symmetry

symmetry

transformation (change)
conservation (invariance)

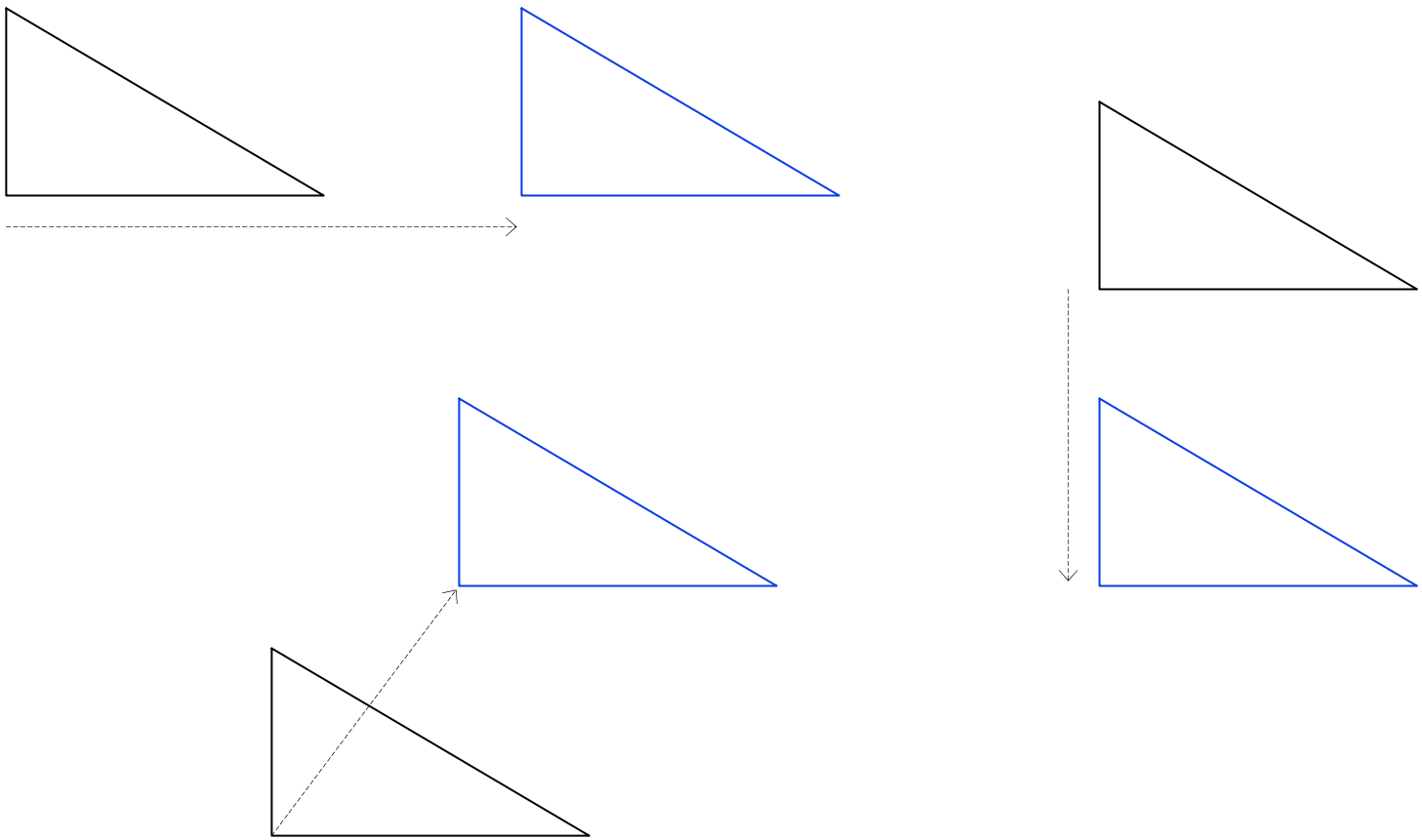
2D transformations

translation
rotation
reflection
glide reflection
(scale)

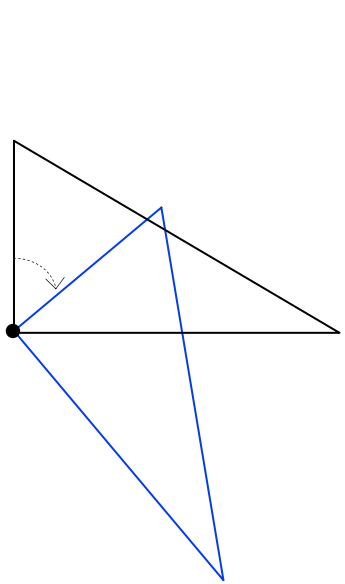
3D transformations

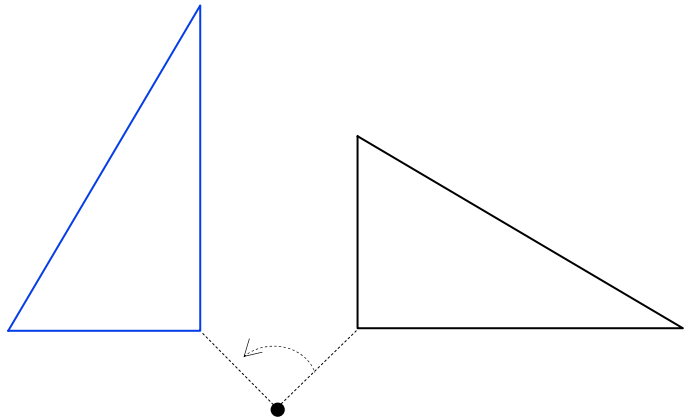
translation
rotation
screw rotation
reflection
glide reflection
rotor reflection
(scale)

