16.50 Lecture 9

Subject: Solid Propellant Gas Generators; Stability; Grain designs

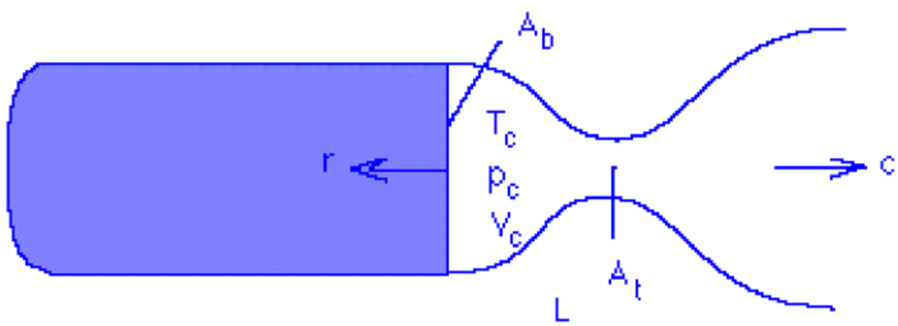
We have thus far discussed two models for the nozzle flow in rocket engines, the Channel Flow Model and the Two Dimensional Isentropic Model. Now we will introduce a model for the source of the hot gases in Solid Propellant Rockets.

Gas Generators

The distinguishing feature of a solid propellant rocket is that the fuel and oxidizer are pre-mixed. Typically, a modern propellant consists of:

polybutadiene	14%
aluminum	16%
ammonium perchlorate	70%

It is prepared as a physical mixture of NH_4 ClO_4 grains and aluminum powder in the plastic matrix



The mixture reacts at the surface of the propellant grain to produce hot gas. The surface regresses(i.e. decomposes and evaporates) at a rate:

$$\dot{r} = ap_c^n$$

where a and n are empirical coefficients. This is an empirical rule. There is no straightforward way to derive it from first principles. The physical processes involved are quite complex, including heat transfer to the grain by conduction, convection and radiation, decomposition of the solids, mixing and finally combustion in the gas phase.

Typical values of the empirical constants are:

$r = 6.8x10^{-3}$ @ $6.9x10^6$ N/m^2
n = 0.15
r = 1,775 kg/m^3

and as we shall discuss, such a propellant gives a specific impulse

I_s = 260 - 265 sec (at ground)
I_s =280 – 295 sec (in vacuum)

Once the rocket is built and ignited, it has a mind of its own. That is p_c, T_c, r and the mass flow are all set by the propellant properties and geometryof the grain. For design purposes we want to find the relationship between these performance parameters and the geometry.

First assume the rocket is operating in steady state, then

$$\dot{m}_p = rA_b\rho_p = \frac{p_c A_t}{c^*}$$

and using the above expression for the regression rate,

$$ap_c^n A_b\rho_p = \frac{p_c A_t}{c^*}$$

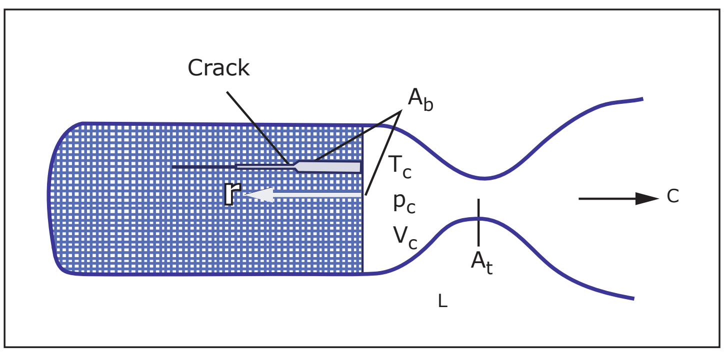
where A_b is the "burning area", ie the surface area of the grain that is regressing. Solving for p_c gives

$$p_c^{1-n} = ac^*\rho_p \frac{A_b}{A_t} \qquad (1)$$

So given the propellant properties we can compute the chamber pressure, and the mass flow of the propellant. Combined with a nozzle model, this enables us to compute the thrust of the rocket and its specific impulse

Stability

This relationship applies if the operating point is stable, that is if the pressure and mass flow are steady in time. If a, n, c*, ρ_p are all well defined, we can get p_c from it. Note however that p_c depends on A_b and that it can be sensitive to unintended variations in the grain configuration..



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Cracking of the grain or its debonding from the case can lead to increases in the burning area, very high p_c and an "explosion". This is probably the most frequent cause of solid rocket failures.

