Problem Set: Hybrid State Estimation Modeling

Introduction

Congratulations, you've managed to execute a successful rescue mission to Levinia! However, on your way back to Earth, you start to have some troubles with your warp drive. Maybe you shouldn't have hastily assembled the warp reactor during that mission…

Your chief engineer recommends that you use a hybrid discrete/continuous state estimator to monitor the drive. Your task in this assignment is to model the CPHA model used by the estimator.

System Description

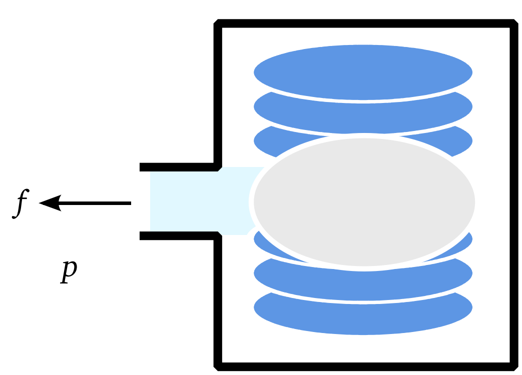
A Federation starship's engine system is composed of five main types of components as well as the ship itself. The components are the warp reactor, plasma conduits, valves, junctions, and nacelles.

Note: The following descriptions often describe flow rate (denoted *f*), and pressure (denoted *p*) through the engine. If your unfamiliar with thinking about fluid systems, there's a nice circuit analogy. Pressure is analogous to voltage (it "drops" across certain elements, just as in a resistor), and flow is analogous to current.

Warp Reactor

The purpose of the warp reactor is to produce charged plasma. It can be either offline, operating in safe mode, or operating nominally. If the reactor is offline, no plasma leaves the reactor and the pressure at the outlet drops to zero. If the reactor is either operating nominally or in safe mode, the pressure and flow rate of the plasma at the output are linearly related; the maximum flow rate is realized when the pressure is zero and the maximum pressure is realized when the flow rate is zero. When the reactor is operating in a safe mode, the maximum pressure is $P_{R,s}$ and the maximum flow rate is $F_{R,s}$. When the reactor is operating nominally, the maximum pressure is $P_{R,n}$ and the maximum flow rate is $F_{R,n}$.

The reactor can be commanded between any of its operating modes and moves to those states reliably.

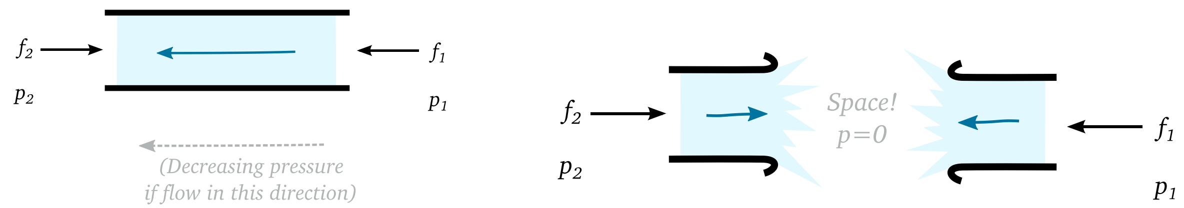


Warp reactor, with output port.

Plasma Conduits

The purpose of a plasma conduit is to carry charged plasma from point to point. If the conduit is undamaged, conservation of mass is observed and the pressure drop across the conduit is proportional to the flow rate by the constant factor $R_{C,n}$. If the conduit is destroyed, it acts as two separate conduits which both leak into space with resistance $0.1\ R_{C,n}$. In this case, the exposed end of each conduit are at a fixed pressure of zero since they're exposed to space.

A conduit has a 1% chance of becoming destroyed at any point in time (from either combat or just normal wear and tear). Once a conduit is destroyed, it is permanently destroyed.

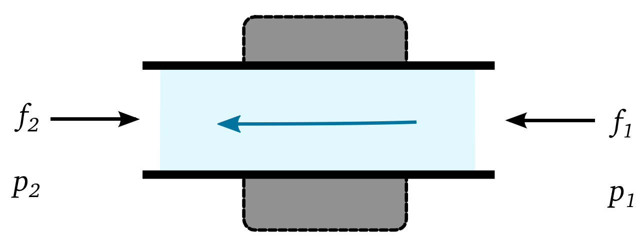


At left: an undamaged plasma conduit, with two ports. Note the direction of the pressure drop if the flow is in the indicated arrow direction. At right: a destroyed plasma conduit, leaking plasma into space.

