16.881
HW#5 Design for Additivity
Air Gap Problem
Proposed Solution

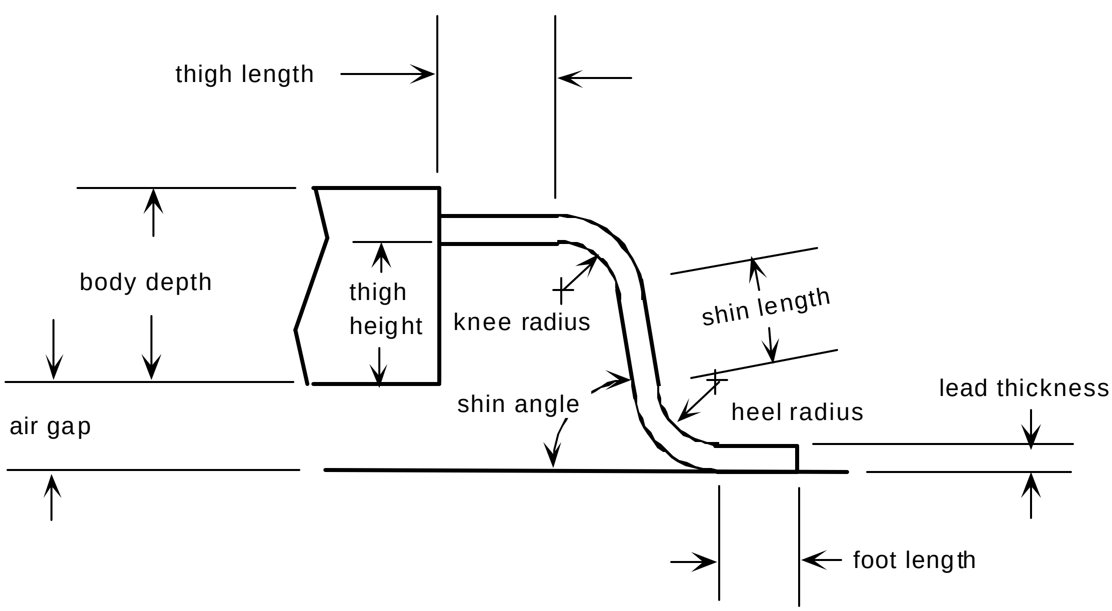
ORIGIN := 1

Objectives:

- Explore the effect of the choice of system response on the accuracy of an additive model of a system
- Reinforce material from earlier sessions

Assignment

The figure below depicts a side view of an electronic package. The ribbon leads are formed in a die into a leg shape (the industry uses a set of anthropomorphic terms as defined in Figure 1). The problem is that the yield strength of the leads varies by $\pm 10\%$ about its nominal value of 200Mpa (assume the band is $\pm 3\sigma$). This tends to make the spring-back of the ribbon lead during the forming process inconsistent and hence the air gap is inconsistent. This is a problem as the air gap is filled with thermally conductive material. If the air gap is too small, the fill material will overflow from the bottom of the package and foul the contacts. If the air gap is too great, there will be insufficient area covered by conductive material. You have been given the task of making this process more robust to the variation in yield strength of the lead material and thereby reducing quality problems. See the next page for details.



You have been told that **you may vary** the following parameters within the following ranges:

Thickness of the lead material t = 0.1mm to 0.2mm
Initial radius of the knee bend R_i = 1mm to 2mm
Initial knee angle Θ_o = 80° to 120° (see Fig. 2 below)
Elastic modulus of the lead material E = 90GPa to 110GPa
Shin length = 1mm - 4mm

The other parameters of the problem are **fixed**:

Air gap (desired) = 0.5 mm ± 0.2mm (Δ_o=0.2mm)
Cost to rework a ribbon lead A_o=$0.50
Thigh length = 2mm
Body depth = 2mm
Thigh height = 5.3mm
Foot length = 2 mm
Shin angle is always equal to knee angle
Heel radius is always equal to knee radius

To simplify your analysis, you may wish to neglect the spring-back in the heel bend and focus on only the spring-back in the knee. You may assume that the springback of the knee bend is governed by the equation

$$\frac{R_i}{R_f} = 4\left(\frac{R_i Y}{Et}\right)^3 - 3\left(\frac{R_i Y}{Et}\right) + 1$$