We have assumed that there is a steady p_c. To determine whether this is a stable operating point, we first construct a model that allows unsteady behavior of p_c. In first approximation, unsteadiness implies a variation in the mass stored in the chamber, which modifies the mass balance we used to determine p_c.

$$\frac{d}{dt}(\rho_c V_c) = \dot{r} A_b\rho_p - \frac{p_c A_t}{c^*}$$

$$= ap_c{}^n A_b \rho_p - \frac{p_c A_t}{c*}$$

$$\frac{d}{dt}(\rho_c V_c) = \rho_c \frac{dV_c}{dt} + V_c \frac{1}{RT_c}\frac{dp_c}{dt}; \quad T_c \approx const$$

$$= \rho_c \dot{r} A_b + V_c \frac{1}{RT_c}\frac{dp_c}{dt}$$

$$= \rho_c a p_c^n A_b + \frac{V_c}{RT_c}\frac{dp_c}{dt}$$

Substituting above, and noting that $\rho_p >> \rho_c$,

$$\frac{V_c}{RT_c}\frac{dp_c}{dt} = (\rho_p - \rho_c) A_b a p_c^n - \frac{p_c A_t}{c^*} \approx \rho_p A_b a p_c^n - \frac{p_c A_t}{c^*}$$

Now suppose the chamber pressure consists of a steady part plus one that varies in time,

$p_c = p_{co} + \delta p_c(t)$ where $\delta p_c << p_{co}$

$$\frac{V_c}{RT_c}\frac{d\delta p_c}{dt} = a(p_{co} + \delta p_c)^n A_b \rho_p - \frac{(p_{co} + \delta p_c) A_t}{c*}$$

$$= a p_{co}{}^n (1 + \frac{\delta p_c}{p_{co}})^n A_b \rho_p - \frac{p_{co} A_t}{c*}(1 + \frac{\delta p_c}{p_{co}})$$

$$\approx a p_{co}{}^n (1 + n\frac{\delta p_c}{p_{co}}) A_b \rho_p - \frac{p_{co} A_t}{c*}(1 + \frac{\delta p_c}{p_{co}})$$

The zero'th order terms on the right hand side cancel out, leaving

$$\frac{V_c}{RT_c}\frac{d\delta p_c}{dt} = \rho_p a p_{co}{}^n A_b n(\frac{\delta p_c}{p_{co}}) - \frac{p_{co} A_t}{c*}(\frac{\delta p_c}{p_{co}})$$

$$= \left[a p_{co}{}^{n-1} A_b n \rho_p - \frac{A_t}{c*} \right] \delta p_c$$

$$\frac{1}{\delta p_c}\frac{d\delta p_c}{dt} = \frac{RT_c}{V_c}[a p_{co}{}^{n-1} A_b n \rho_p - \frac{A_t}{c*}]$$

So we have <u>divergence</u> if the quantity in brackets >0, <u>stability</u> if it is <0. Simplifing the expression by use of Eq. (1):

$$a p_{co}^{n-1} A_b n \rho_p - \frac{A_t}{c*} = \rho_p \frac{a A_b n}{a c* \rho_p \frac{A_b}{A_t}} - \frac{A_t}{c*} = (n-1)\frac{A_t}{c*}$$

$$\frac{1}{\delta p_c}\frac{d\delta p_c}{dt} = -\frac{RT_c}{V_c}\frac{A_t}{c*}(1-n)$$

So we have:

stability for n < 1

instability for n > 1

The time scale for growth is $\frac{V_c c^*}{(1-n)RT_c A_t}$, and using $\rho_c = p_c/(RT_c)$ and $\dot{m} = p_c A_t / c^*$,

$$t_{growth} \approx \frac{1}{1-n}\frac{\rho_c V_c}{\dot{m}} \approx \frac{t_{residence}}{1-n}$$

For typical values, this time is about 1 millisecond, which is of course short compared to most rocket burning times, so we conclude that if n<1 the chamber pressure will relax from a perturbation in a time of order of 1 millisecond. If n>1 the rocket will explode in about the same time.

