$

$$E[(A_m^x)^2] = E[(A_m^y)^2] = \frac{K^2-1}{3} = \frac{M-1}{3}, \quad K = \sqrt{M}$$

$$\overline{E}_s = \frac{2(M-1)}{3} E_g$$

$$Transmitted\ energy = \frac{\overline{E}_s}{2} = \frac{(M-1)}{3} E_g$$

$$E_b\,(QAM) = Energy\,/\,bit = \frac{(M-1)}{3Log_2(M)} E_g$$

- **Compare to PAM: E_b increases with M, but not nearly as fast as PAM**

$$E_b(PAM) = \frac{(M^2-1)}{6Log_2(M)} E_g$$

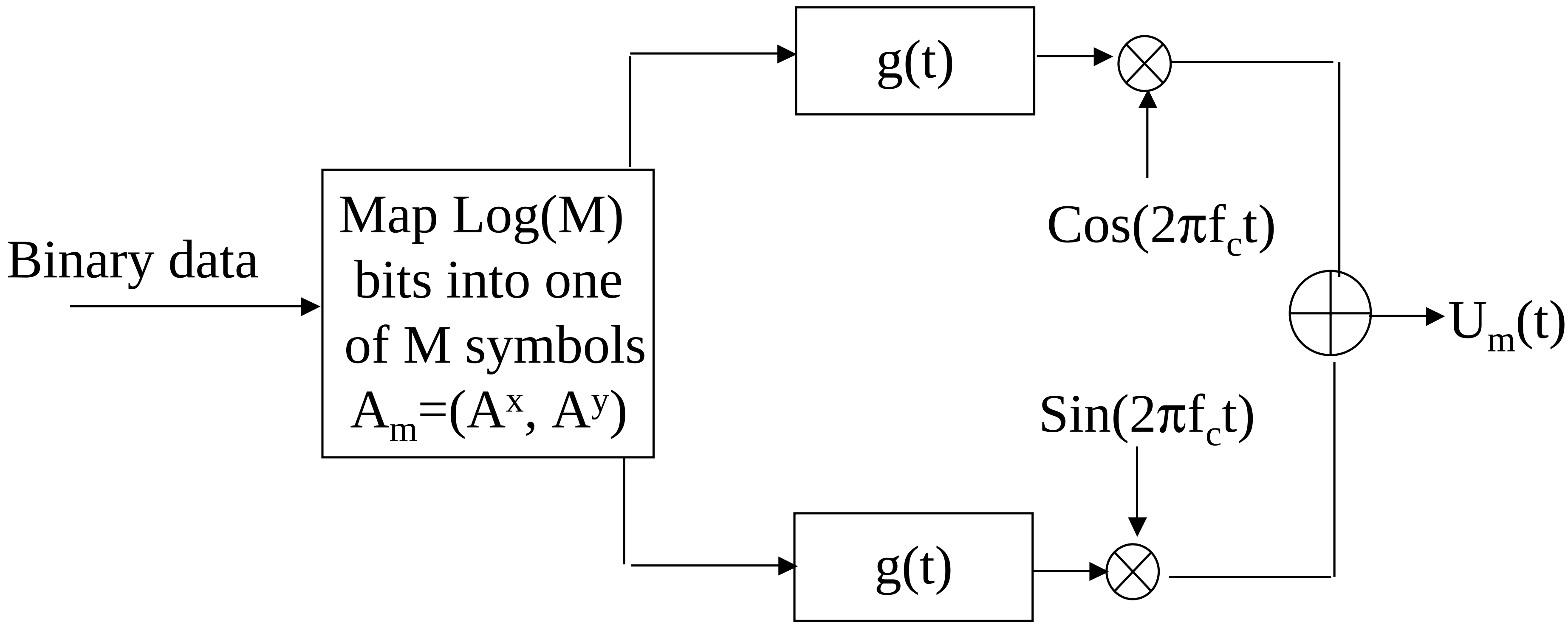
Bandpass QAM

- **Modulate the two dimensional signal by multiplication by orthogonal carriers (sinusoids): Sine and Cosine**
  - **This is accomplished by multiplying the A^x component by Cosine and the A^y component by sine**
  - **Typically, people do not refer to these components as x,y but rather A^c or A^s for cosine and sine or sometimes as A^Q, and A^I for quadrature or in-phase components**