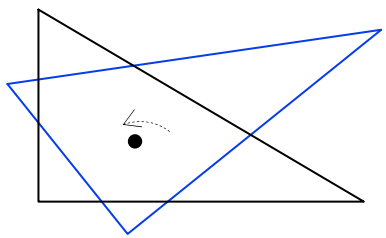
translation



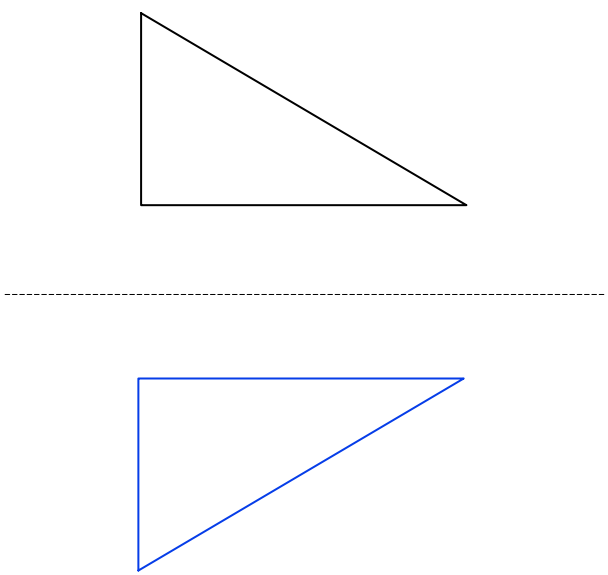
rotation

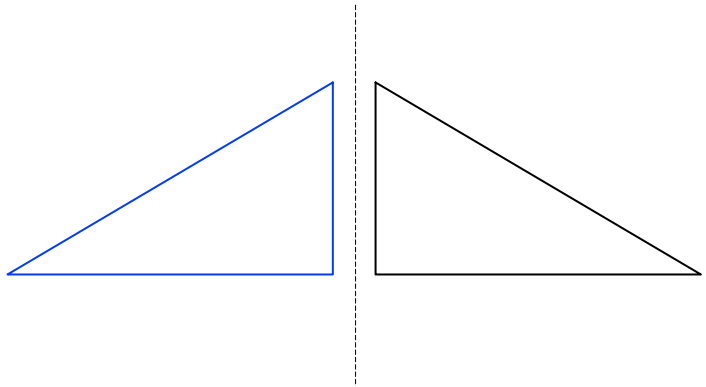


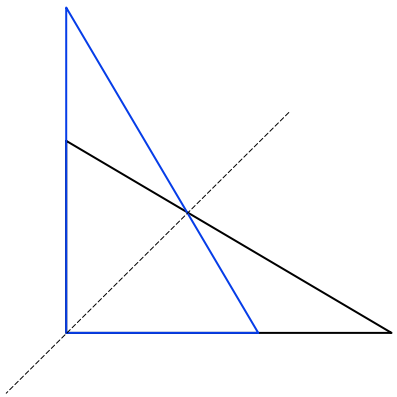




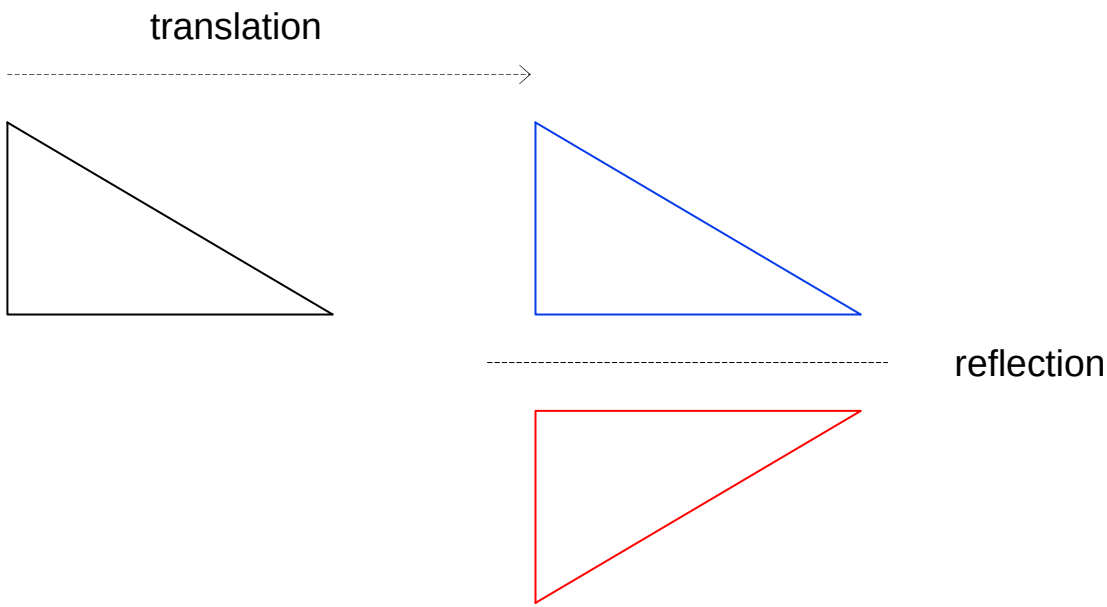
reflection





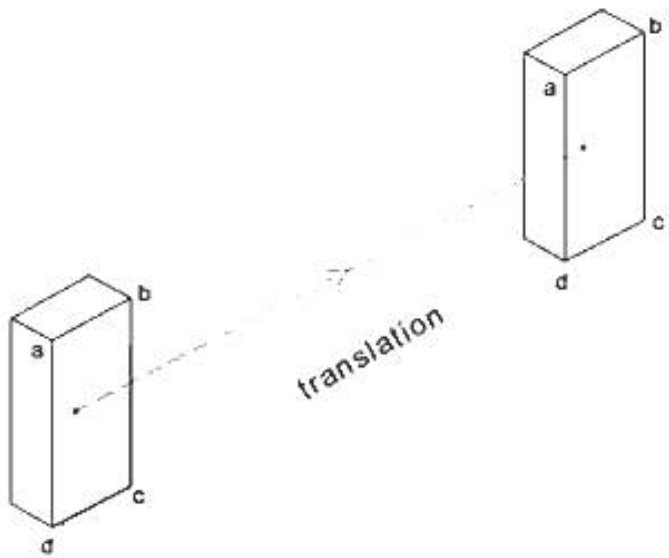


glide reflection

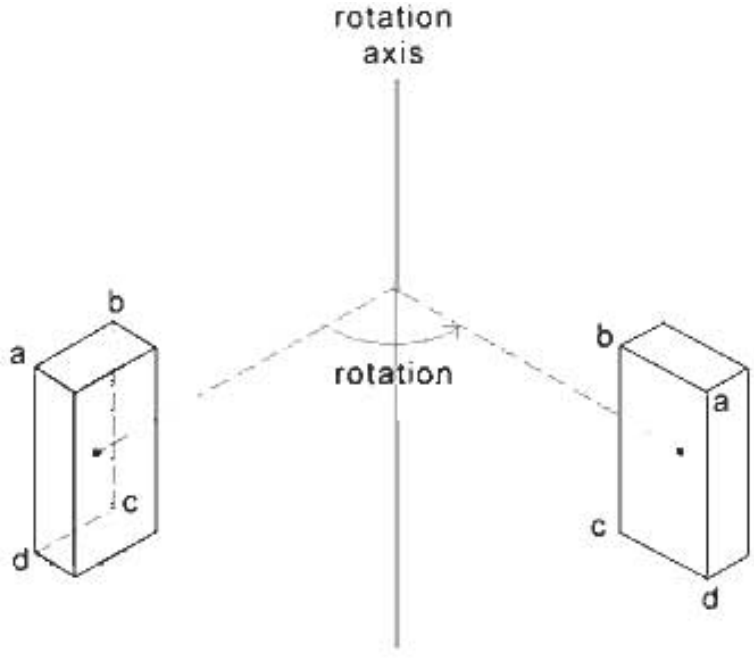


3D transformations

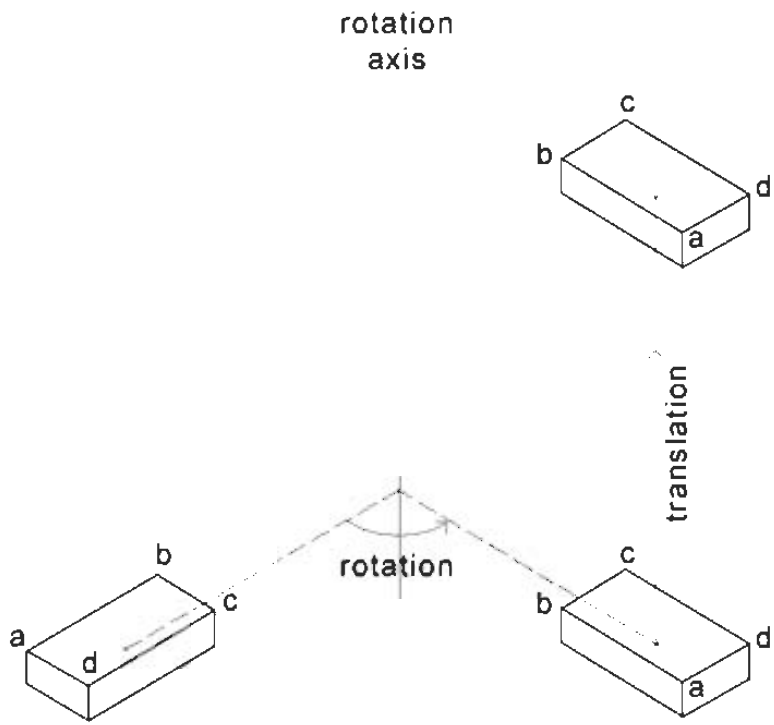
translation
rotation
screw rotation
reflection
glide reflection
rotor reflection
(scale)



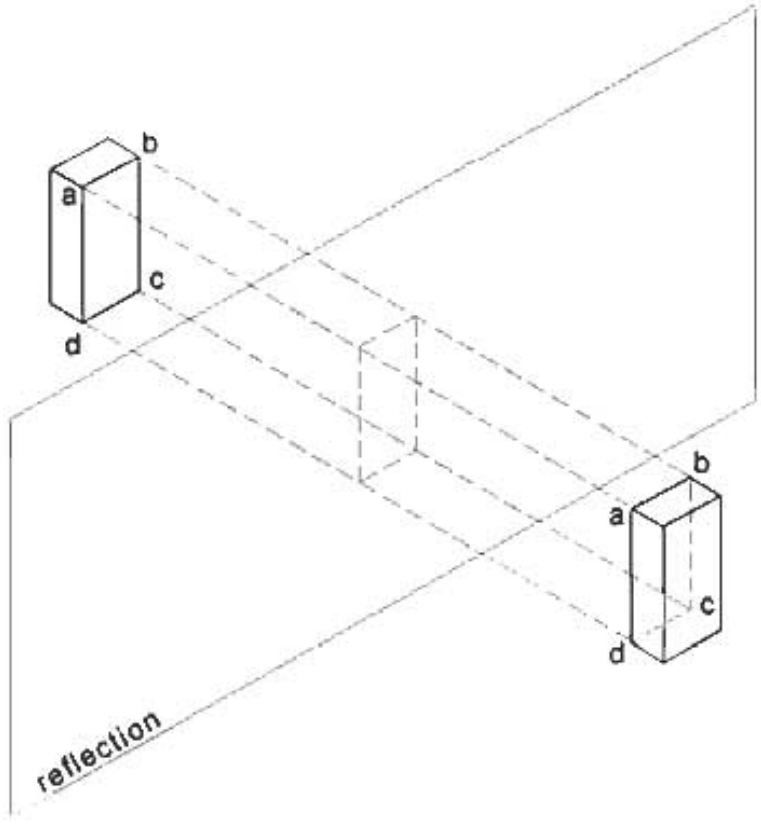
TRANSLATION



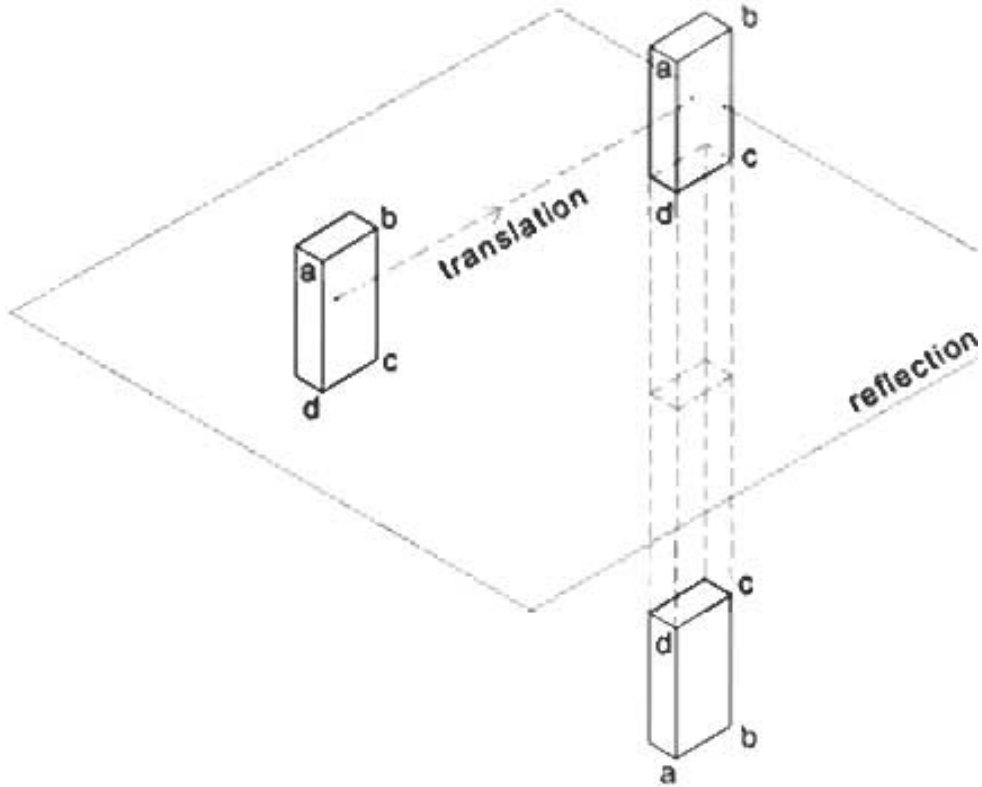
ROTATION



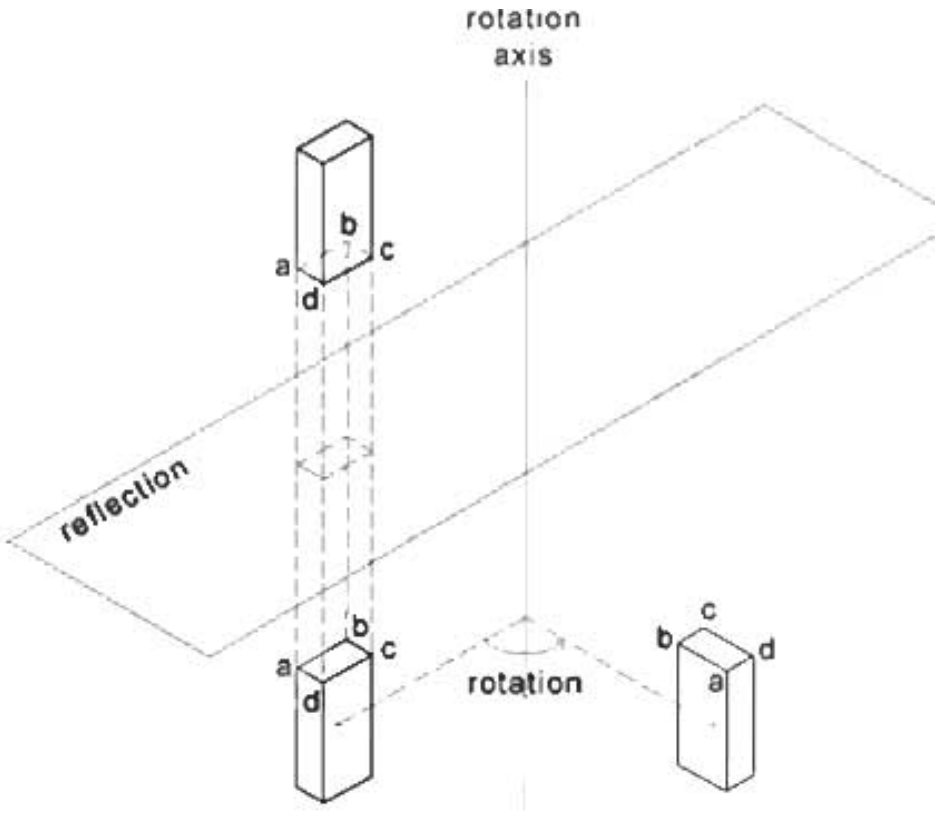
SCREW ROTATION



REFLECTION



GLIDE REFLECTION



ROTOR REFLECTION

2D	symmetry type	transformations
	point	rotations
	frieze	translations (one direction)
	wallpaper	translations (two directions)