- Estimate the quality loss in the system if each control factor is at the middle of its allowable range.
- Evaluate the significance of interaction between the control factors t and R_i if **variance in air gap** is defined as the response of the system.
- Evaluate the significance of interaction between the control factors t and R_i if **percent conforming to air gap specification** is defined as the response of the system.
- Evaluate the significance of interaction between the control factors t and R_i if **20 log(mean air gap/variance in air gap)** is defined as the response of the system.
- How does the choice of initial knee angle affect the robustness of this system? Support your conclusion with some common sense engineering reasoning or a more formal model of the system.
- Which control factor settings will you choose?
- What is the quality loss in the system at your chosen settings?

a) Quality loss with control factors at the middle of the range

Define the values of the parameters given in the problem statement at the middle of their range.

$Y := 200 \cdot 10^{6} \cdot Pa$	Yield strength
$t := 0.15 \cdot mm$	Lead thickness
$R_i := 1.5 \cdot mm$	Initial radius
$\theta_i := 100 \cdot deg$	Initial shin angle (before spring back). Remeber, this is equal to the knee and heel initial angles. You should be able to derive this from the figure above.
$E := 100 \cdot 10^{9} \cdot Pa$	Young's modulus
$SL := 2.5 \cdot mm$	Shin length
$TH := 5.3 \cdot mm$	Thigh height. Note the change from the original problem statement.
$A_o := 0.5$	Cost to rework the lead (dollars)
$\Delta_o := 0.2 \cdot mm$	Half tolerance width for air gap
$m := 0.5 \cdot mm$	Target value for air gap

Set up a model of air gap as a function of the design parameters.

$$R_f(R_i, Y, t, E) := \frac{R_i}{4 \cdot \left(\frac{R_i \cdot Y}{E \cdot t}\right)^3 - 3 \cdot \frac{R_i \cdot Y}{E \cdot t} + 1}$$

Final radius based on the formula I gave you in the problem statement.

$$\theta_f(\theta_i, R_i, Y, t, E) := \frac{R_i}{R_f(R_i, Y, t, E)} \cdot \theta_i$$

Final angle of the knee bend, heel bend and shin based on a relationship yopu should be able to reason out for yourselves.

$$\theta_f(\theta_i, R_i, Y, t, E) = 94.003 \, deg$$

$$AG(\theta_i, R_i, SL, t, Y, E) := 2 \cdot R_f(R_i, Y, t, E) \cdot \left(1 - \cos\left(\theta_f(\theta_i, R_i, Y, t, E)\right)\right) + SL \cdot \sin\left(\theta_f(\theta_i, R_i, Y, t, E)\right) + t - TH$$

This is all the modeling you really need to do, however, you may want to go one step further and use the shin length as a scaling factor to put the mean right on target. Here's how you'd do that:

$$SLopt(\theta_i, R_i, SL, t, E) := root\left[\left(AG(\theta_i, R_i, SL, t, Y, E) - 0.5 \cdot mm\right), SL\right]$$

Compute the shin length that puts the air gap on target when yiled strength is on target.

$$AGopt(\theta_i, R_i, SL, t, Y, E) := AG(\theta_i, R_i, SLopt(\theta_i, R_i, SL, t, E), t, Y, E)$$

$$AGopt(\theta_i, R_i, SL, t, Y, E) = 0.5 \, mm \qquad AGopt(\theta_i, R_i, SL, t, Y \cdot 1.1, E) = 0.49 \, mm$$

Estimate the mean air gap.

$\mu_{AG} := AG(\theta_i, R_i, SL, t, Y, E)$

$\mu_{AG} = 0.758\,\text{mm}$

Estimate the variance in air gap.

Estimate the variance in the noise factor, yield strength (Y) as 1/3 of the specification width.

$$\sigma_Y := \frac{10\% \cdot Y}{3}$$

Remember the formula where variance of a response is the sum of the variances for the noise factor times their partial derivatives squared? Here I just take the partial numerically using the tolerance width as the step size.

$$VAR_{AG}(\theta_i, R_i, SL, t, E) := \left(\frac{AGopt(\theta_i, R_i, SL, t, Y + 10\% \cdot Y, E) - AGopt(\theta_i, R_i, SL, t, Y, E)}{10\% \cdot Y}\right)^2 \cdot \sigma_Y^{\,2}$$

$$\sqrt{VAR_{AG}(\theta_i, R_i, SL, t, E)} = 3.366 \times 10^{-3}\,\text{mm}$$

Average quality loss

$$Q := \frac{A_o}{\Delta_o^{\,2}} \cdot \left[VAR_{AG}(\theta_i, R_i, SL, t, E) + (\mu_{AG} - m)^2\right]$$