Valves

Valves are used to shut off the flow of charged plasma. If the valve is open, the plasma flows through it unimpeded with no pressure drop across it. If the valve is closed, no flow passes through it.

Valves can't be destroyed, but they can get stuck open or stuck closed. At any time step, there is a 2% chance that a valve will become stuck in whatever state it is currently in. For example, if the valve is open, there is that small percent change that it could become stuck open. Valves can be commanded to open and closed (but don't respond when stuck). A valve in a stuck configuration stays that way forever.



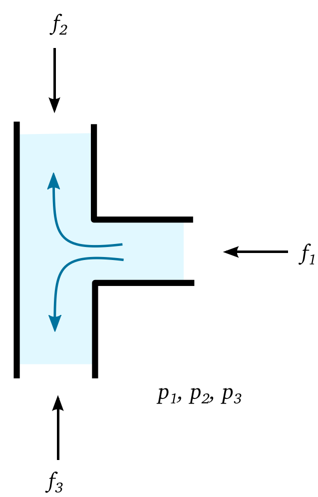
A valve, which has two ports. This picture shows the valve in an open configuration, but you can imagine it getting stuck open or stuck closed.

Junctions

Note: This component has already been modeled as a PHA for you, as an example. See the end of the problem set.

The purpose of a junction is to divide flow. If a junction is undamaged, conservation of mass is observed and there is no resistance to the flow through the junction. If a junction is destroyed, everything connected to it is exposed to space (where the pressure is zero). The pressure everywhere in a junction is the same.

At any time, a junction has a 0.1% chance of being destroyed (either through battle or normal wear and tear).



A junction, which divides flow. It has three ports.

Nacelles

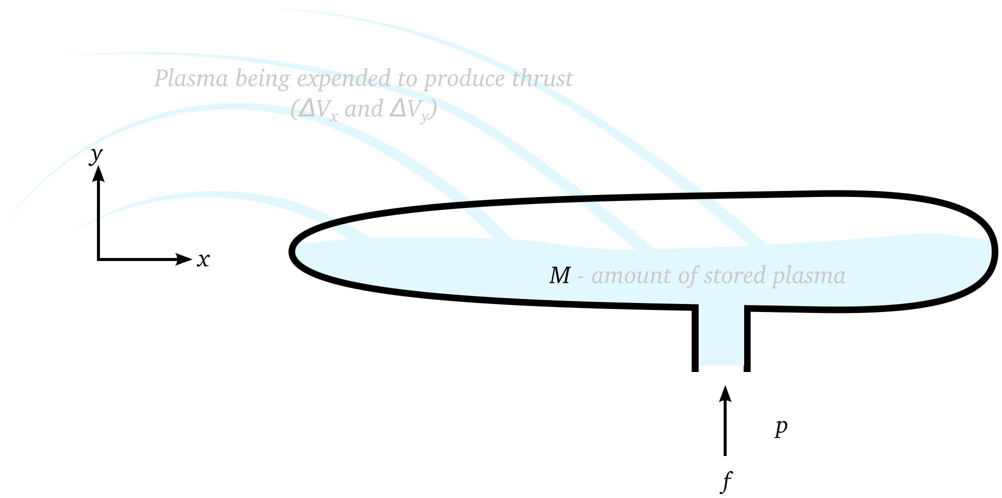
The purpose of the nacelles is to store charged plasma and generate thrust. It can be thought of a tank that fills from its inlet and expends plasma to produce thrust, which we model as changes in velocity. The amount of charged plasma stored in this “tank” is denoted by M. The nacelle takes a continuous command input: the commanded thrust to produce in the x and y directions. We denote output generated thrusts as ΔV_x and ΔV_y respectively. For each unit of ΔV_x or ΔV_y produced, $C_{\Delta V}$ units of plasma are expended from the nacelle. If the nacelle is offline or damaged, no ΔV is produced (even if thrust is commanded). If the nacelle is damaged, then the amount of plasma stored in the tank is emptied to space and stays at 0.

The internal pressure grows linearly with the amount of plasma it contains by the factor ρ_N. When a nacelle is damaged, the internal pressure drops to zero (because the plasma is exposed to space).

Note that nacelles can drain through their inlet connections if the conditions are right!

A nacelle is offline until C_{on} units of plasma are stored, at which point it automatically becomes online. A nacelle automatically goes offline if the amount of plasma stored drops below $C_{off} < C_{on}$. A nacelle has a 0.1% chance of being destroyed and once it is destroyed, it is permanently destroyed.