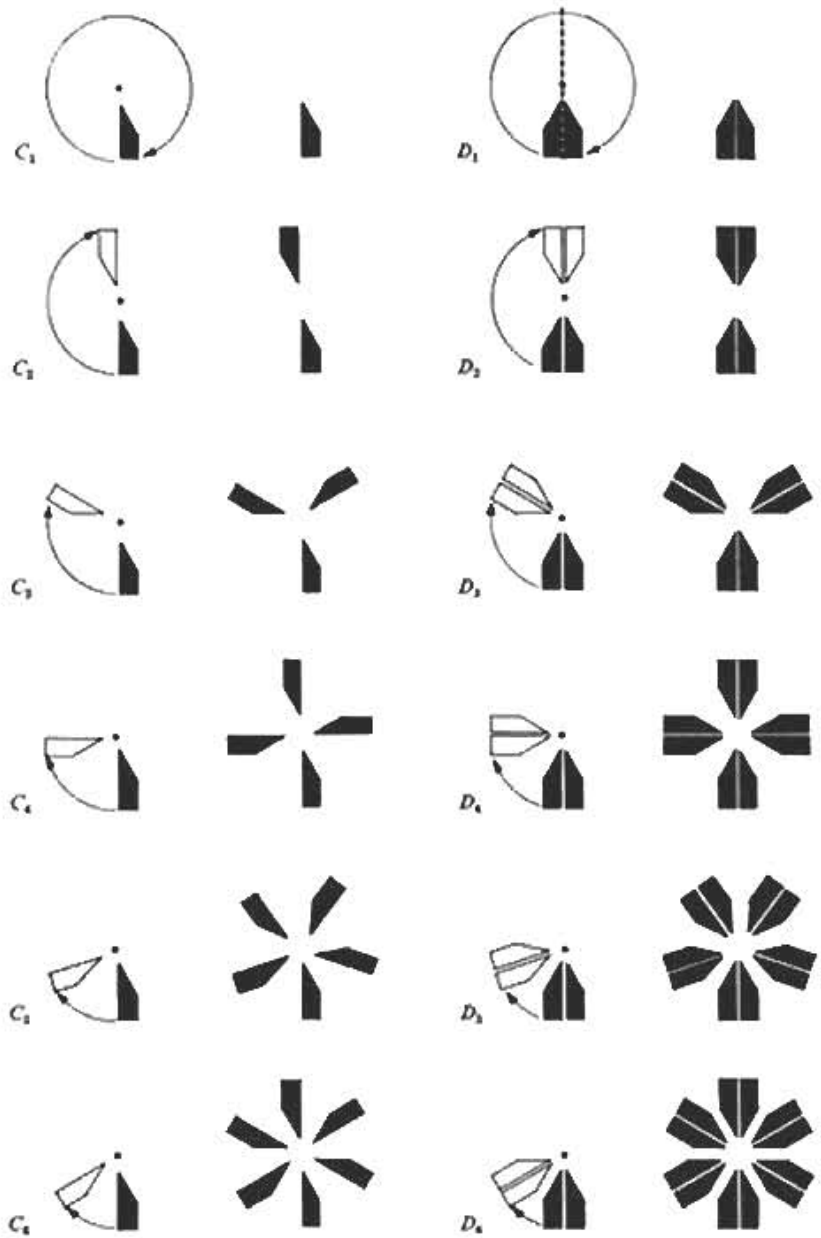
3D	symmetry type	transformations
	point	rotations
	rod	translations (one direction)
	layer	translations (two directions)
	space	translations (three directions)

2D (finite) point symmetry

rotations about a point

optional: reflections across an axis through the rotation point

The two point groups: the cyclic group, Cn, consists of rotations about a single point, O, through $2\pi/n$, while the dihedral group, Dn, includes a reflection through O.



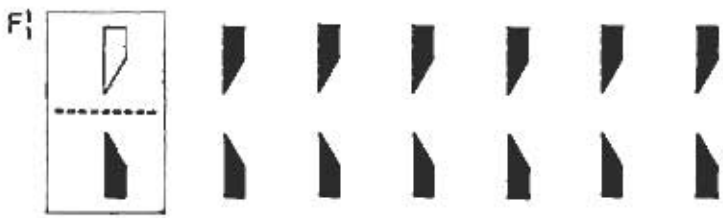
2D (infinite) frieze symmetry

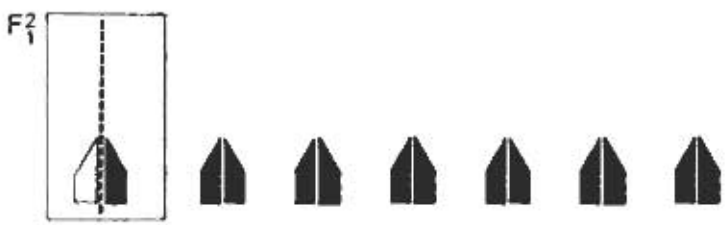
translations along a line

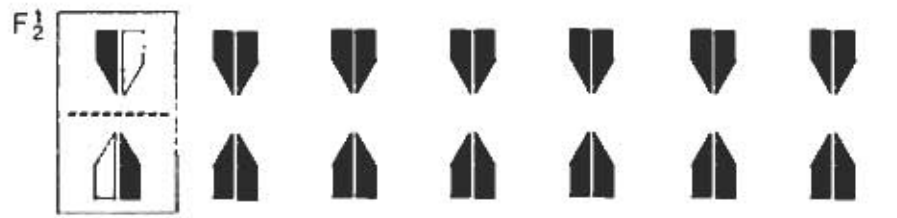
optional: rotations, reflections, glide reflections

The seven frieze groups:

F_1, translation along one axis
F_1^1, reflection in the axis
F_1^2, reflection perpendicular to the axis
F_1^3, reflection and glide translation
F_2, halfturn
F_2^1, reflection in the axis
F_2^2, glide reflection, or reflection in pairs of axes



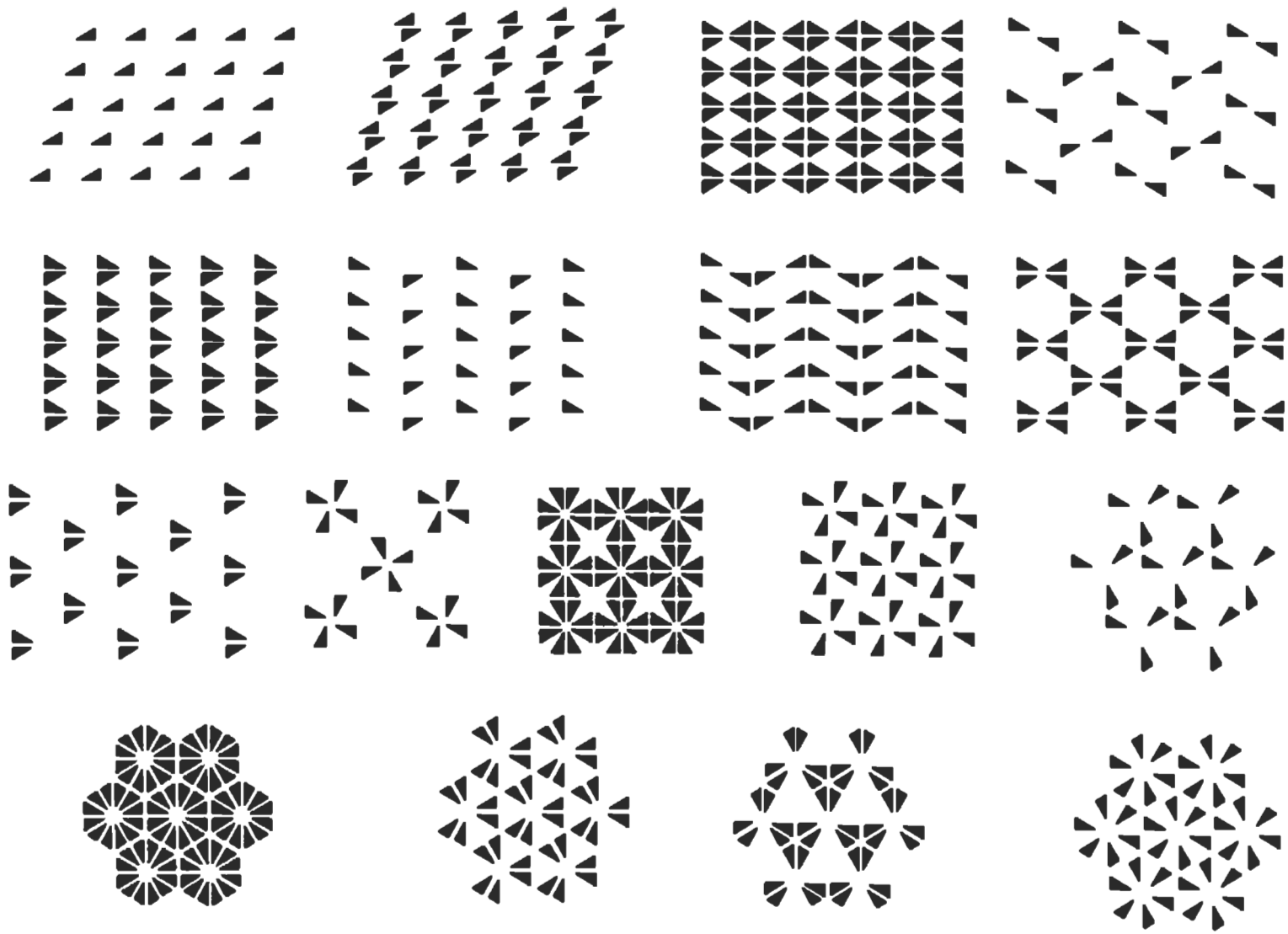




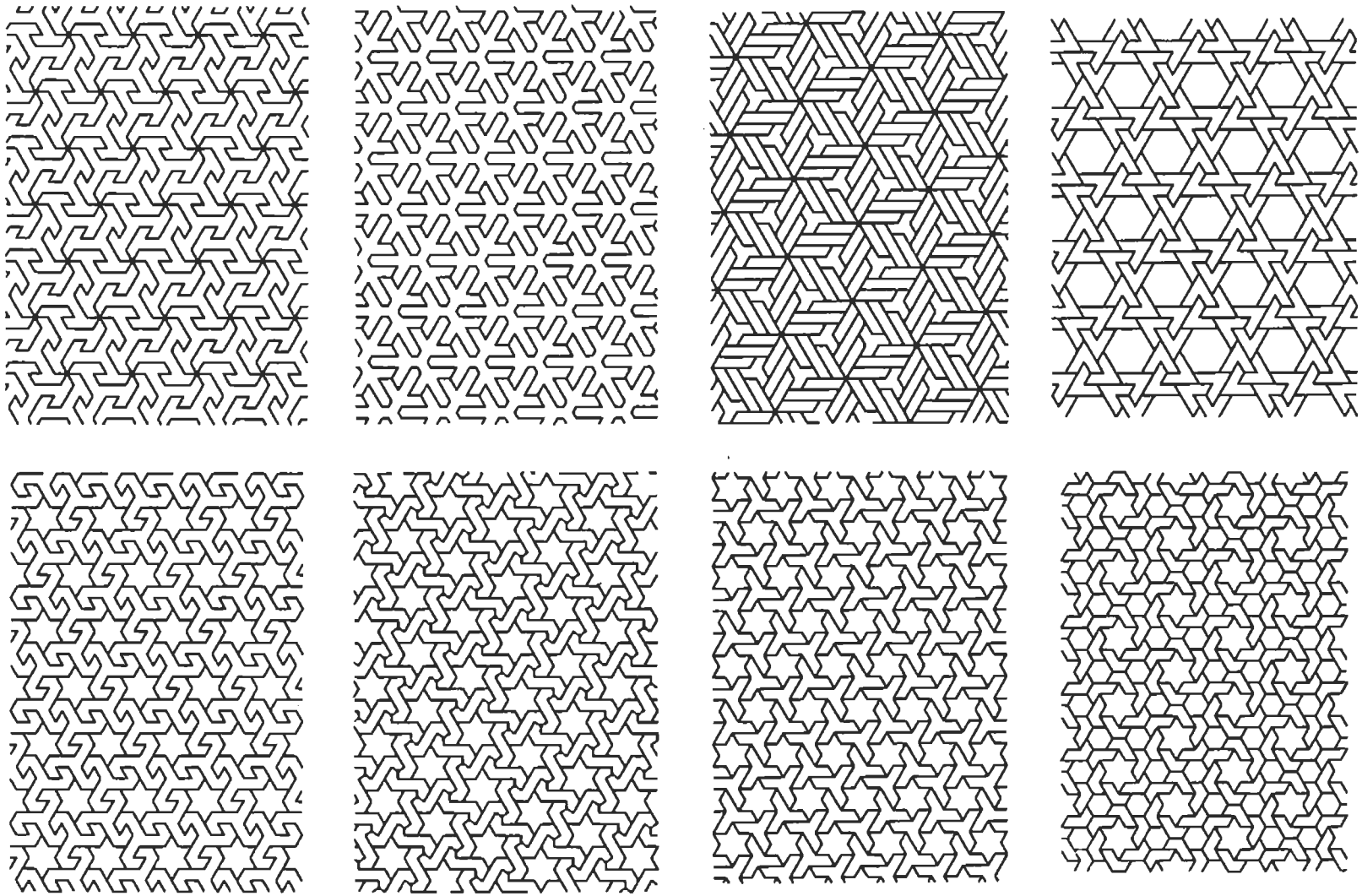
2D (infinite) wallpaper symmetry

translations along two lines (two directions)