$Q = 0.833$ dollars

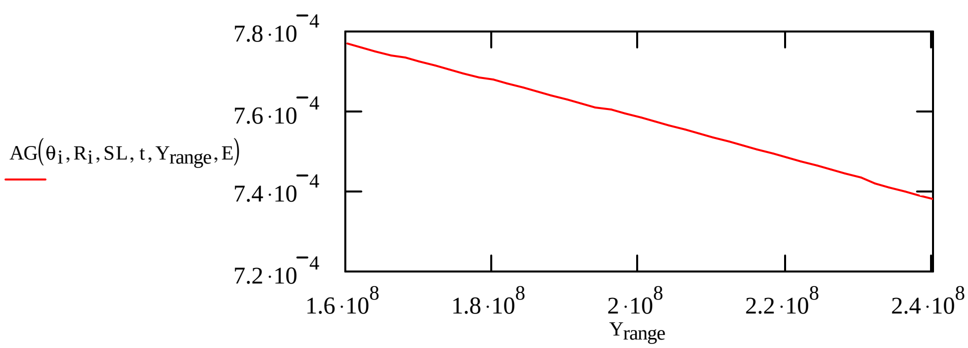
This is the answer I was looking for. However, I forgot to say "average" quality loss, so if you gave an answer that only included the mean shift and not the variance, then that's OK. The mean shift obviously dominated anyway. For example:

$$\frac{A_o}{\Delta_o^{\,2}}(\mu_{AG} - m)^2 = 0.833$$

Now remeber, the formula above for estiamting variance assumes a linear relationship between noise and response. Is this assumption valid in this case?

$Y_{range} := 0.8 \cdot Y, 0.81 \cdot Y .. 1.2 \cdot Y$

Looks pretty linear to me!



By the way, there is another way to compute the variance. Monte Carlo simulation.
Just generate random numbers from a population as defined in the problem statement and run those numbers through your model.

$number\_of\_trials := 1000$

$$Y_{random} := rnorm\left[number\_of\_trials, 200 \cdot 10^6, \frac{10 \cdot \% \cdot \left(200 \cdot 10^6\right)}{3}\right]$$

$trial := 1 .. number\_of\_trials$

$$AG_{random_{trial}} := AG\left(\theta_i, R_i, SL, t, Y_{random_{trial}} \cdot Pa, E\right)$$

$$stdev\left(AG_{random}\right) = 3.181 \times 10^{-3}\, mm \qquad \sqrt{VAR_{AG}\left(\theta_i, R_i, SL, t, E\right)} = 3.366 \times 10^{-3}\, mm$$

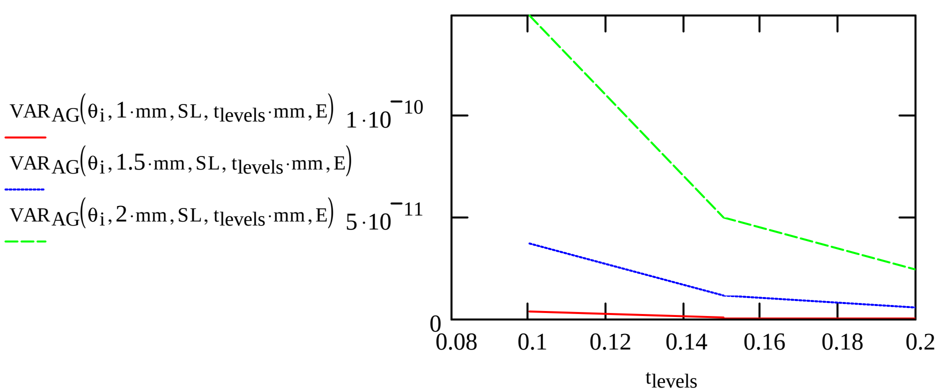
You should get almost exactly the same answer.

b) Estimate the significance of interaction between the control factors t and R if variance in air gap is defined as the response of the system.

Remeber, if you're looking for interactions, you need to set at least two levels for control factors and make an interaction plot. I'll define my levels as the bottom and top of my control factor ranges.

