> Welcome to 16.90 iSession ...

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...etc...

...etc...

$$\frac{\partial u}{\partial t} = -\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}$$

$$\frac{u^{n+1} - u^{n}}{\Delta t} = A u^{n}$$ explicit

$$\frac{u^{n+1} - u^{n}}{\Delta t} = A u^{n+1}$$

A is finite difference $\left(-\frac{\partial}{\partial x} - \frac{\partial}{\partial y}\right)$

$$\left(\frac{I}{\Delta t} - A\right) u^{n+1} = \frac{u^{n}}{\Delta t}$$

u_{11} u_{12} u_{13} . . — $u_{1 n_x}$

u_{21} u_{22} - — $u_{2 n_x}$

$\vdots$

$u_{n_y 1}$ $u_{n_y 2}$

Scalar Conservation Laws

$$\frac{\partial u}{\partial t} + \frac{\partial F(u)}{\partial x} = S$$

differential form

$$\int_L^R \frac{\partial u}{\partial t} + \frac{\partial F(u)}{\partial x}\, dx = \int_L^R S \, dx$$

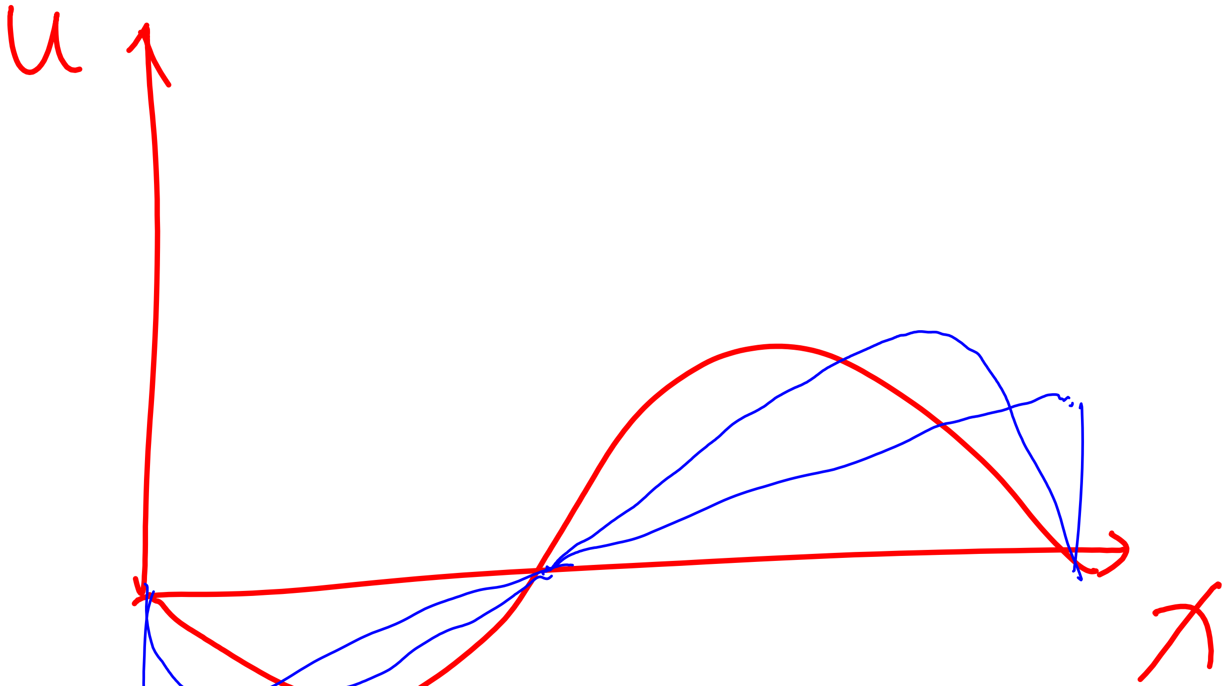
$$\Rightarrow \frac{d}{dt}\int_L^R u \, dx = -F(u)\Big|_R + F(u)\Big|_L + \int_L^R s \, dx$$

