16.50 Lecture 30

**Topics: Film cooling; Thermal stresses; Impingement
Cooling; How to do cooling design**

Film cooling:

With internal cooling the heat flux through the blades' surface increases as the external
gas temperature increases. That is large $T_r - T_w$ implies large q_w. This large heat flux
must be conducted through the blade material according to

$$q_w = k\frac{\Delta T_w}{t}$$

where t is the thickness of blade skin.
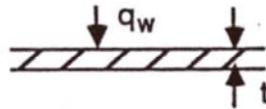
So for a given back-side T, a small t is needed if the surface facing the hot gas is to be
maintained at an acceptable level. As noted in the last lecture, it is also necessary to
match the internal cooling precisely to external heat load to prevent a large T_w variation.
All of these difficulties can be mitigated somewhat by the technique known as film
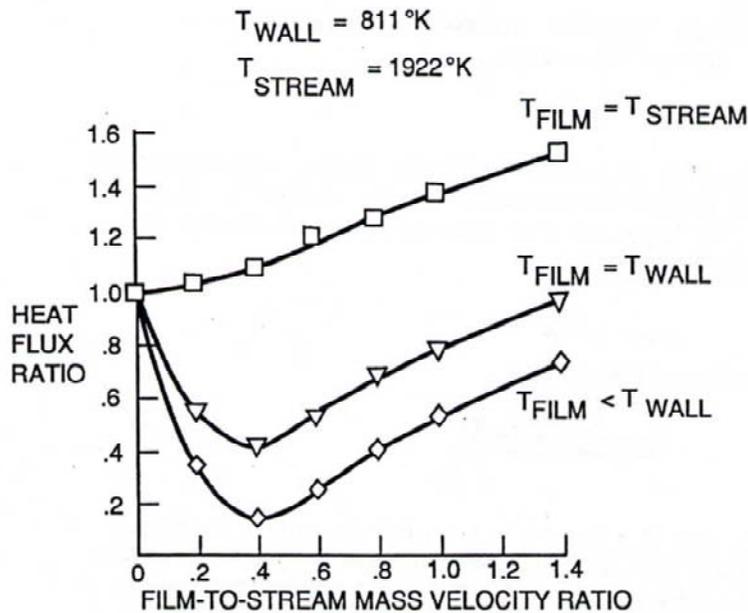cooling, which is illustrated in the figure:
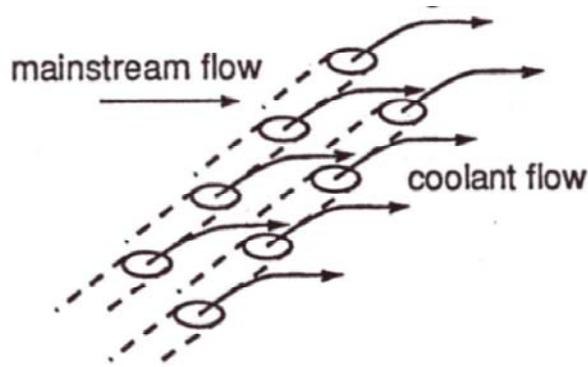
From Kerrebrock, Jack L. *Aircraft Engines and Gas Turbines*. 2nd edition. MIT
Press, 1992. © Massachusetts Institute of Technology. Used with permission.

Essentially the idea is to introduce cooling air into the boundary layer over the surface of
the blade, reducing the effective temperature of the gas heating the surface. This lowers
the heat flux, making less cooling air necessary. In addition the cooling air cools the wall
in passing through the small holes.

The performance of film cooling is related to the "mass velocity ratio"

$$m = \frac{\rho_f u_f}{\rho_s u_s}$$

where f refers to the film coolant and s to the free stream flow:

T$_{WALL}$ = 811°K

T$_{STREAM}$ = 1922°K

T$_{FILM}$ = T$_{STREAM}$

T$_{FILM}$ = T$_{WALL}$

T$_{FILM}$ < T$_{WALL}$

HEAT FLUX RATIO

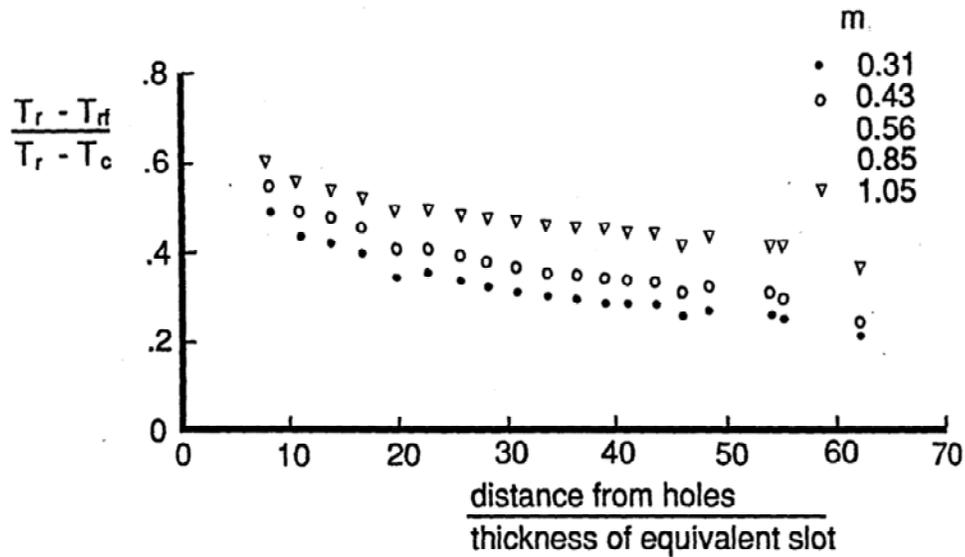FILM-TO-STREAM MASS VELOCITY RATIO

From Kerrebrock, Jack L. *Aircraft Engines and Gas Turbines*. 2nd edition. MIT Press, 1992. © Massachusetts Institute of Technology. Used with permission.

As indicated in the figure, which shows heat transfer rate versus the (dimensionless) magnitude of the cooling air flow, the cooling air actually has two effects on the heat transfer to the wall, which can be separated by varying the cooling air temperature relative to the wall temperature. If the cooling air is at the wall temperature its effect is to increase the heat transfer, by stirring the boundary layer. But if it is cooler than the wall, this effect is partially offset by the reduction of the effective hot gas temperature, so that there is a "best" cooling air flow rate.

To make these trends quantitative for design purposes, we define a Cooling Effectiveness by the relation

$$\eta_{ad} = \frac{T_r - T_{rf}}{T_r - T_c}$$

where T$_r$ is the "adiabatic recovery temperature", i.e. the temperature the wall would reach if adiabatic in the absence of film cooling, T$_{rf}$ is the recovery temperature in the presence of film cooling, and T$_c$ is the coolant temperature, which is the lowest T$_{rf}$. The cooling effectiveness so defined applies to a specific geometrical arrangement of cooling holes, and also on the cooling mass flow rate. A set of data for a single row of holes is shown in the figure below. The effectiveness decreases downstream of the row of holes, and in general increases with cooling flow, at least for the range shown.

The graph shows $\frac{T_r - T_{rf}}{T_r - T_c}$ on the vertical axis (0 to .8) versus $\frac{\text{distance from holes}}{\text{thickness of equivalent slot}}$ on the horizontal axis (0 to 70).

Legend — $\dot{m}$:
- 0.31
- 0.43
- 0.56
- 0.85
- ▽ 1.05

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Now with this cooling effectiveness, how do we estimate the heat transfer rate to the wall? The usual approach is to assume that the conventional film coefficient h, or the equivalent Stanton number St, can be applied, with an effective gas temperature given by the cooling effectiveness, Thus,

$$h = \frac{q_w}{T_{rf} - T_w}$$

where h is that for a normal boundary layer, and $T_{rf} - T_w = T_r - T_w - \eta_{ad}(T_r - T_c)$. We then calculate

$$q_w = \rho u c_p (T_{rf} - T_w) S_t$$

where h or St are correlated in the usual way to the Reynold's number and Prandtl number.