$t_{levels} := 0.1, 0.15 .. 0.2$

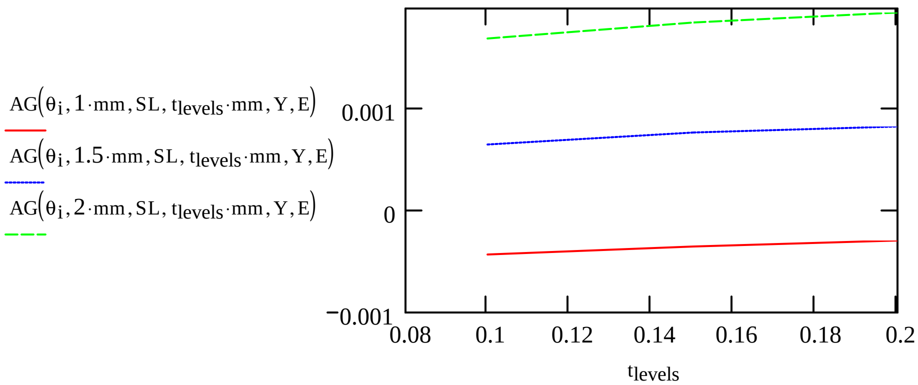
$R_{levels} := 1, 1.5 .. 2$



The variance is not very additive at all in the variables R and t. To make a quantitative assessment look at the difference in slope divided by the average slope:

$$\frac{\left(VAR_{AG}(\theta_i, 2 \cdot mm, SL, 0.2 \cdot mm, E) - VAR_{AG}(\theta_i, 2 \cdot mm, SL, 0.1 \cdot mm, E)\right) - \left(VAR_{AG}(\theta_i, 1 \cdot mm, SL, 0.2 \cdot mm, E) - VAR_{AG}(\theta_i, 1 \cdot mm, SL, 0.1 \cdot mm, E)\right)}{\left[\frac{\left(VAR_{AG}(\theta_i, 2 \cdot mm, SL, 0.2 \cdot mm, E) - VAR_{AG}(\theta_i, 2 \cdot mm, SL, 0.1 \cdot mm, E)\right) + \left(VAR_{AG}(\theta_i, 1 \cdot mm, SL, 0.2 \cdot mm, E) - VAR_{AG}(\theta_i, 1 \cdot mm, SL, 0.1 \cdot mm, E)\right)}{2}\right]} = 1.906$$

I find this significant because the air gap itself is very additive with respect to t and R.



c) Estimate the significance of interaction between the control factors t and R if percent conforming is defined as the response of the system.

Because I've got a scaling factor that keeps my mean exactly on target, all I need to consider is the way that variance affects the percent conforming. If I assume that the air gap is normally distributed, then percent conforming is given by:

$$\int_{-D}^{D} \frac{1}{\sigma\sqrt{2\cdot\pi}} \cdot e^{\frac{-x^2}{2\cdot\sigma^2}} \, dx$$ which comes out to $$\mathrm{erf}\left(\frac{1}{2}\cdot\frac{\sqrt{2}}{\sigma}\cdot D\right)$$

$$VAR_{AG}(\theta_i, R_i, SL, t, E)$$

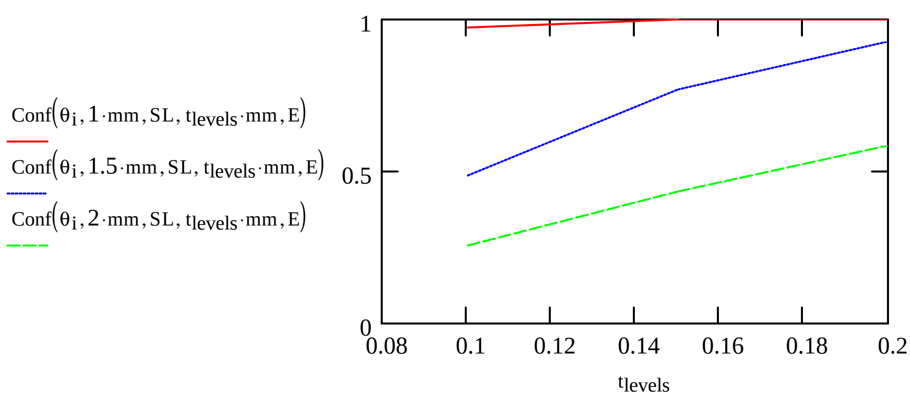
$$Conf(\theta_i, R_i, SL, t, E) := \mathrm{erf}\left(\frac{1}{2}\cdot\frac{\sqrt{2}}{\sqrt{VAR_{AG}(\theta_i, R_i, SL, t, E)}}\cdot\Delta_o\right)$$