optional: rotations, reflections, glide reflections



The seventeen wallpaper groups



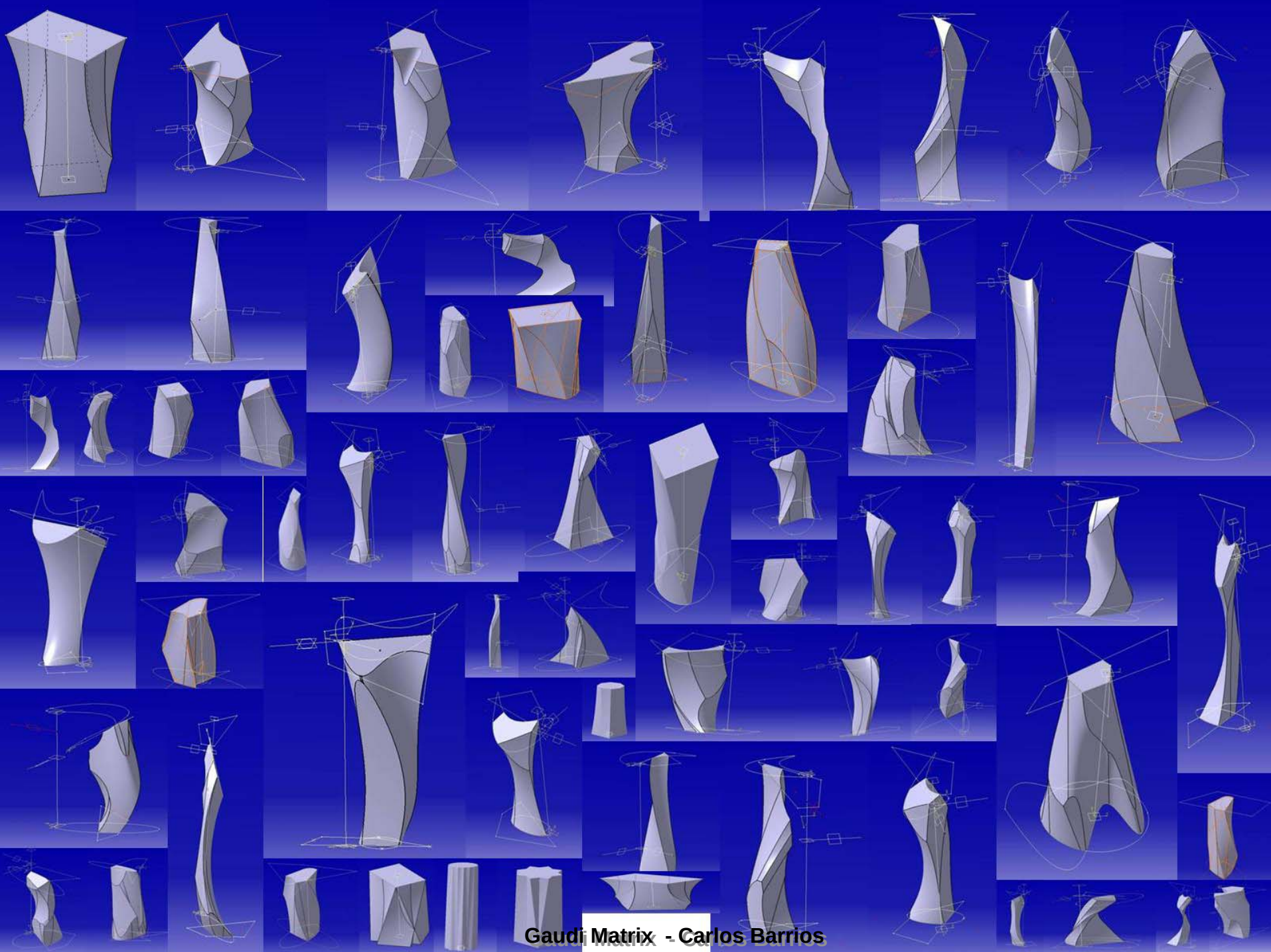
3D space

point rotations
optional: reflections, rotor reflections

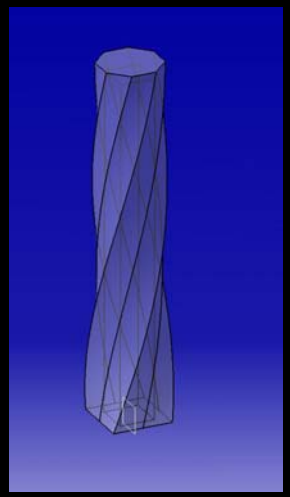
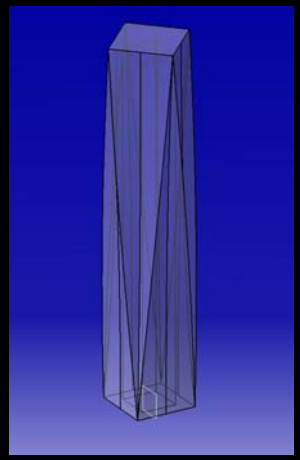
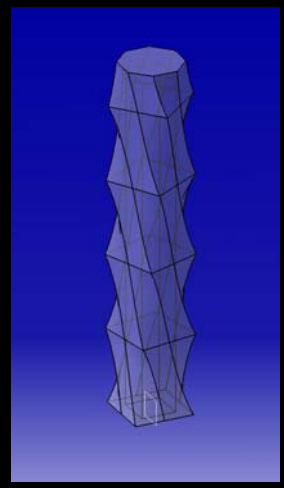
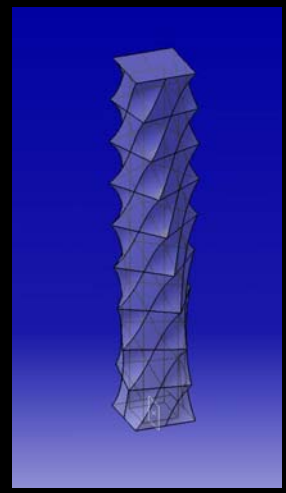
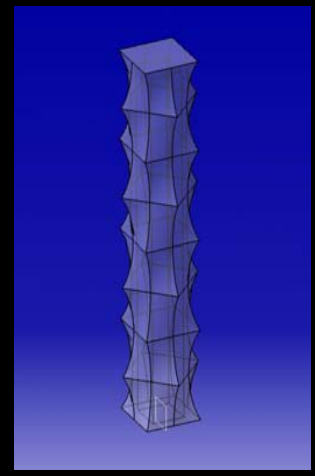
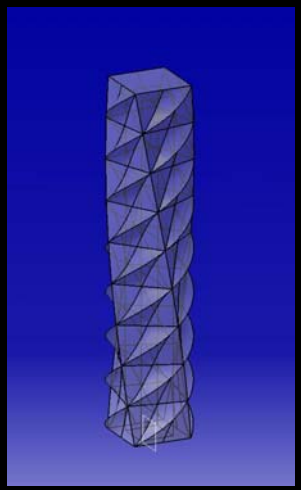
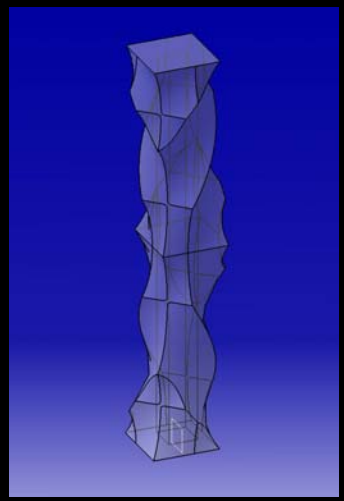
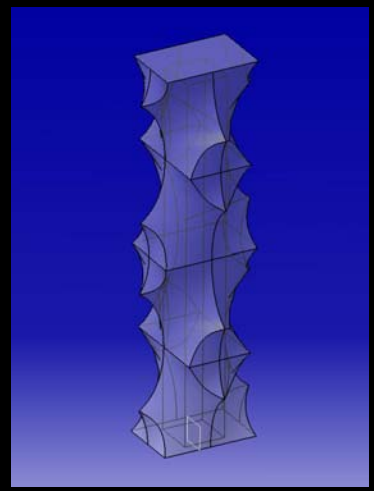
rod translations (one direction)
optional: rotations, reflections, rotor reflections
screw rotations, glide reflections

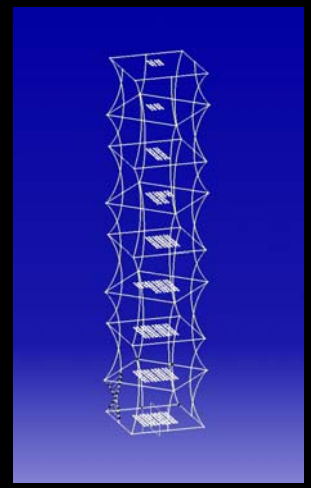
layer translations (two directions)
optional: rotations, reflections, rotor reflections
screw rotations, glide reflections

space translations (three directions)
optional: rotations, reflections, rotor reflections
screw rotations, glide reflections



Gaudi Matrix - Carlos Barrios



twisted tower prototypes - Carlos Barrios

3D space

point rotations
optional: reflections, rotor reflections

rod translations (one direction)
optional: rotations, reflections, rotor reflections
screw rotations, glide reflections

layer translations (two directions)
optional: rotations, reflections, rotor reflections
screw rotations, glide reflections

space translations (three directions)
optional: rotations, reflections, rotor reflections
screw rotations, glide reflections

symmetry group

set of *transformations* that leave an object invariant

(looking exactly the same -- same position, size, and orientation -- before and after the transformation)