- **The transmitted signal, corresponding to the m^th symbol is:**

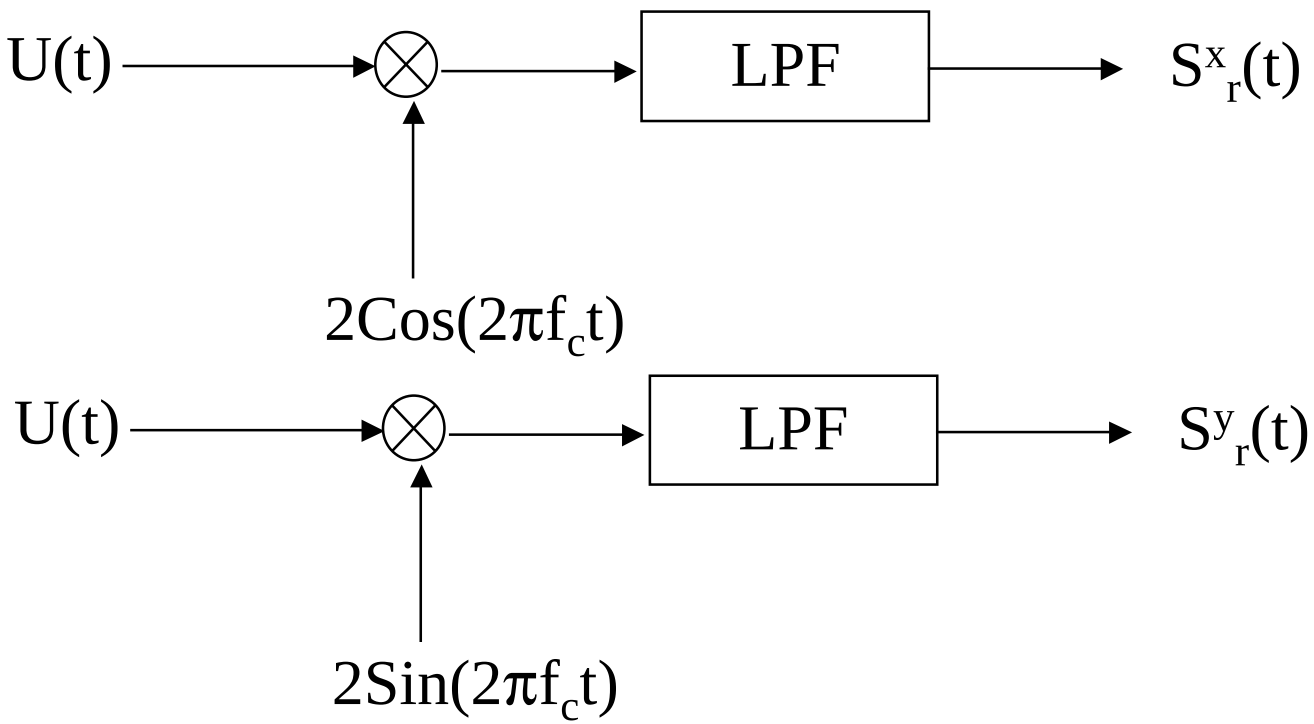
$$U_m(t) = A_m^x g(t) Cos(2\pi f_c t) + A_m^y g(t) Sin(2\pi f_c t), \; m = 1..M$$

Modulator



Binary data
Map Log(M) bits into one of M symbols $A_m=(A^x, A^y)$
g(t)
g(t)
$Cos(2\pi f_c t)$
$Sin(2\pi f_c t)$
$U_m(t)$

Demodulation: Recovering the baseband signals



- Over a symbol duration, $Sin(2\pi f_c t)$ and $Cos(2\pi f_c t)$ are orthogonal
  - As long as the symbol duration is an integer number of cycles of the carrier wave ($f_c = n/T$) for some n
- When multiplied by a sine, the cosine component of U(t) disappears and similarly the sine component disappears when multiplied by cosine

Demodulation, cont.

$$U(t)2Cos(2\pi f_c t) = 2A^x g(t)Cos^2(2\pi f_c t) + 2A^y g(t)\cos(2\pi f_c t)\sin\left(2\pi f_c t\right)$$

$$Cos^2(\alpha) = \frac{1+\cos(2\alpha)}{2}$$

$$\Rightarrow U(t)2Cos(2\pi f_c t) = S^x(t) + S^x(t)\cos(4\pi f_c t) \rightarrow LPF \ \Rightarrow S^x(t) = A^x g(t)$$

Similarly,

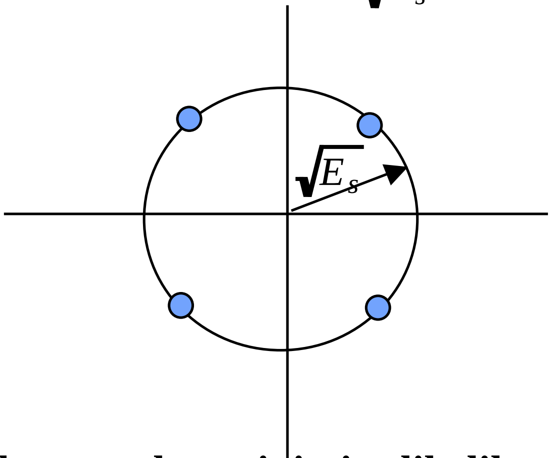
$$U(t)2Sin(2\pi f_c t) = 2A^x g(t)Cos(2\pi f_c t)Sin\left(2\pi f_c t\right) + 2A^y g(t)\sin^2\left(2\pi f_c t\right)$$

$$Sin^2(\alpha) = \frac{1-\cos(2\alpha)}{2}$$

$$\Rightarrow U(t)2Sin(2\pi f_c t) = S^y(t) - S^y(t)\cos(4\pi f_c t) \rightarrow LPF \ \Rightarrow S^y(t) = A^y g(t)$$