$$Conf(\theta_i, R_i, SL, t, E) = 1$$

The tolerances on this product are so wide compared to the variance that the percent conforming is basically 100% no matter what control factor settings you choose. Unfortunately, this doesn't help to make the point I intended to make. Just for fun, I'll show you what the answer would look like if I had set the tolerances 50 times tighter.

$$Conf(\theta_i, R_i, SL, t, E) := \mathrm{erf}\left(\frac{1}{2}\cdot\frac{\sqrt{2}}{\sqrt{VAR_{AG}(\theta_i, R_i, SL, t, E)}}\cdot\frac{\Delta_o}{50}\right)$$

$$Conf(\theta_i, R_i, SL, t, E) = 0.765$$



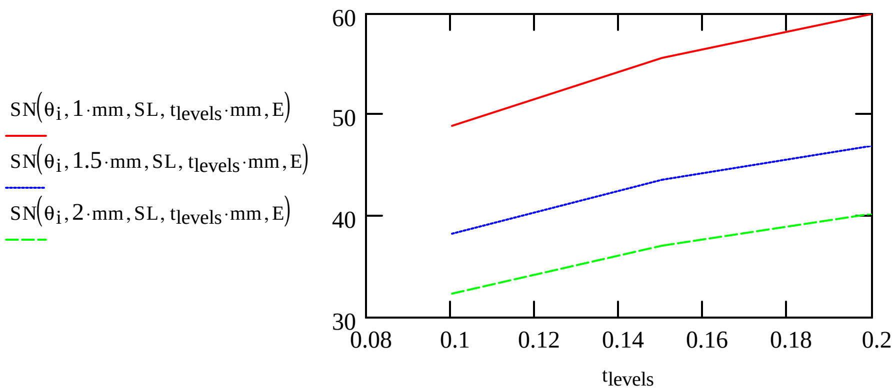
Not very additive!

d) Estimate the significance of interaction between the control factors t and R if 10*log(mean squared air gap/variance in air gap) is defined as the response of the system. (I realize that I forgot the "squared" part in the homework. If you computed something different, then you'll get credit.)

$t_{levels} := 0.1, 0.15 .. 0.2$

$R_{levels} := 1, 1.5 .. 2$

$$SN(\theta_i, R_i, SL, t, E) := 10 \cdot \log\left(\frac{AGopt(\theta_i, R_i, SL, t, Y, E)^2}{VAR_{AG}(\theta_i, R_i, SL, t, E)}\right)$$



The S/N ratio is quite additive in the variables R and t. To make a quantitative assessment look at the difference in slope divided by the average slope:

$$\frac{\left(SN(\theta_i, 2 \cdot mm, SL, 0.2 \cdot mm, E) - SN(\theta_i, 2 \cdot mm, SL, 0.1 \cdot mm, E)\right) - \left(SN(\theta_i, 1 \cdot mm, SL, 0.2 \cdot mm, E) - SN(\theta_i, 1 \cdot mm, SL, 0.1 \cdot mm, E)\right)}{\left[\frac{\left(SN(\theta_i, 2 \cdot mm, SL, 0.2 \cdot mm, E) - SN(\theta_i, 2 \cdot mm, SL, 0.1 \cdot mm, E)\right) + \left(SN(\theta_i, 1 \cdot mm, SL, 0.2 \cdot mm, E) - SN(\theta_i, 1 \cdot mm, SL, 0.1 \cdot mm, E)\right)}{2}\right]} = -0.338$$

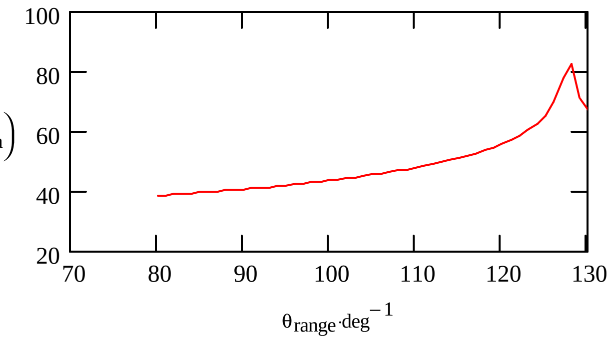
The S/N ratio tamed the interactivity of the problem very well. The main effects are about 10X the interactions.

e) How does the choice of initial knee angle affect the robustness of this system?