order of symmetry group

number of transformations in the group

basic grammar

shapes

↓

spatial relations

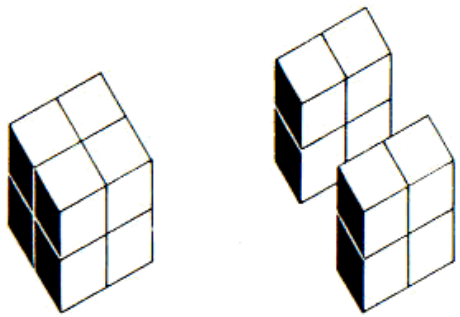
↓

rules

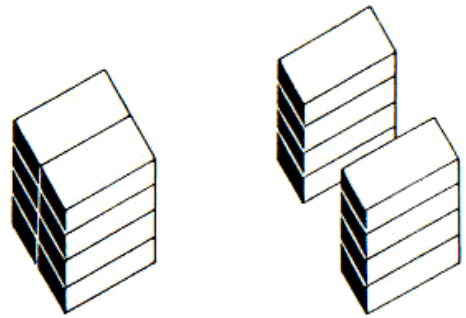
↓

shape grammar

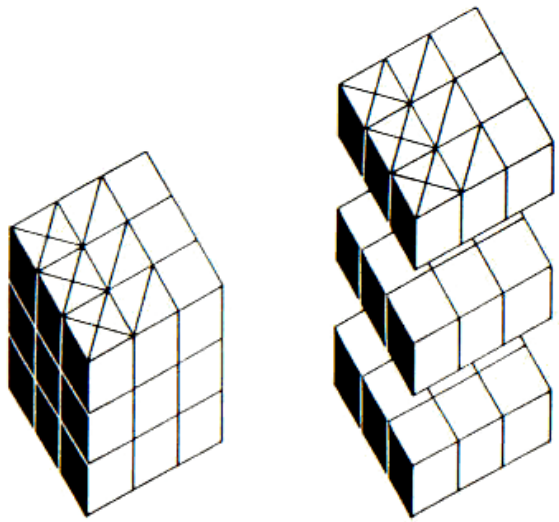
Froebel building gifts



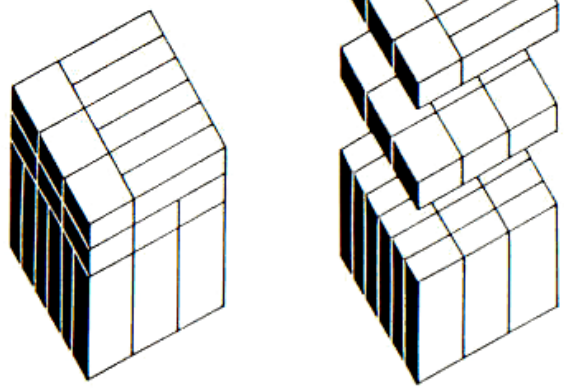
Gift 3



Gift 4

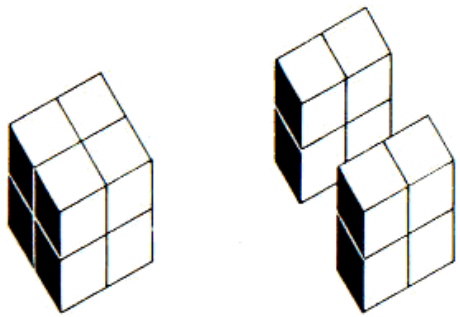


Gift 5

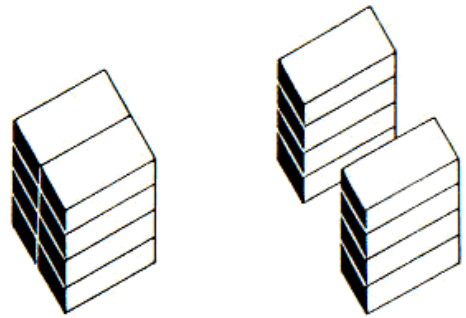


Gift 6

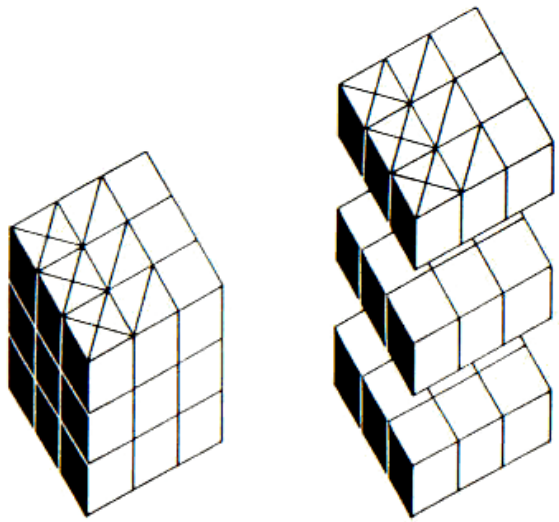
Froebel building gifts



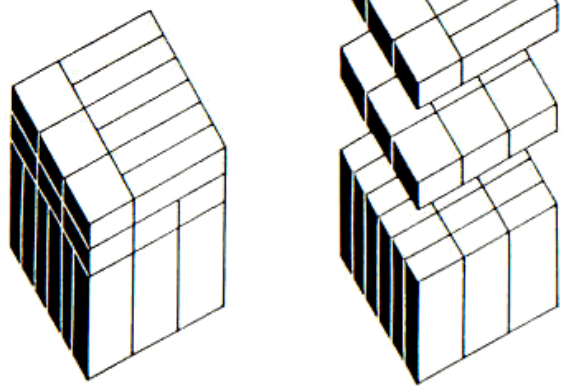
Gift 3



Gift 4



Gift 5



Gift 6

basic grammar

shapes

↓

spatial relations

↓

rules

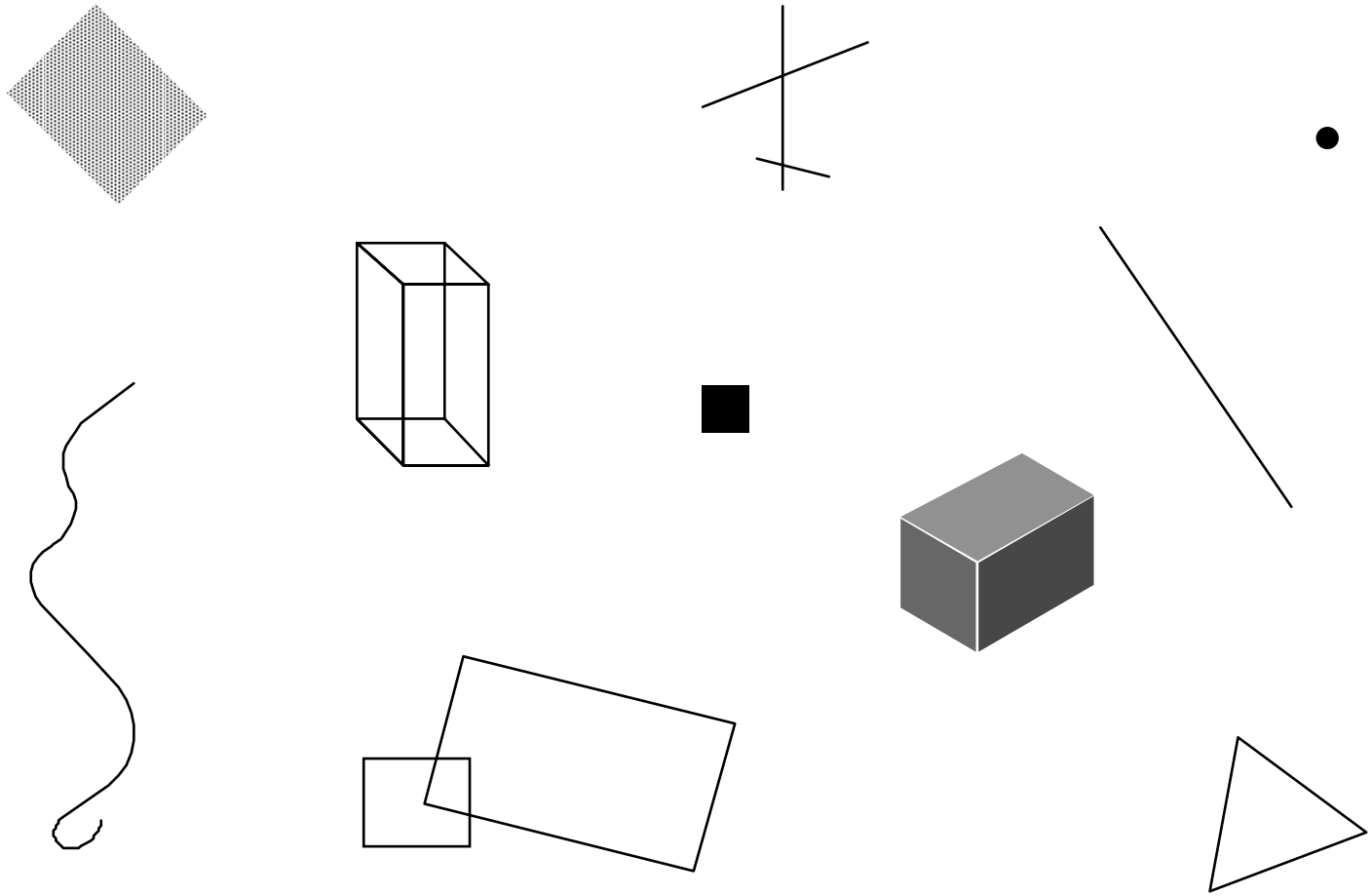
↓

shape grammar

shape

arrangement of basic elements in space

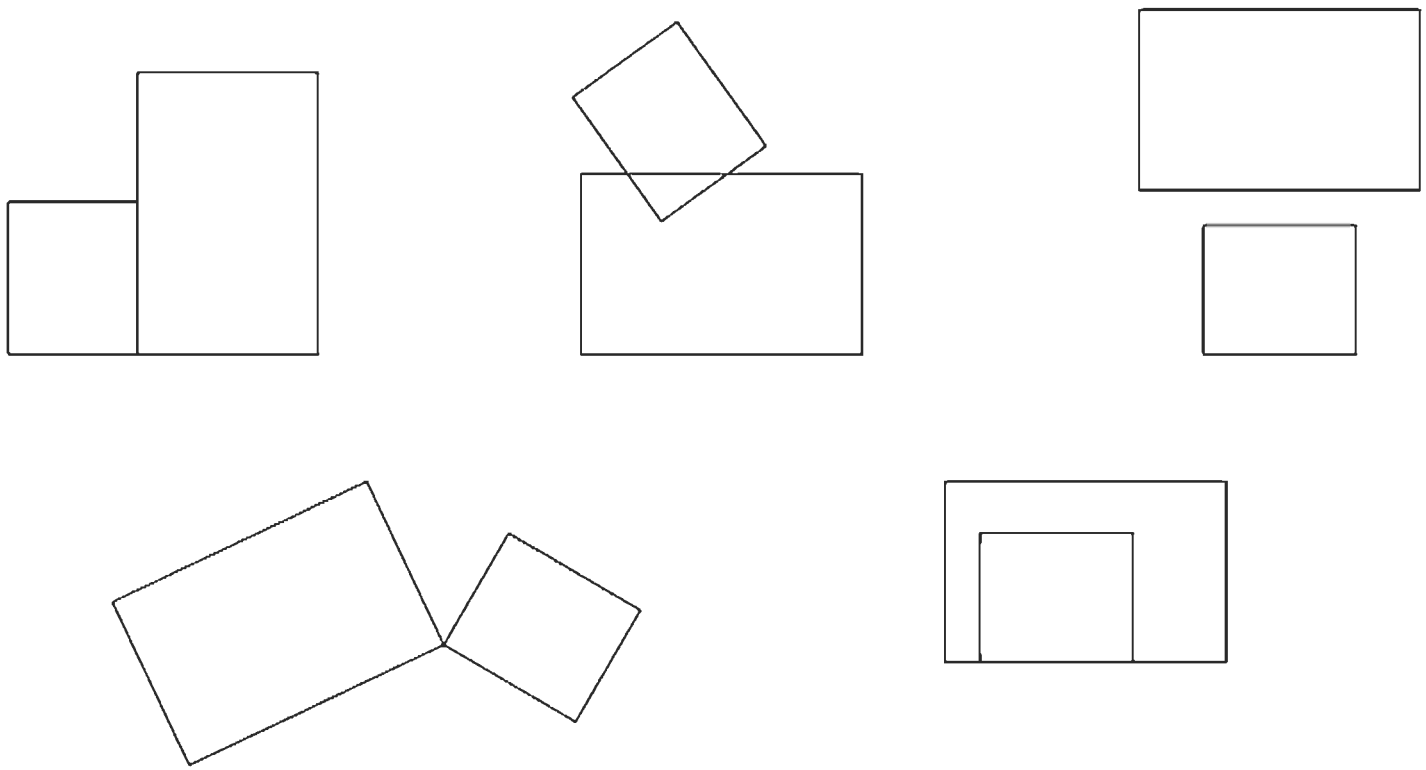
shapes



spatial relation

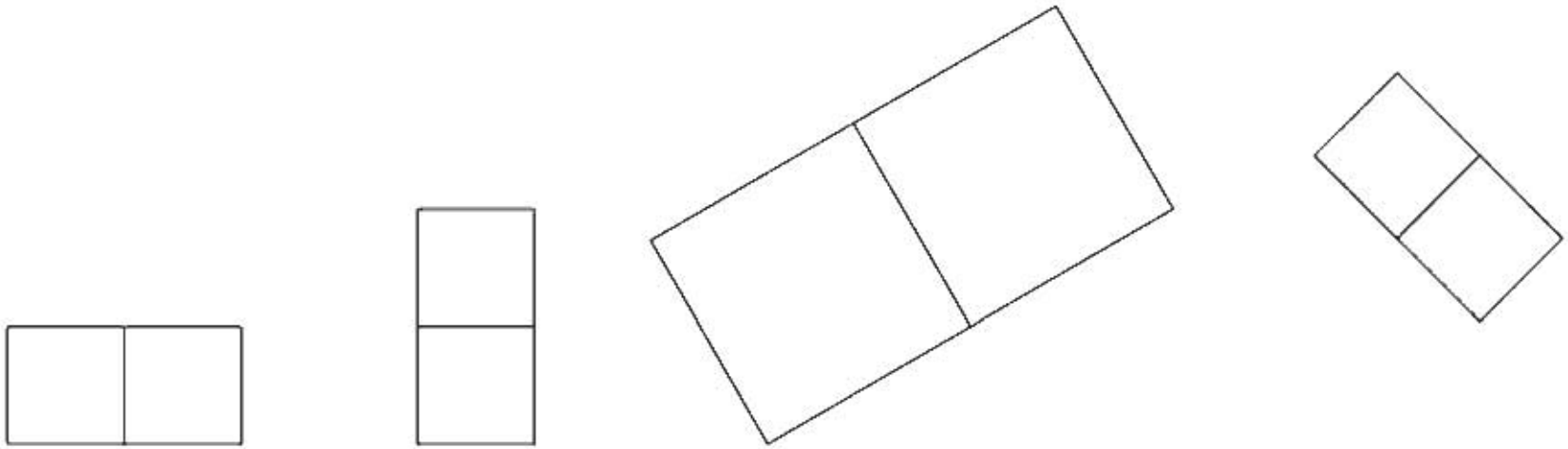
an arrangement of shapes (in 2D or 3D)