Impingement cooling

Because the heat transfer rate is largest at the leading edge of the blade where the boundary layer thickness is least, special measures are sometimes called for to cool this region. One is Impingement Cooling, illustrated in the figure:
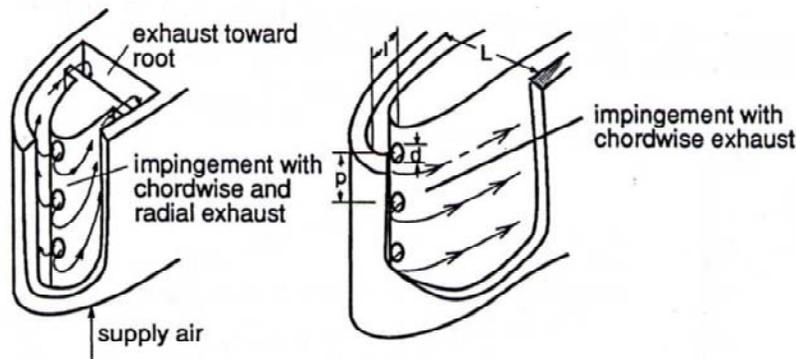
From Kerrebrock, Jack L. *Aircraft Engines and Gas Turbines*. 2nd edition. MIT Press, 1992. © Massachusetts Institute of Technology. Used with permission.

For the scheme shown the cooling rate can be correlated as

$$Nu_{stag} = 0.44 \, \text{Re}_d^{.7} \left(\frac{d}{p} \right)^{.8} e^{-0.85(\frac{l}{d})(\frac{d}{p})(\frac{d}{L})^{0.4}}$$

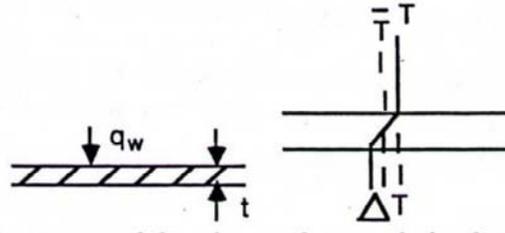where the Reynolds's number is that for the impinging jet

$$\text{Re}_d = \frac{(\rho u)_{Jet} \, d}{\mu}$$

and the Nusselt Number similarly is based on the jet diameter d

$$Nu = \frac{h \, d}{k_{fluid}}$$

Thermal stresses

For a thin skin supporting a temperature difference due to a heat flux,

a thermal stress is generated by the tendency of the hot side of the wall to expand relative to the colder side. To see this we can note that in general the strain in the material due to a combination of a normal stress σ_c and a temperature difference from some mean temperature is

$$\varepsilon = (1 - v)\frac{\sigma_c}{E} + \alpha(T - \bar{T})$$

If the plate extends to infinity in both directions, then the strains on both the hot and cold sides will be the same (note however that this assumes the plate does not curve or bulge). In this approximation the stress, called the Thermal Stress, is found by equating the two strains:

$$(1 - v)\frac{\sigma_H - \sigma_C}{E} = -\alpha(T_H - T_C)$$

If the plate is unloaded, force balance imposes $\sigma_H = -\sigma_C$, and the temperature difference is related to the heat flux through the plate by $T_H - T_C = t \, q_w / k$. Combining and solving for the cold-side stress (a positive tension), we obtain

$$\sigma_C = \frac{\alpha E}{2k(1 - v)}(q_w t)$$

The quantity $\alpha E / k$ is termed the "thermal stress susceptibility". Clearly we would like it to be small. It is proportional to the thermal expansion coefficient, and to the elastic

modulus and inversely to the thermal conductivity. Unfortunately the oxidation-resistant Nickel and Cobalt alloys favored for turbine blades have relatively low thermal conductivity and high modulus, so thermal stress is a controlling problem for turbine blades. It results in cracking an ultimately limits the blade life.

A related material quantity is the "Figure of Merit", $Z = \dfrac{(1-v)\sigma_{Ult.}}{E\alpha}$ $(in\ K)$, which is a measure of the allowable temperature difference across the plate. A few values are:

Material	Cu	SS 302	Ti	Alloy Steel
Z (K)	35	111	136	234

The Z value is low for copper, but, of course, even a small temperature difference can drive a large heat flux through copper.

How to design cooled blades

It is presumptuous to engage this subject at the tail end of a one-hour lecture, because this is one of the most difficult areas of gas turbine design. But the intent here is simply to outline the way one might proceed, as follows:

a) Estimate the q_w over the blade surface for specified T_w.
b) Find the thermal stresses in the skin.
c) Find the reduction in q_w required to limit the stresses to acceptable values, hence the η_{ad} required of the film cooling.
d) Find the arrangement of cooling holes and cooling air flow for film cooling, to give the required effectiveness.
e) Find the internal cooling airflow to absorb the residual q_w.

16.50 Introduction to Propulsion Systems
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