Set all the control factors in the middle of their allowable range, and look at the effect of initial angle.

$\theta_{range} := 80 \cdot deg, 81 \cdot deg .. 130 \cdot deg$

$SN(\theta_{range}, 1.5 \cdot mm, SL, 0.15 \cdot mm, 100 \cdot 10^9 \cdot Pa)$



Think about this. Why did the S/N ratio spike around 128 degrees? Physically, what is going on here? Spring back tends to cause the air gap to decrease under normal conditions. But with initial knee angle greater than 90 degrees, the shin tends to kick down increasing air gap. At just the right value of initial knee angle, the two effects will be balanced causing very low sensitivity to variation in yeild strength.

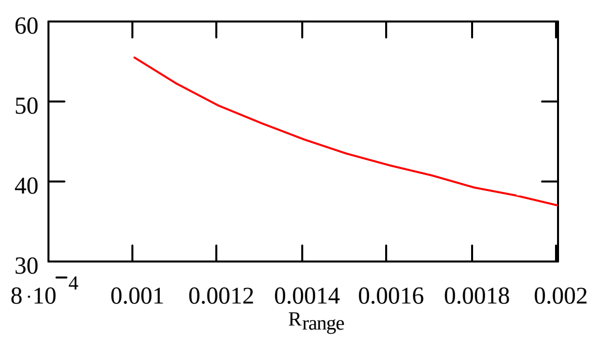
Therefore, our robustness to yield strength problems can be extremely high at the right initial angle setting. But what would the die look like? Would this be a serious difficulty?
The other concern here is that the spike is narrow. Also, I suspect that it moves when the control factor settings change. There is a real risk of missing this robustness opportunity with an orthoganal array based experiment.

f) Which control factor settings will you choose?

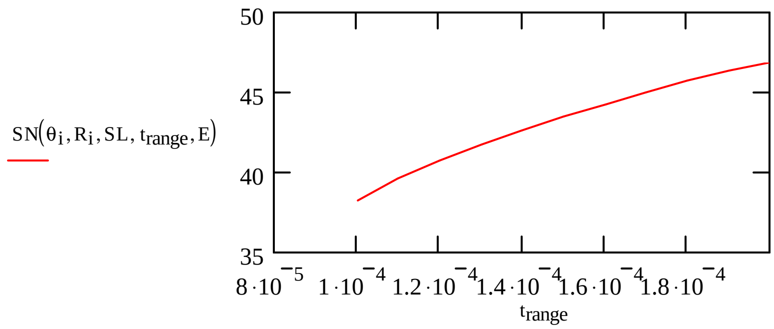
As long as we have a simulation up and running, we may as well study the effect of the control factors individually before proceeding.

$R_{range} := 1 \cdot mm, 1.1 \cdot mm .. 2 \cdot mm$

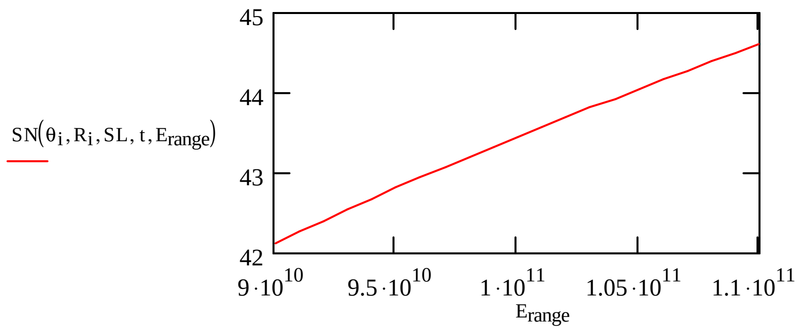
$SN(\theta_i, R_{range}, SL, t, E)$



$t_{range} := 0.1 \cdot mm, 0.11 \cdot mm .. 0.2 \cdot mm$



$E_{range} := \left(90 \cdot 10^{9} \cdot Pa\right), 91 \cdot 10^{9} \cdot Pa .. 110 \cdot 10^{9} \cdot Pa$