Grain Designs

The burning time is set by mission requirements, in particular by the acceleration level of the vehicle. For example for an acceleration of 2 "g's" and a propellant fraction of 0.7,

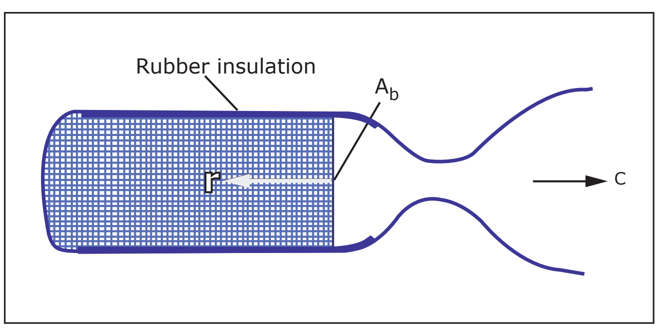
$$t = \frac{c}{a_o}[1-\frac{m(t)}{m(o)}] \approx \frac{2500}{2(9.8)}[.7] \approx 89\sec$$

The regression rate as quoted above is about 0.7 cm/s, so in the direction normal to the burning the thickness of the grain must be about 56 cm.

Grain Designs

The geometry of the grain must be chosen so as to meet the above requirements. The simplest grain configuration is what is termed "end burning".

a) End burning:



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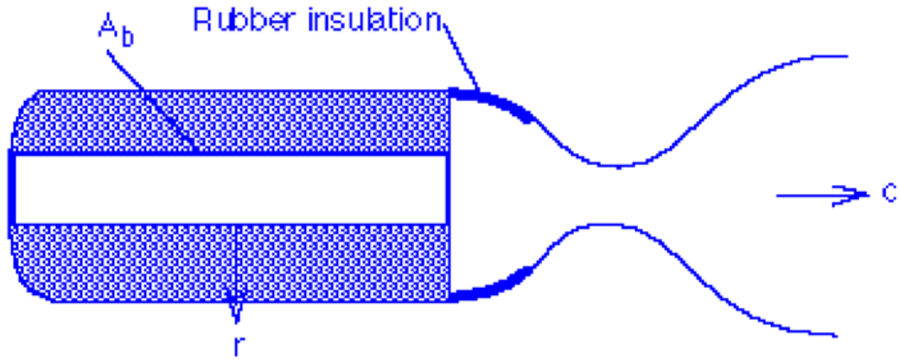
Here the grain regresses axially across the end facing the nozzle. The outer (cylindrical) surfaces of the grain are generally bonded to the case with an intermediate layer of insulating material such a rubber and coated with a substance that inhibits decomposition. As the grain recesses it leaves the layer of rubber, which protects the case from the hot combustion gases. This type of grain works well if the size of the grain needed for the application results in a length that matches the desired burning time. But if the two do not match then a design must be found that gives the right burning length for the particular grain mass that is required for the mission.

b) Radial burning:

A configuration that shortens the burning time for a given grain mass is the "radial burning" grain:

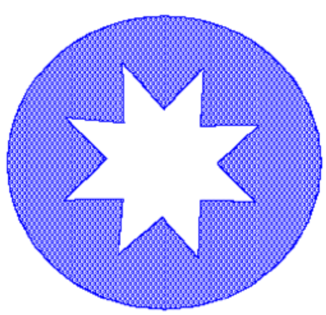


This configuration has the disadvantage that the burning area increases as the grain regresses, resulting in an increase of chamber pressure with time, a situation that is undesirable for three reasons. First, the case must be designed structurally for the maximum pressure and it is used inefficiently for much of the burn time. Second, at least for launch systems, we want a high chamber pressure at the beginning of the burn to provide a high nozzle pressure ratio in the presence of the high atmospheric pressure. And finally, if the thrust increases with time it exacerbates the increase in acceleration due to the decreasing mass of the vehicle that results from the propellant consumption, and results in very high g loads at the end of burning.

To get around these difficulties, many large rocket engines use a "star" grain configuration that has the objective of maintaining the burning area constant as the grain regresses, or even decreasing it.

c) "Star" grain

These grains have a series of points protruding inward, as shown in the sketch, such that as the points burn off, they keep the area roughly constant. In a first approximation one can see that the periphery of the "star" should be equal to the outer(circular) periphery of the grain, so that the burning area is equal at beginning and end. Detailed geometric constructions have been developed that keep the area very nearly constant throughout the burn.



c) Segmented Grains

The normal process for manufacturing solid propellant grains consists of mixing the ingredients in a batch process (essentially a big food mixer), then pouring it into the case, where the rubber matrix cures. Because of limits on the size of the mixer and for safety and transportation reasons, the amount that can be poured into a single case is limited. So for very large rocket motors such as the Space Shuttle Solid Rocket Boosters, the grain is made up of several axial "segments" that are poured separately then assembled to make the complete motor. It was a failure of one of the case joints that led to the Challenger accident.