Today’s Topics

- Monte Carlo estimators for
  - Mean
  - Probability
  - Variance
- Monte Carlo termination criteria
- Bootstrapping

Reminder: MCS Steps

1. Define input pdfs
2. Draw N random input samples; conduct deterministic simulation for each one
3. Analyze outputs

Main Idea for Today

Run MCS

↓

compute estimates of probabilistic outputs (e.g. mean, var, probabilities)

↓

how good are those estimates

Estimator for the mean

Output y

mean μ_y

var σ_y^2

samples $\{y_i\}_{i=1}^{N}$

$$\hat{\mu}_y \equiv \bar{y} = \frac{1}{N}\sum_{i=1}^{N} y_i$$

$$\bar{y} \sim N(\mu_y, \sigma_{\bar{y}}^2)$$

"unbiased estimator"

$$\sigma_{\bar{y}} = \frac{\sigma_y}{\sqrt{N}}$$

"std error"

Estimating a probability

$P\{A\}$ – probability event A occurs

$$\hat{P}_A = \frac{N_A}{N}$$ ← # times A occurs

$$\hat{P}_A \sim N\left(P\{A\}, \sigma_{\hat{p}}^2\right)$$

↑ unbiased est

↑ $\sigma_{\hat{p}} = \sqrt{\frac{P\{A\}(1-P\{A\})}{N}}$ std. error

Estimating variance

Output y Samples $\{y_i\}_{i=1}^{N}$

Var σ_y^2

$$s_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{y})^2$$

$$E[s_y^2 - \sigma_y^2] = 0 \rightarrow \text{unbiased est.}$$

In general $\sigma_{s_y^2}$ not known.

Termination criteria for MCS

e.g. Terminate sampling when error in estimating the mean is less than $\pm\varepsilon$ with 95% confidence

Since $(\bar{y} - \mu_y) \sim N(0, \sigma_{\bar{y}}^2)$

then $P\left\{ -\frac{2\sigma_y}{\sqrt{N}} \le \bar{y} - \mu_y \le \frac{2\sigma_y}{\sqrt{N}} \right\} = 0.95$

$\rightarrow$ need $\frac{2\sigma_y}{\sqrt{N}} \le \varepsilon \longrightarrow N \ge \frac{4\sigma_y^2}{\varepsilon^2}$

$$N \geq \frac{4}{\varepsilon^2} \frac{1 - P\{A\}}{P\{A\}} \approx \frac{4}{\varepsilon^2} \frac{1}{P\{A\}}$$

Estimating low-probability events

Often want

$$-\varepsilon P\{A\} \leq \left(\hat{p}_A - P\{A\}\right) \leq \varepsilon P\{A\}$$