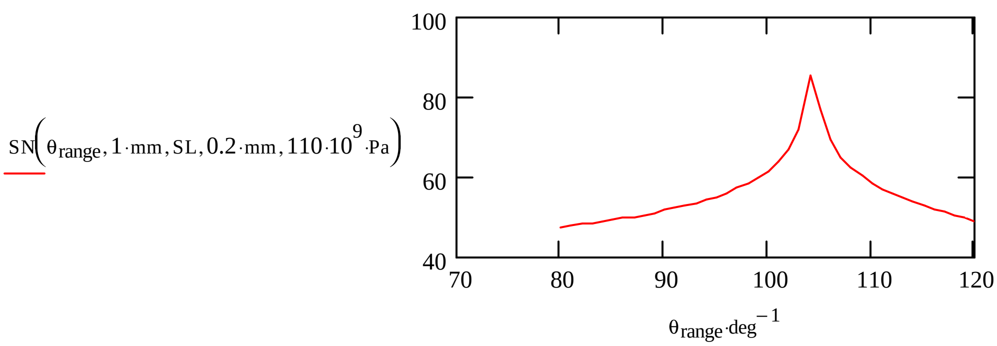


Remeber, we discovered that the additive model holds for R and t. It happens that it will also hold for E. Given that this is true, we can simply select the best values of these control factors individually based on the graphs above. Right? Pick the highest E, the highest thickness, the largest initial angle, and the tightest radius.

You could have anticipated these results based on physical intuitons or by inspecting the equations you developed. If a sheet of metal is extremely thin compared to the bend radius, it will spring back more. Also, if the stiffness of the material is high compared to the yield strength, the material will spring back more. If the total spring back is lower, it is reasonable to assume that the variation in spring back will also be lower.

Now, set the values for R, t, and E to thier max values and search for just the right initial bend angle:

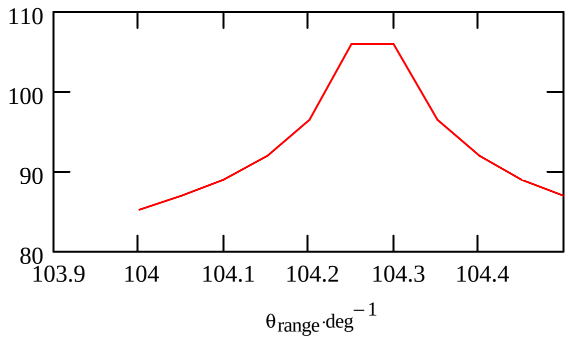
$\theta_{range} := 80 \cdot deg, 81 \cdot deg .. 120 \cdot deg$



Zoom in for a closer look.

$\theta_{range} := 104 \cdot deg, 104.05 \cdot deg .. 104.5 \cdot deg$

$SN\left(\theta_{range}, 1 \cdot mm, SL, 0.2 \cdot mm, 110 \cdot 10^{9} \cdot Pa\right)$



$SN\left(104.3 \cdot deg, 1 \cdot mm, SL, 0.2 \cdot mm, 110 \cdot 10^{9} \cdot Pa\right) = 105.971$

Even a full factorial experiment with three levels of each factor will miss the maximum opportunity for robustness to some extent since it is unlikely that you'd think to try 104 degrees as a factor level for radius.

g) What is the quality loss at the optimal settings?

Average quality loss

$R_i := 1 \cdot mm \qquad E := 110 \cdot 10^{9} \cdot Pa$

$t := 0.2 \cdot mm \qquad \theta_i := 104.3 \cdot deg$

$$VAR_{AG}(\theta_i, R_i, SL, t, E) := \left(\frac{AGopt(\theta_i, R_i, SL, t, Y + 10\% \cdot Y, E) - AGopt(\theta_i, R_i, SL, t, Y, E)}{10\% \cdot Y}\right)^2 \cdot \sigma_Y^{\;2}$$

$$\sqrt{VAR_{AG}(\theta_i, R_i, SL, t, E)} = 2.514 \times 10^{-6} \, mm$$

$$Q := \frac{A_o}{\Delta_o^{\;2}} \cdot \left(VAR_{AG}(\theta_i, R_i, SL, t, E)\right)$$

$Q = 7.902 \times 10^{-11}$ dollars

Admittedly, some other noises aside from yield strength would begin to dominate at this point. You have to take this solution with a big grain of salt.