Note: To aid you in your modeling, we recommend modeling the continuous input commanded thrust with 4 seemingly-redundant variables instead of 2. These are u_{+x}, u_{-x}, u_{+y}, and u_{-y}. To command forward thrust in the +x direction, $u_{+x} > 0$ and $u_{-x} = 0$. To command backwards thrust in the –x direction, $u_{+x} = 0$ and $u_{-x} > 0$. This guarantees that both of these variables will always be greater than or equal to 0.



A nacelle, which converts charge plasma to thrust, has a single inlet port. Pressure and plasma build up in the Nacelle. Plasma leaves the nacelle when thrust is generated.

Ship

Everything is rigidly attached to the ship. The position *x, y* and velocity V_x, V_y of the ship changes based on the thrust provided by the nacelles. The ship has sensors that observe its location, but not its velocity.

In this section, please assume that the total thrust produced by all nacelles combined is ΔV_x and ΔV_y.

Part A – PHA Models

Model each of the components described above (except the junction, because we modeled it for you) as a PHA. Recall that a PHA is described by the following:

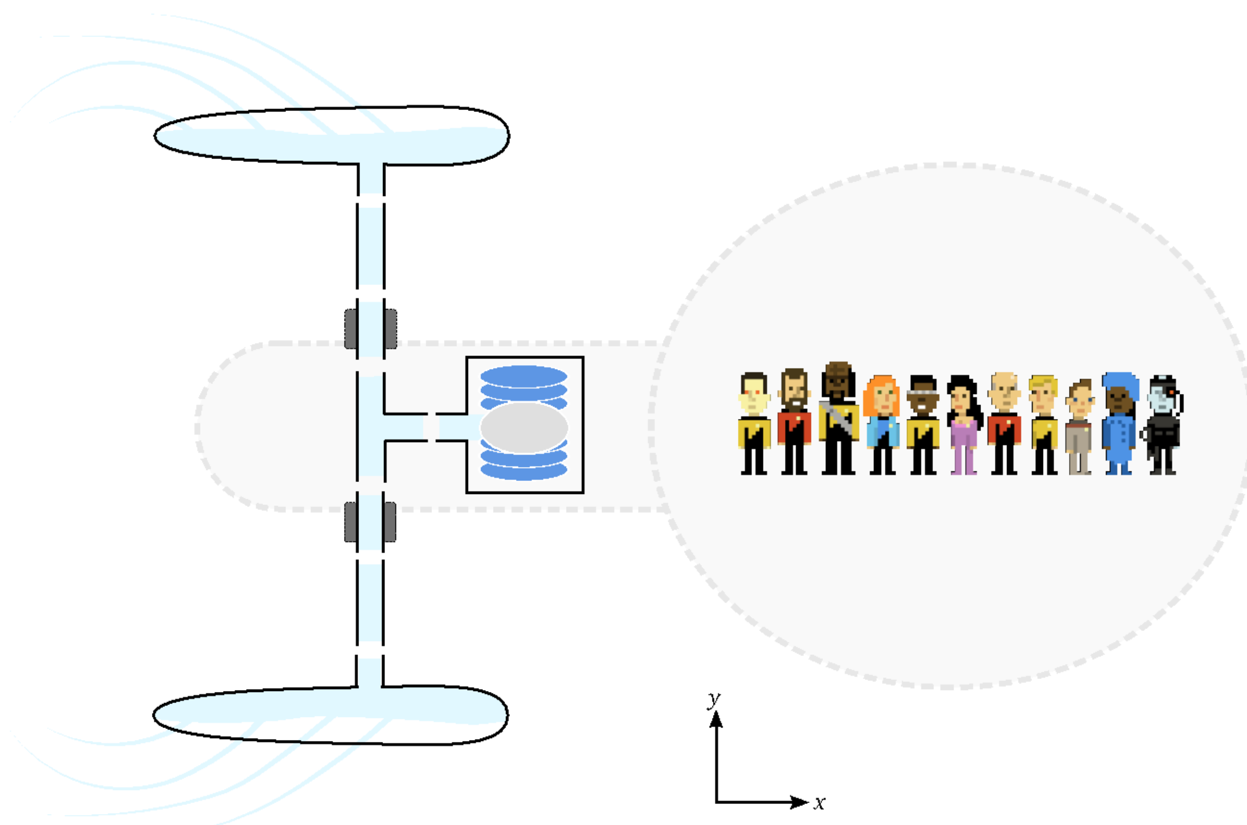
- modes,
- real-valued state variables,
- real-valued input/output (I/O) variables,
- discrete valued command variable (and its domain),
- guarded transition functions for each mode,
- and a set of algebraic and / or difference equations for each mode.