$R-L$ Integral form $R-L$

$$\frac{d}{dt}\overline{u}$$

Characteristic Lines – Smooth Solution

Shockwaves – Shock speed



$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0$$

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(\frac{u^2}{2}\right) = 0$$

Finite Volume: Cell Average, Numerical Flux

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}\left(\frac{u^2}{2}\right) = 0$$

$$\frac{\partial u}{\partial t} + \frac{\partial F(u)}{\partial x} = 0 \qquad \text{where } F(u) = \frac{u^2}{2}$$

$$\bar{u}_k := \frac{1}{\Delta x_k}\int_{L_k}^{R_k} u(x,t)\,dx$$

size of control volume $R_k - L_k$

uniform mesh $\Delta x_k = \Delta x$

$$L_k = (k-1)\cdot\Delta x$$

$$R_k = k\cdot\Delta x$$

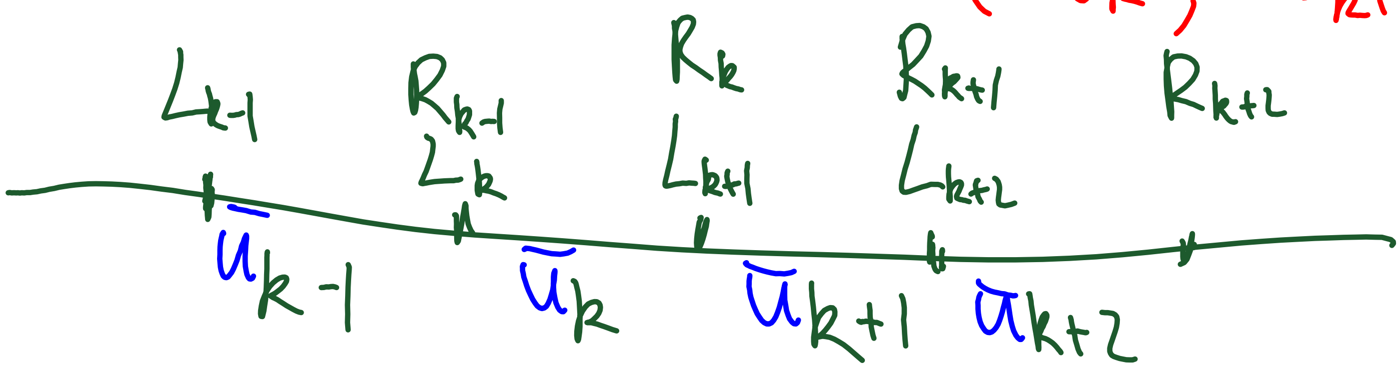
$$\frac{d}{dt}\bar{u}_k = \frac{1}{\Delta x}\frac{d}{dt}\int_{L_k}^{R_k} u(x,t)\,dx$$

$$= \frac{1}{\Delta x}\left(-F\big|_{R_k} + F\big|_{L_k} + \cancel{\int_{L_k}^{R_k} s\,dx}\right)$$

Finite Volume approximation:

$$F \text{ at } L_k \approx F(\bar{u}_{k-1}, \bar{u}_k)$$

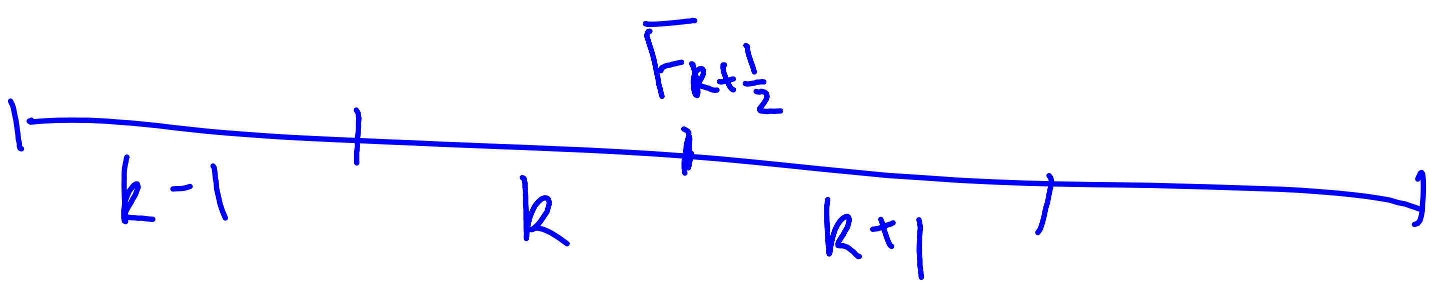
$$F \text{ at } R_k \approx F(\bar{u}_k, \bar{u}_{k+1})$$



First Order Upwind Scheme

$$F_{k+\frac{1}{2}} := F\big|_{R_k} = F\big|_{L_{k+1}}$$

$$F_{k+\frac{1}{2}} = \begin{cases} F(\bar{u}_k) = \dfrac{\bar{u}_k^2}{2} & \text{if } \dfrac{\bar{u}_{k+1}+\bar{u}_k}{2} > 0 \\ F(\bar{u}_{k+1}) = \dfrac{\bar{u}_{k+1}^2}{2} & \text{else} \end{cases}$$



Shock capturing of Finite Volume