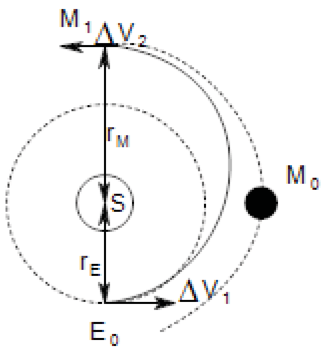
# Homework 2: Chemical vs. Electrical Thrusters

## a) Chemical:

We start outside the sphere of influence (SOI) of Earth, and end outside the SOI of Mars, so no "escape" or "capture" $\Delta Vs$ are involved. The whole motion is under the Sun's influence alone.




**To enter the transfer orbit:**

$$\Delta V_1 = v_{perihelion} - v_{c,Earth} = \sqrt{\frac{\mu_S}{r_E}}\left(\sqrt{\frac{2r_M}{r_E+r_M}} - 1\right) \qquad (1)$$

**Known and calculated values:**

$\mu_S = 1.327E20 \ \frac{m^3}{s^2}$

$r_E = 1.496E11 \ m$

$r_M = 1.5237 * r_E = 2.279E11 \ m$

$v_{c,Earth} = 29{,}780 \frac{m}{s}$

$v_{c,Mars} = 24{,}130 \frac{m}{s}$

**Substituting values:**

$$\Delta V_1 = 29{,}780\left(\sqrt{\frac{2*1.5237}{2.5237}} - 1\right) = 2{,}945 \frac{m}{s}$$

**To enter circular orbit near Mars:**

$$\Delta V_2 = v_{c,Mars} - v_{apohelion} = \sqrt{\frac{\mu_S}{r_M}}\left(1 - \sqrt{\frac{2r_E}{r_E+r_M}}\right) \qquad (2)$$

$$\Delta V_2 = 24{,}130\left(1 - \sqrt{\frac{2}{2.5237}}\right) = 2{,}649 \frac{m}{s}$$

**Total $\Delta V$:**

$\Delta V = \Delta V_1 + \Delta V_2 = 5.549 \frac{m}{s}$ **(Chemical)**

**Transfer duration:**

The transfer time is ½ the orbital time in the transfer ellipse.

**Semiaxis:** $a = \frac{r_M + r_E}{2} = 1.8877E11\ m$

$\Delta t = \frac{1}{2} * 2\pi * \frac{a^{3/2}}{\sqrt{\mu_S}}$ (3)

$\Delta t = 2.237E7\ s = \frac{2.237E7}{86400} = 259\ days$

$\frac{\boldsymbol{M_{pay}}}{\boldsymbol{M_0}} = \boldsymbol{e}^{-\frac{\Delta V}{c}} - \varepsilon$ (4)

$M_{pay} = 20{,}000\left(e^{-\frac{5{,}547}{4{,}500}}\right) - 0.05 = 4{,}770\ kg$

## b) Electrical:

### The propulsive $\Delta V$ is now:

$\Delta V = v_{c,E} - v_{c,M} = 29{,}780 - 24{,}130 = 5{,}650 \frac{m}{s}$ **(Electric Propulsion)**

This is only slightly more than the chemical $\Delta V$; for transfers to larger radii, the difference is more noticeable.

For optimization of the low-thrust mission, define **non-dimensional variables**:

$\mu = \frac{M_{pay}}{M_0}$ (5)

$v = \frac{\Delta V}{c}$ (6)

$\lambda = \frac{\alpha a_0 \Delta V}{2\eta}$ (7)

$\varepsilon = \frac{M_{str}}{M_0}$ (8)

$\alpha = 10 \frac{kg}{kW} = 0.01 \frac{kg}{W}$ is the specific mass (per unit power) of the power and propulsion equipment.

$a_0$ is the initial acceleration.

### Combining expressions:

$\mu = e^{-v} - \frac{\lambda}{v} - \varepsilon$ (9)

To find the best specific impulse $c$, we have differentiate with respect to $v$:

$-e^{-v} + \frac{\lambda}{v^2} = 0$

$\lambda = v^2 e^{-v}$ (10)

**Substituting values:**

$\lambda = \frac{0.01*5650}{2*0.7} = 40.39a_0$

For each value of $a_0$ we then need to solve (by trial and error) the equation:

$40.39a_0 = v_{opt}^2 e^{-v_{opt}}$ (11)

Once $v_{opt}$ is known, we calculate:

$c_{opt} = \frac{\Delta V}{v_{opt}} = \frac{5650}{v_{opt}}$

**The implied transfer time follows from:**

$\Delta t = \frac{\mu_{prop}}{\dot{m}} = \frac{M_0\left(1-e^{\frac{-\Delta V}{c}}\right)}{F/_C} = \frac{c}{a_0}\left(1 - e^{-\frac{\Delta V}{c}}\right) \cong \frac{\Delta V}{a_0}\ if\ \frac{\Delta V}{c} \ll c$ (12)

**The power per unit initial mass:**

$\frac{P}{M_0} = \frac{Fc/_{2\eta}}{M_0} = \frac{a_0 c}{2\eta}$ (13)

**Finally, the payload mass is:**

$\mu_{pay} = \mu_{opt} M_0 = M_0\left(e^{-v_{opt}} - \frac{\lambda}{v_{opt}} - \varepsilon\right)$ (14)

**The results are tabulated below for a range of initial accelerations:**

| $a_0\left[m/_{s^2}\right]$ | $\lambda$ | $v_{opt}$ | $c_{opt}[m/_s]$ | $\Delta t[days]$ | $\frac{P}{M_0}\left[W/_{kg}\right]$ | $M_{pay}[kg]$ | $P[MW]$ |
|---|---|---|---|---|---|---|---|
| 1E-4 | 4.039E-3 | 0.06567 | 86052 | 633.3 | 6.149 | 16498 | 0.123 |
| 2E-4 | 8.078E-3 | 0.09421 | 60004 | 322.2 (chem) | 8.572 | 15486 | 0.1714 |
| 4E-4 | 0.01616 | 0.1361 | 41536 | 152.9 | 11.867 | 14082 | 0.2373 |
| 6E-4 | 0.02423 | 0.1694 | 33371 | 100.3 | 14.302 | 13024 | 0286 |

**We see several important things here:**

a) For any specific power $P/_{M_0} \geq 10\ W/_{kg}$, the transfer is *faster than chemical.*

b) The payload delivered is *3-4 times greater* than with chemical.

c) The specific impulse is in the range from 3,400*s* to 8,700*s*.

d) The required power is from 120-290 KW, possible with large solar arrays.

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