Do not worry about modelling system noise and bounds on the real-valued variables (for example, no need to model that the volume of plasma in the nacelle stays positive). To get you started, the end of this pset has the PHA model for a junction as an example.

You may assume that the time step (Δt) is a known constant. You may also assume that the ΔV_x and ΔV_y produced by nacelles is instantaneously applied. Additionally, when modeling the nacelles, make the approximation that the flow rate at its inlet is constant over the time step.

Part B – CPHA Model

Assume that your ship has one reactor, one junction, two valves, two nacelles, and two conduits. The junction is connected to the reactor, the other ends of the junction are connected to valves, each valve is connected a conduit and the conduits are connected to a nacelle. Assume also that both of the nacelles take the same commanded thrust inputs.



A schematic showing how the components are connected together to form the warp drive in the starship.

Answer the following questions.

- Make a diagram similar to the image above, except labeling every continuous state and I/O variable in the entire system.
- Assume the following combination of modes: one conduit is destroyed, it's valve is stuck open, it's nacelle is online; the other conduit is undamaged, it's valve is open, it's nacelle is destroyed; the junction is undamaged and the reactor is nominal
    1. Write all of the difference equations and algebraic equations expressing the dynamics of the system, in terms of the variables above. (Note that this should simply involve plugging the appropriate variables into the equations you already defined earlier in this problem set for the appropriate modes).
    2. Write a series of "connection" equations, 2 equations per connection point, describing constraints on how the various components are connected together. (For example, the flow into this valve equals the flow out of that component… etc). Be sure to keep track of the directions of your arrows.
    3. Your combined system has six continuous state variables $\boldsymbol{x}$, and four continuous input variables $\boldsymbol{u}$ (that are not "intermediate" and can't be solved out). What are they?
    4. Express the system-level described by all your equations above in matrix form. Again, do not worry about modeling the noise. Your equations should be of the general form:

$$\boldsymbol{x}_{t+1} = \boldsymbol{A}\boldsymbol{x}_t + \boldsymbol{B}_u\boldsymbol{u}_{t+1} + \boldsymbol{B}_c$$

$$\boldsymbol{y}_{t+1} = \boldsymbol{C}\boldsymbol{x}_{t+1} + \boldsymbol{D}_u\boldsymbol{u}_{t+1} + \boldsymbol{D}_c$$

Extra credit!

Repeat the above procedure, but instead use the modes: conduits and junction: undamaged; valves: open; reactor: nominal; nacelles: online.

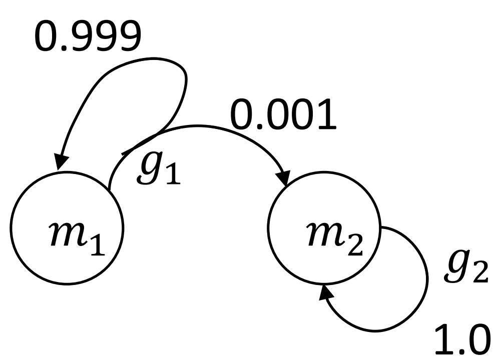
This portion of the problem set is conceptually the same as above, but the algebra is a bit trickier because the Nacelle's are coupled now.

Deliverables

Turn in a **single** PDF containing your answers to Parts A and B. Scanning hand-written notes is acceptable (just be sure they are legible).

Example Modeling – Junction PHA

- Modes:
  - undamaged (m_1)
  - destroyed (m_2)
- Continuous state variables: none.
- Continuous I/O variables:
  - port 1 pressure (p_1)
  - port 1 flow rate, positive in (f_1)
  - port 2 pressure (p_2)
  - port 2 flow rate, positive in (f_2)
  - port 3 pressure (p_3)
  - port 3 flow rate, positive in (f_3)
- Discrete command domain: empty set. (i.e., no command)
- Algebraic equations for modes
  - undamaged
    - $f_1 + f_2 + f_3 = 0$
    - $p_1 = p_2 = p_3$
  - destroyed
    - $p_1 = p_2 = p_3 = 0$
- Transition diagram below, guards g_1 and g_2 are both True. (But in general, they could be logical sentences over the discrete command variable, continuous state variables, etc.).



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