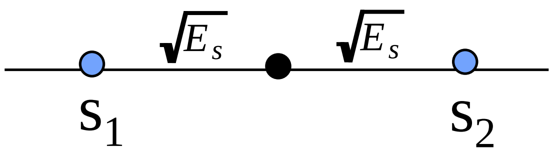
Phase Shift Keying (PSK)

- **Two Dimensional signals where all symbols have equal energy levels**
  - **I.e., they lie on a circle or radius** $\sqrt{E_s}$



- **Symbols are equally spaced to minimize likelihood of errors**

- **E.g., Binary PSK**



- **4-PSK (above) same as 4-QAM**

M-PSK

$$A_i^x = Cos(2\pi i/M),\ A_i^y = Sin(2\pi i/M),\ i = 0,\ldots,M-1$$

$$U_m(t) = g(t)A_m^x Cos(2\pi f_c t) - g(t)A_m^y Sin(2\pi f_c t)$$

$$Notice: Cos(\alpha)Cos(\beta) = \frac{Cos(\alpha-\beta) + Cos(\alpha+\beta)}{2}$$

$$Sin(\alpha)Sin(\beta) = \frac{Cos(\alpha-\beta) - Cos(\alpha+\beta)}{2}$$

$$Hence,\quad U_m(t) = g(t)Cos(2\pi f_c t + 2\pi m/M)$$

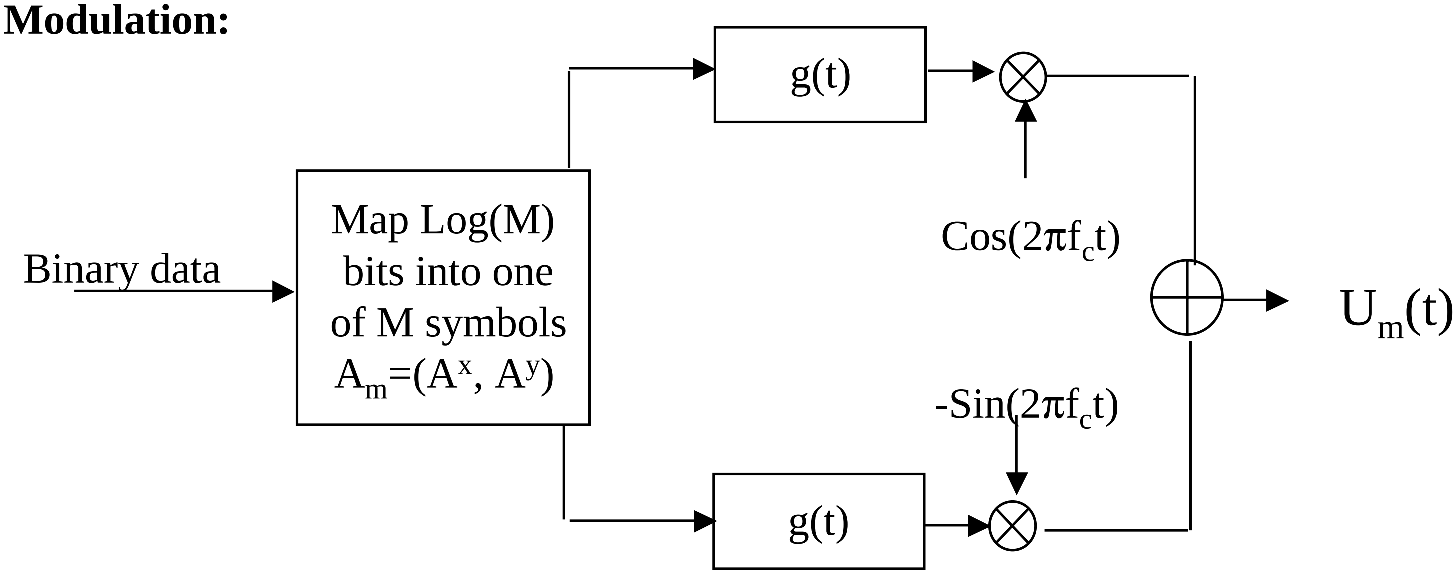
$$\phi_m = 2\pi m/M = phases\ shift\ of\ m^{th}\ symbol$$

$$U_m(t) = g(t)Cos(2\pi f_c t + \phi_m),\ m = 0..M-1$$

M-PSK Summary

- **Constellation of M Phase shifted symbols**
  - **All have equal energy levels**
  - **$Log_2(M)$ bits per symbol**

- **Modulation:**



- **Notice that for PSK we subtract the sine component from the cosine component**
  - **For convenience of notation only. If we added, the phase shift would have been negative but the end result is the same**

- **Demodulation is the same as for QAM**