with some confidence level

e.g. 95% confidence requires

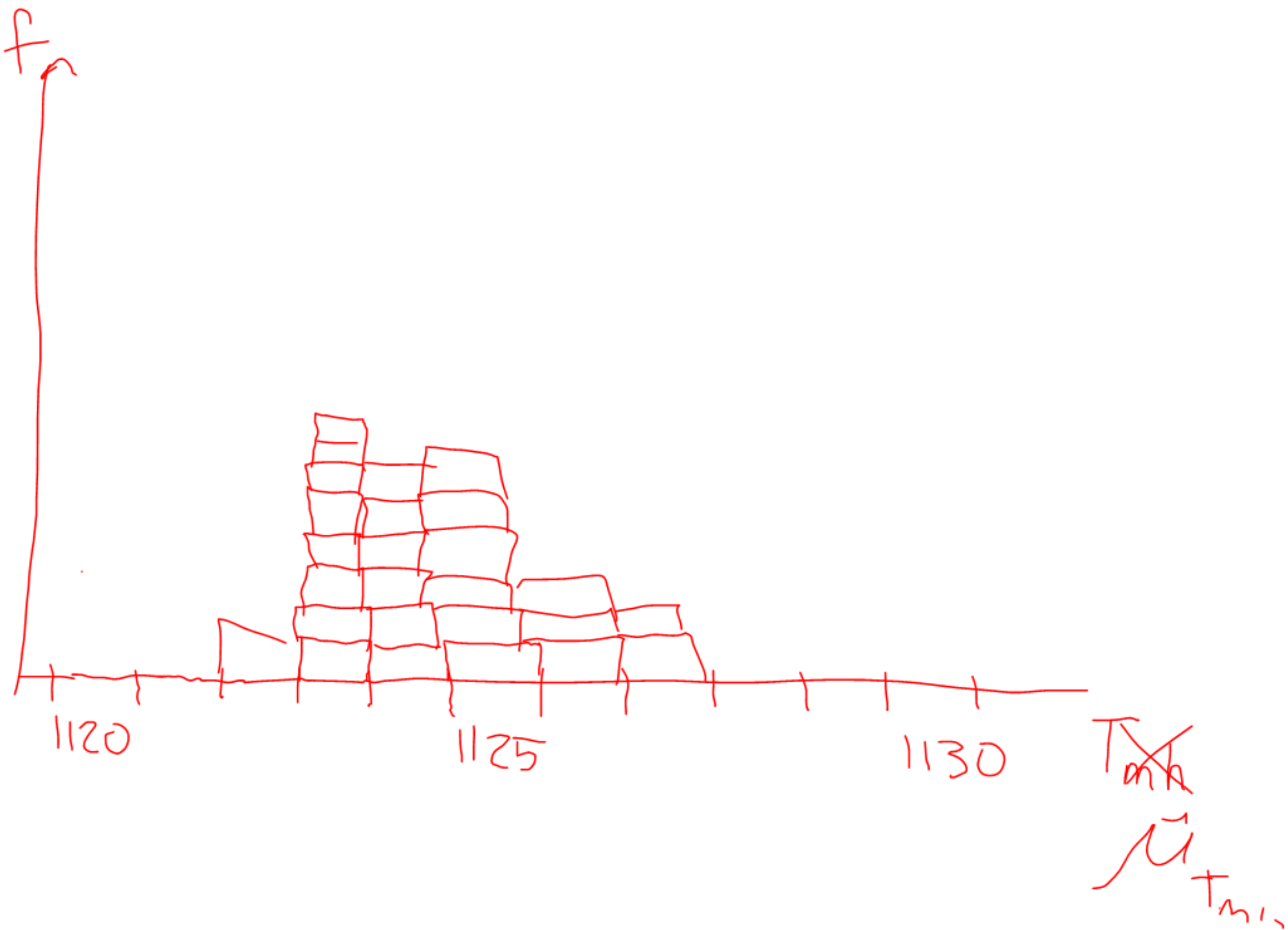
$$\sigma_{\hat{p}} \leq \frac{\varepsilon P\{A\}}{2}$$

$$\frac{P\{A\}(1-P\{A\})}{N} \leq \frac{\varepsilon^2 (P\{A\})^2}{4}$$

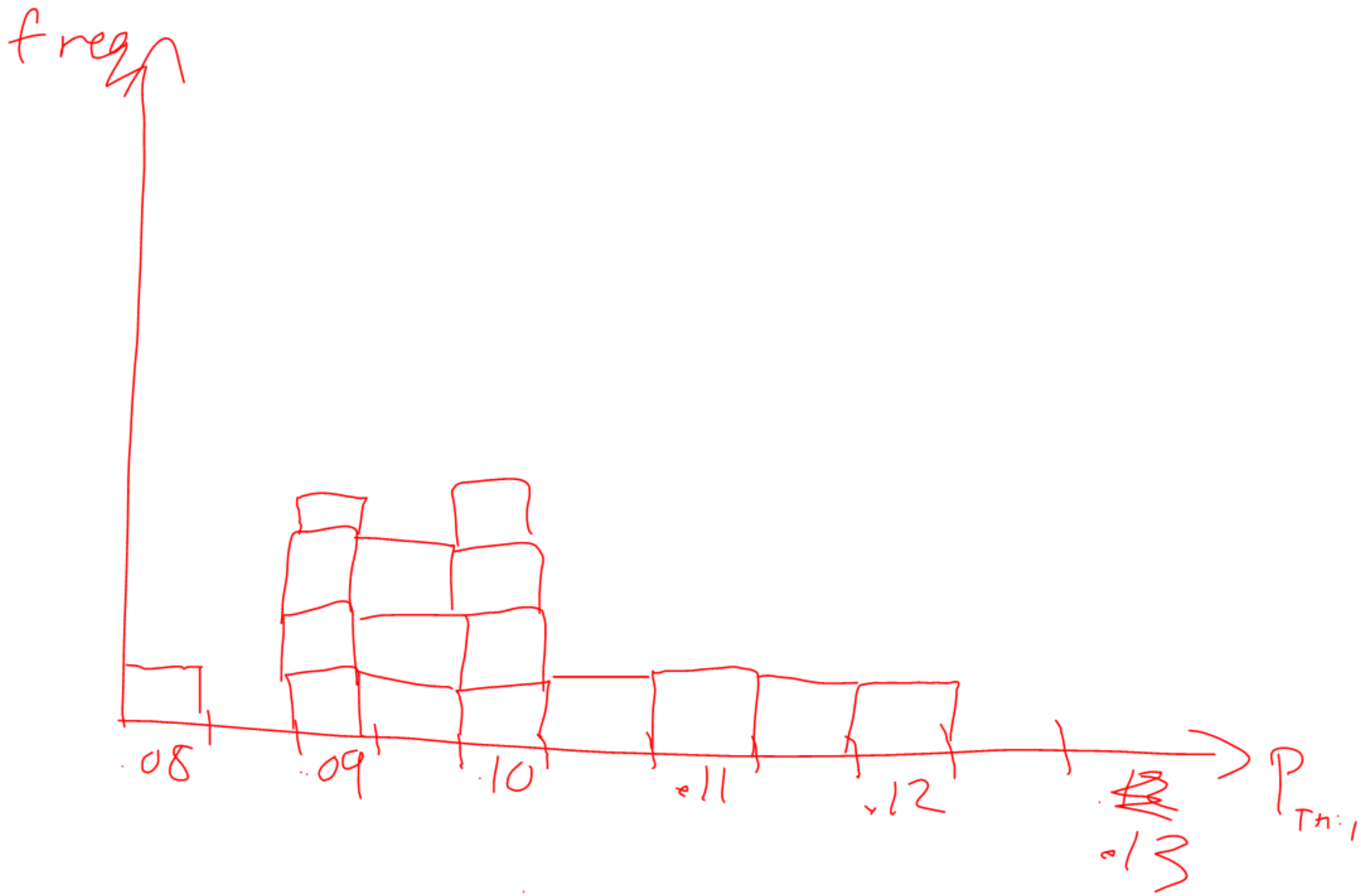
MCS Challenge

- Write a code to carry out MCS for the blade heat transfer problem using the forward code blade1D.m
  - function [Ttbc, Tmh, Tmc, q] = blade1D(hgas, Tgas, ktbc, Ltbc, km, Lm, hcool, Tcool)
- Set the input PDF for L_{TBC} to be U(0.00025,0.00075)
- Use nominal values for other inputs:
  - hgas = 3000; % TBC-gas heat transfer coef. (W/(m^2 K))
  - Tgas = 1500; % Mixed gas temperature (K)
  - ktbc = 1; % TBC thermal conduct. (W/mK)
  - km = 20; % Metal thermal conduct. (W/mK)
  - Lm = 0.003; % Metal thickness (m)
  - hcool = 1000; % Coolant-metal heat transfer coef. (W/(m^2 K))
  - Tcool = 600; % Coolant temperature (K)
- Using N=1000 samples, generate output histograms. Estimate the mean value of T_{mh} and the probability that T_{mh} > 1180 K.
- Plot your mean and probability estimates on the class histograms

Class Histogram—Mean estimate



Class Histogram—Probability estimate



Bootstrapping