spatial relations



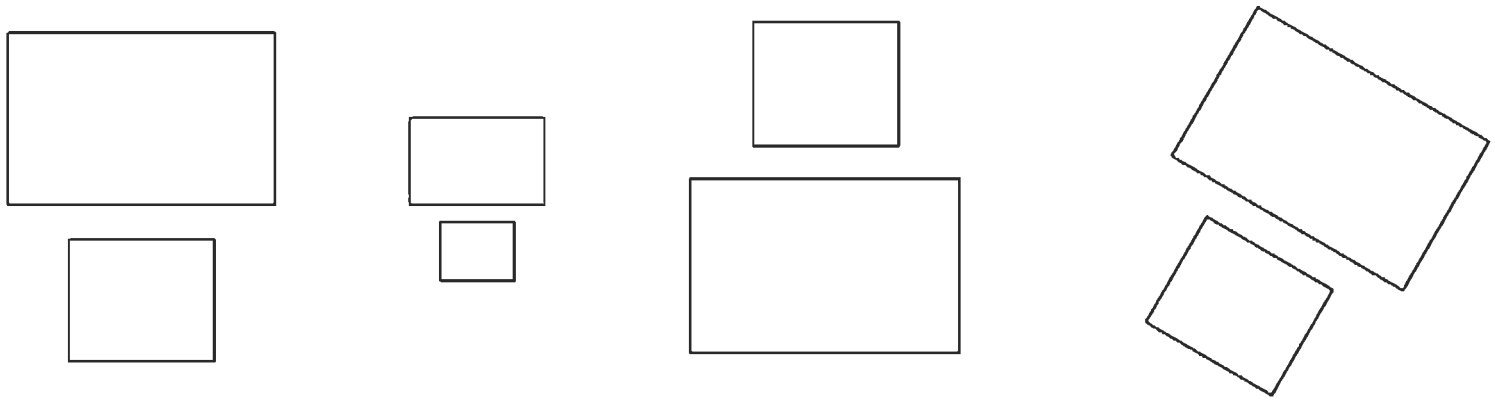
shapes can be arranged in any way to make a spatial relation

a spatial relation is denoted as **A + B**

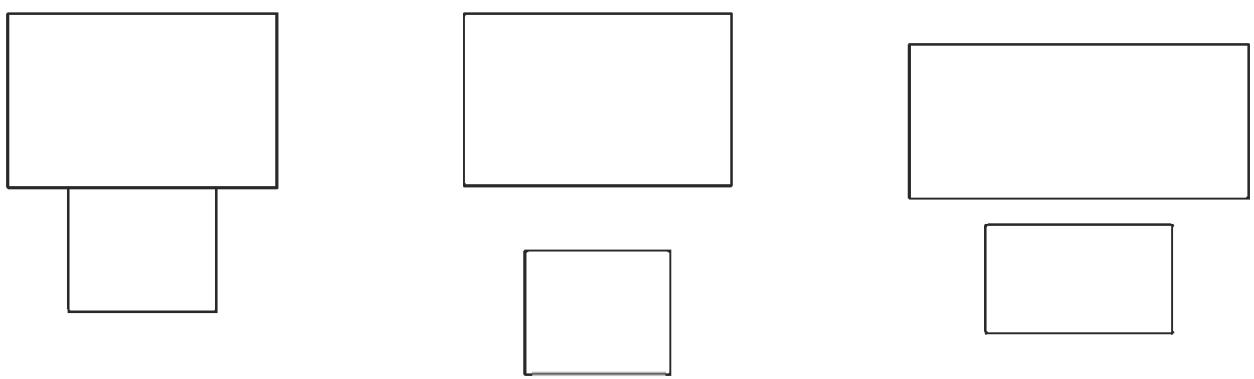


spatial relations that are the same

spatial relations that are different



spatial relations that are the same



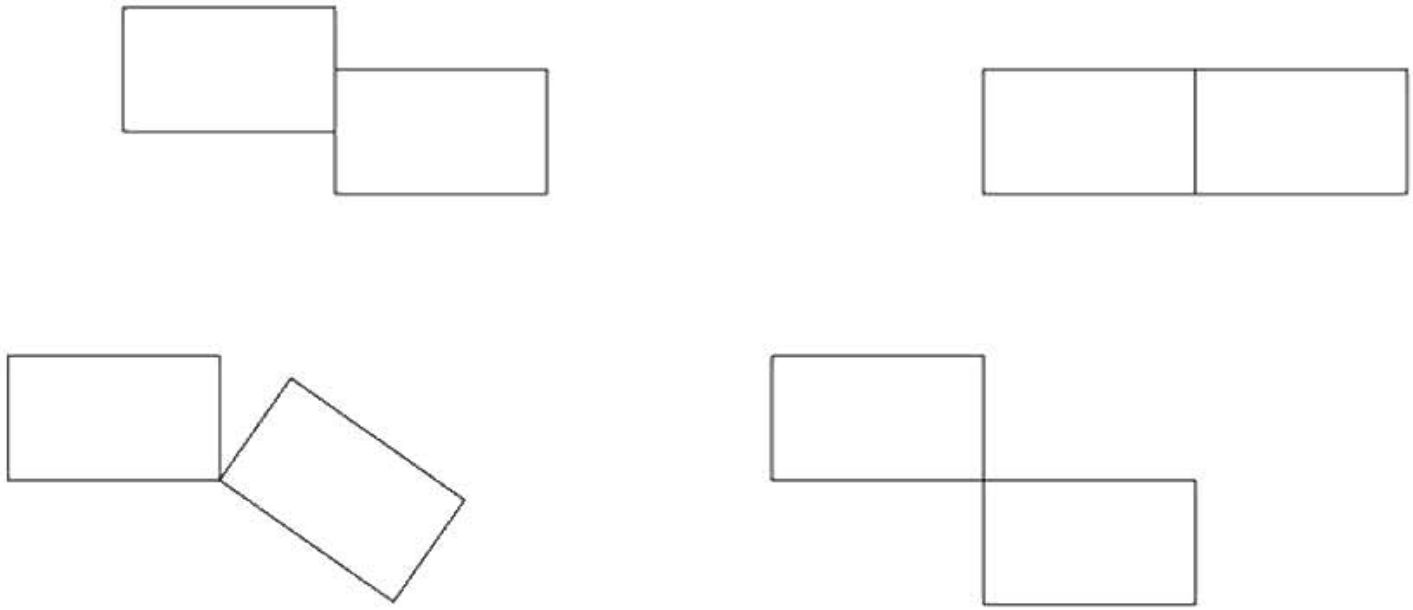
spatial relations that are different

A spatial relation A+B is the **same** as another spatial relation C+D whenever there is a transformation *t* such that:

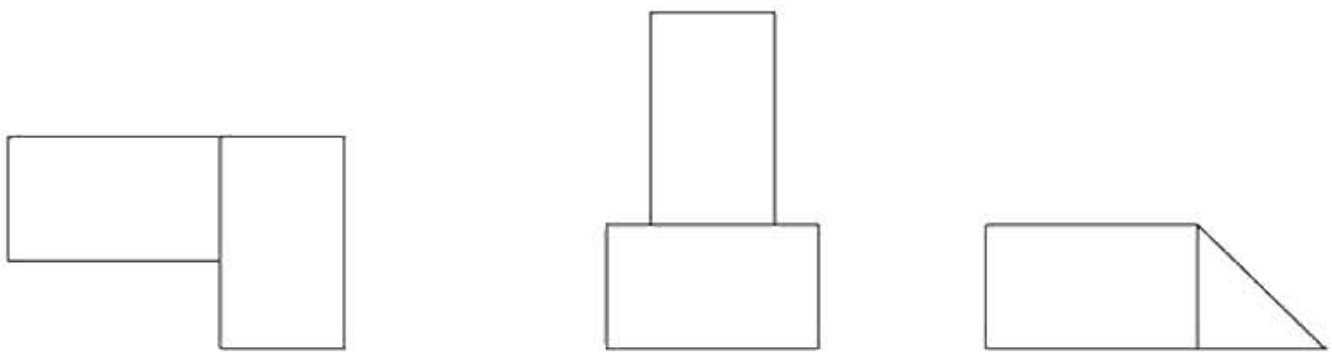
t(A) = C and *t*(B) = D

or

t(A) = D and *t*(B) = C



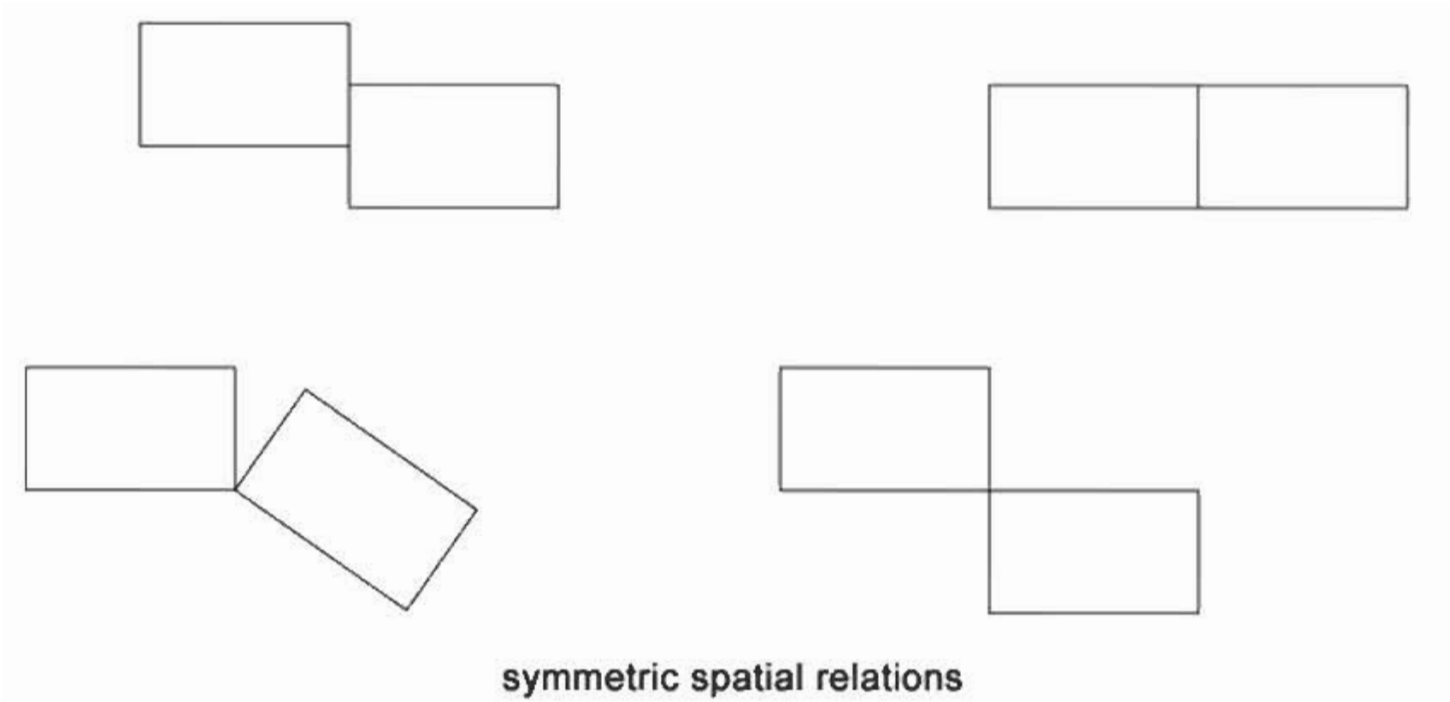
symmetric spatial relations



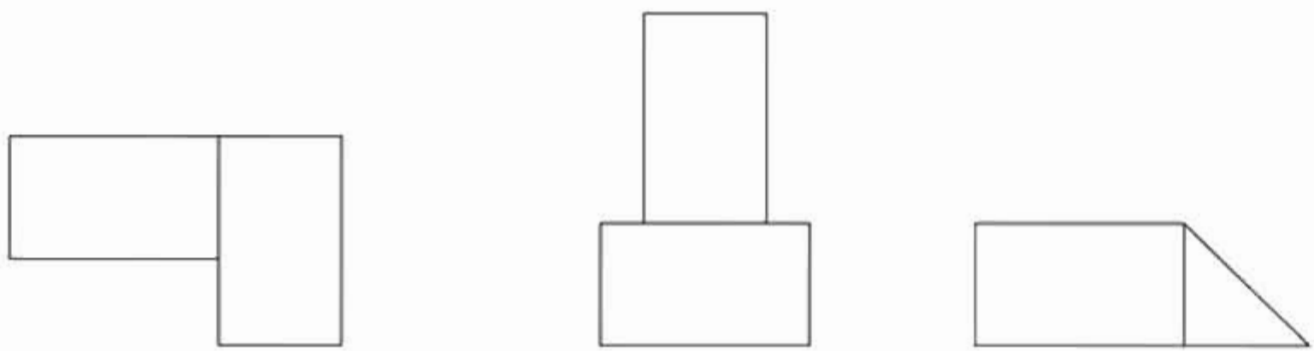
nonsymmetric spatial relations

A spatial relation is A+B is **symmetric**
whenever there is a transformation *t* such that:

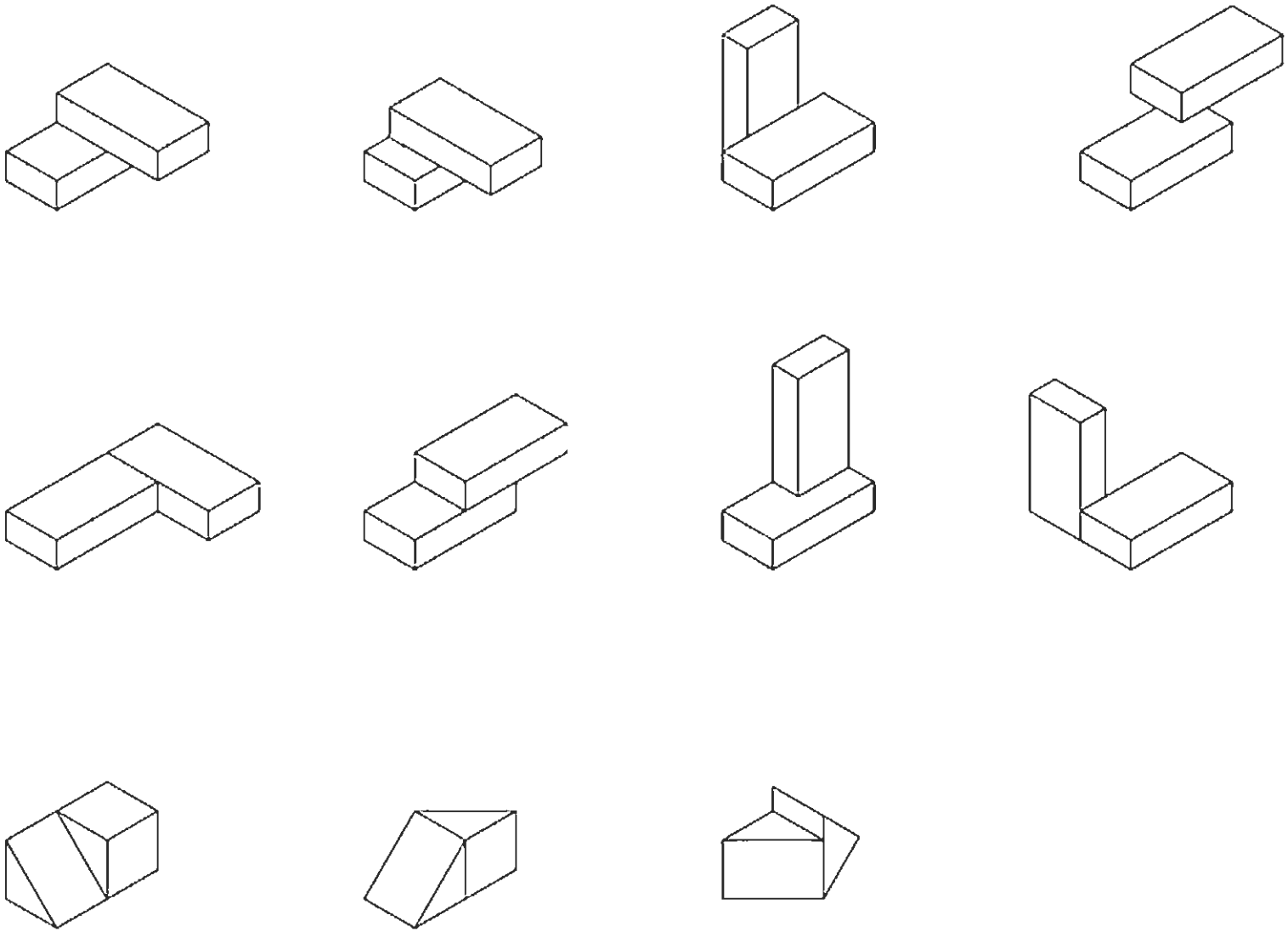
t(A) = B and *t*(B) = A



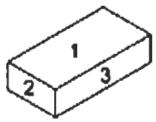
symmetric spatial relations



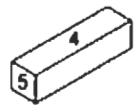
nonsymmetric spatial relations



spatial relations: symmetric and nonsymmetric



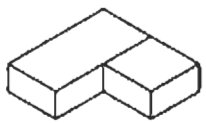
oblong

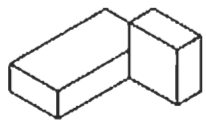


pillar

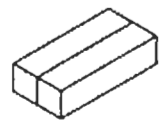
square

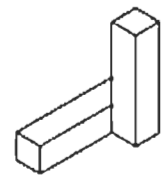
gift 6



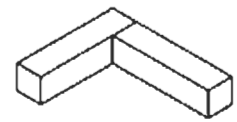


oblong, square: 3, 7

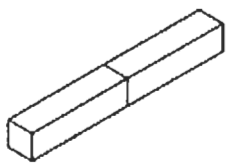




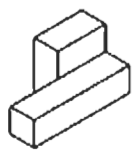
pillar, pillar: 4, 4



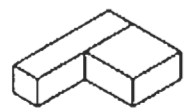
pillar, pillar: 4, 5

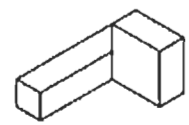


pillar, pillar: 5, 5

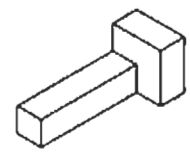


pillar, square: 4, 6

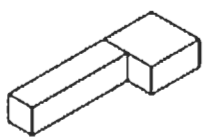




pillar, square: 4, 7

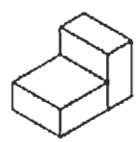


pillar, square: 5, 6

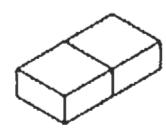


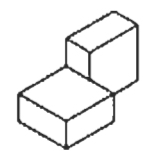
pillar, square: 5, 7

square, square: 6, 6



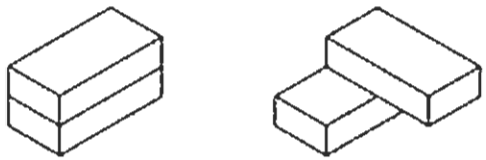
square, square: 6, 7



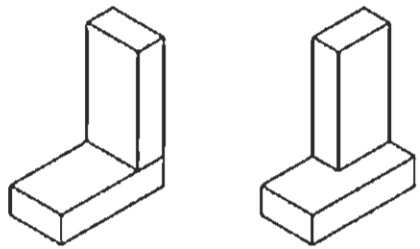


square, square: 7, 7

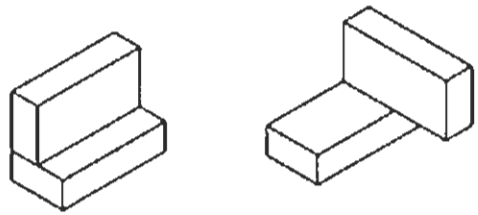
spatial relations for gift 6



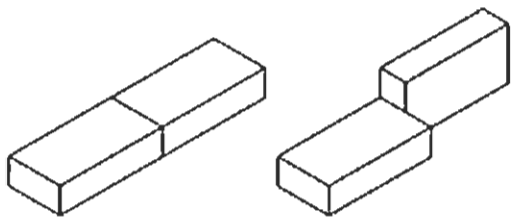
oblong, oblong: 1, 1



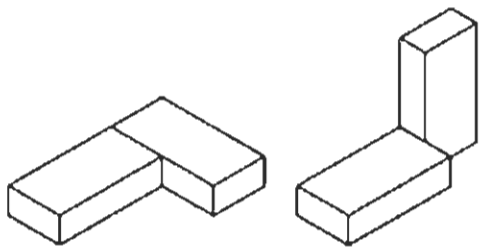
oblong, oblong: 1, 2



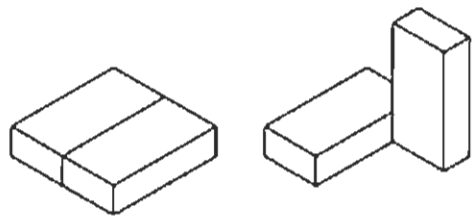
oblong, oblong: 1, 3



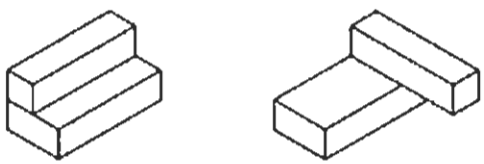
oblong, oblong: 2, 2



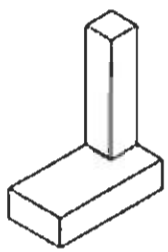
oblong, oblong: 2, 3



oblong, oblong: 3, 3

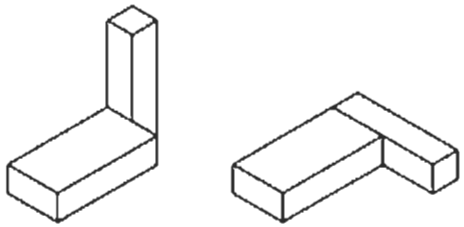


oblong, pillar: 1, 4

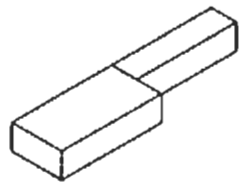


oblong, pillar: 1, 5

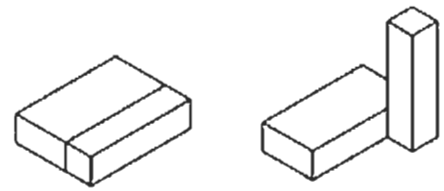
spatial relations for gift 6



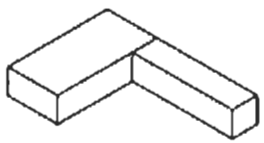
oblong, pillar: 2, 4



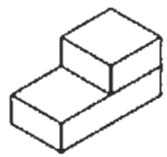
oblong, pillar: 2, 5



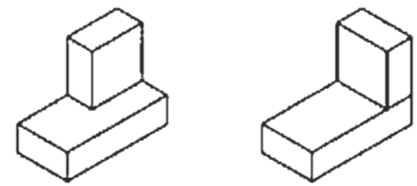
oblong, pillar: 3, 4



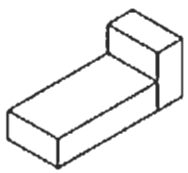
oblong, pillar: 3, 5



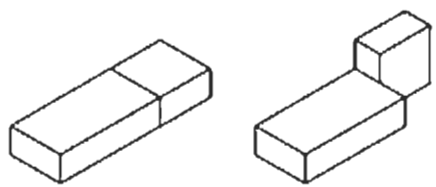
oblong, square: 1, 6



oblong, square: 1, 7



oblong, square: 2, 6



oblong, square: 2, 7

oblong, square: 3, 6

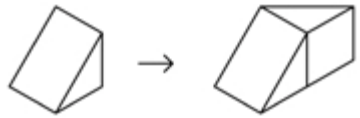
spatial relations for gift 6

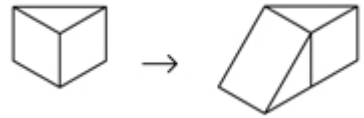
spatial relation A + B

addition rules	subtraction rules
$A \rightarrow A + B$	$A + B \rightarrow A$
$B \rightarrow A + B$	$A + B \rightarrow B$

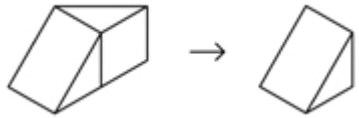
spatial relation

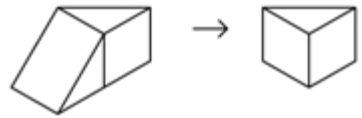
addition rules

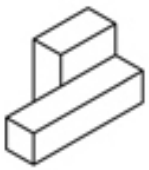




subtraction rules

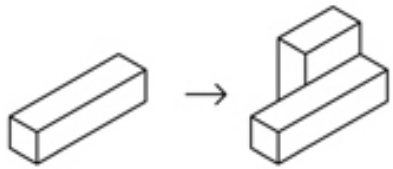


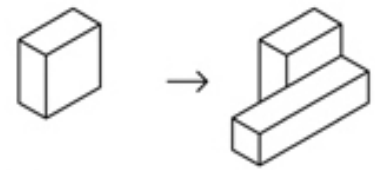




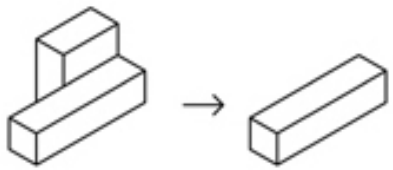
spatial relation

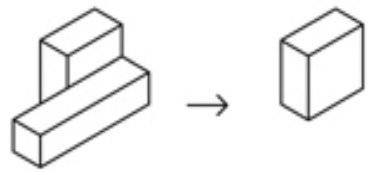
addition rules





subtraction rules





shape rule: $X \rightarrow Y$ **design**

A rule applies to a design:
whenever there is a transformation t that makes the left-side X
a part of the design: $t(X) \leq$ design

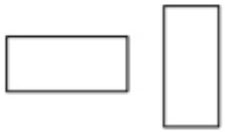
To apply the rule:
first subtract the transformation t of the left-side X from the design,
and then add the same transformation t of the right-side Y to the design.

The result of applying the rule is a new design:
new design = [design - $t(X)$] + $t(Y)$

applying a rule $A \rightarrow A + B$

match the shape A
with a shape in a design

add the shape B to the design
to create the spatial relation A+B

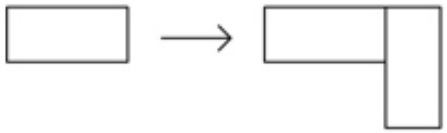


shapes

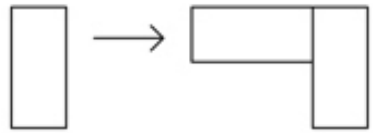
spatial relation

addition rules (+)

1

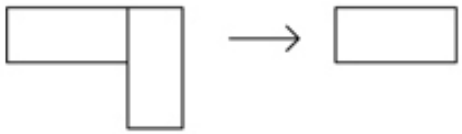


2

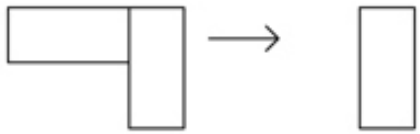


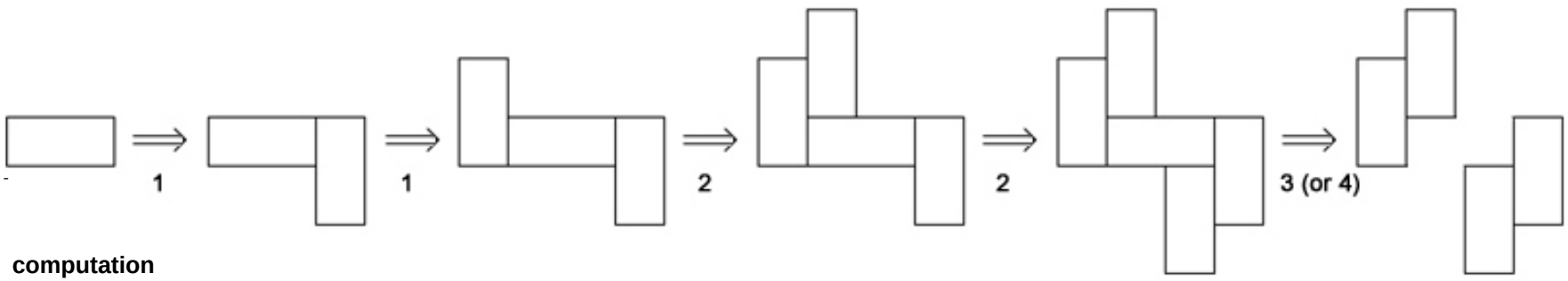
subtraction rules (-)

3

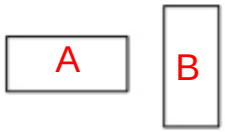


4

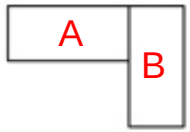




computation

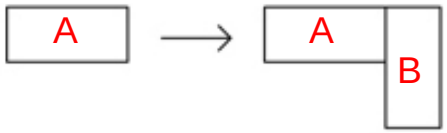


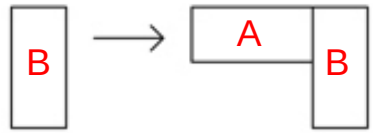
shapes



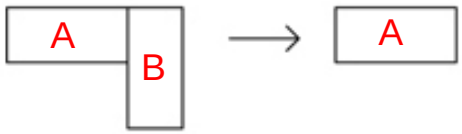
spatial relation

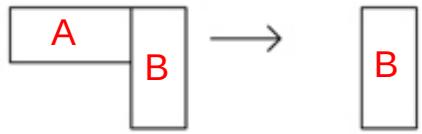
addition rules (+)

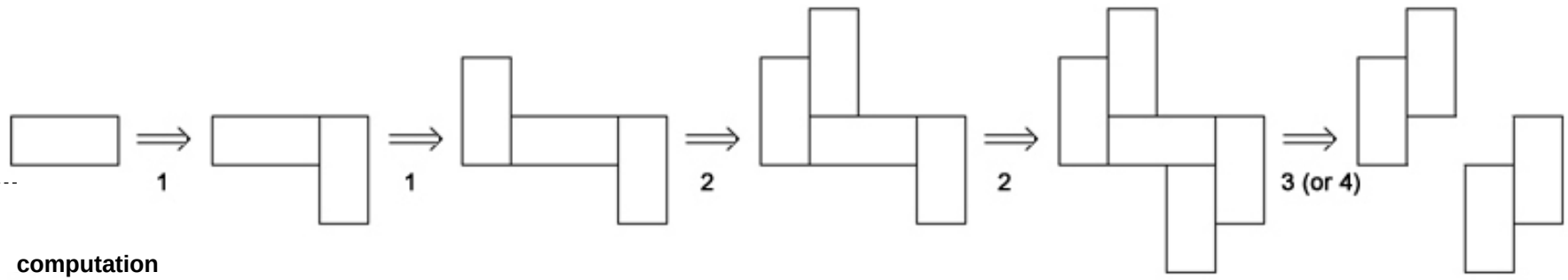
1 

2 

subtraction rules (-)

3 

4 



computation

nondeterminism

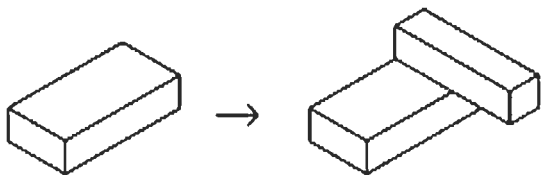
which rule to apply

↓

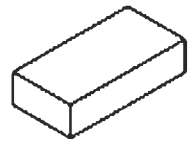
where to apply the rule

↓

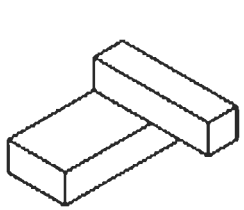
how to apply the rule

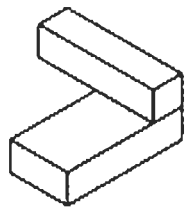


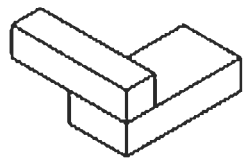
(a)

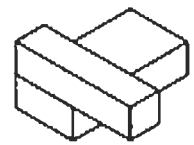


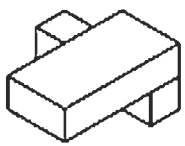
(b)

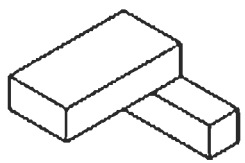


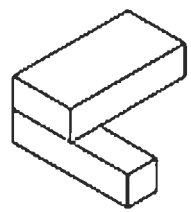


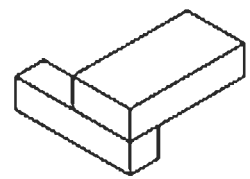












(c)

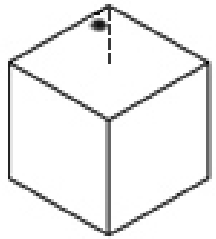
The addition rule in (a) applies to the oblong in (b) under different transformations to generate the different designs in (c).

Questions about nondeterminism

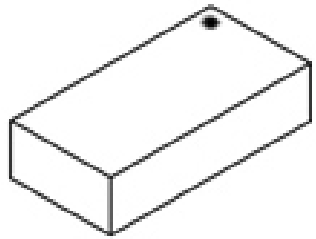
Given a rule and a shape to which it applies:

1 How many different ways does the rule apply (with how many different results)?

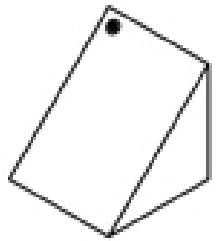
2 Can the rule be restricted to apply in particular ways?



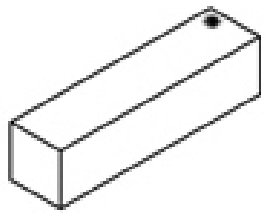
cube (48)



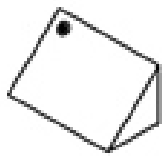
oblong (8)



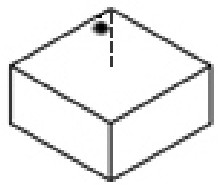
half-cube (4)



pillar (16)

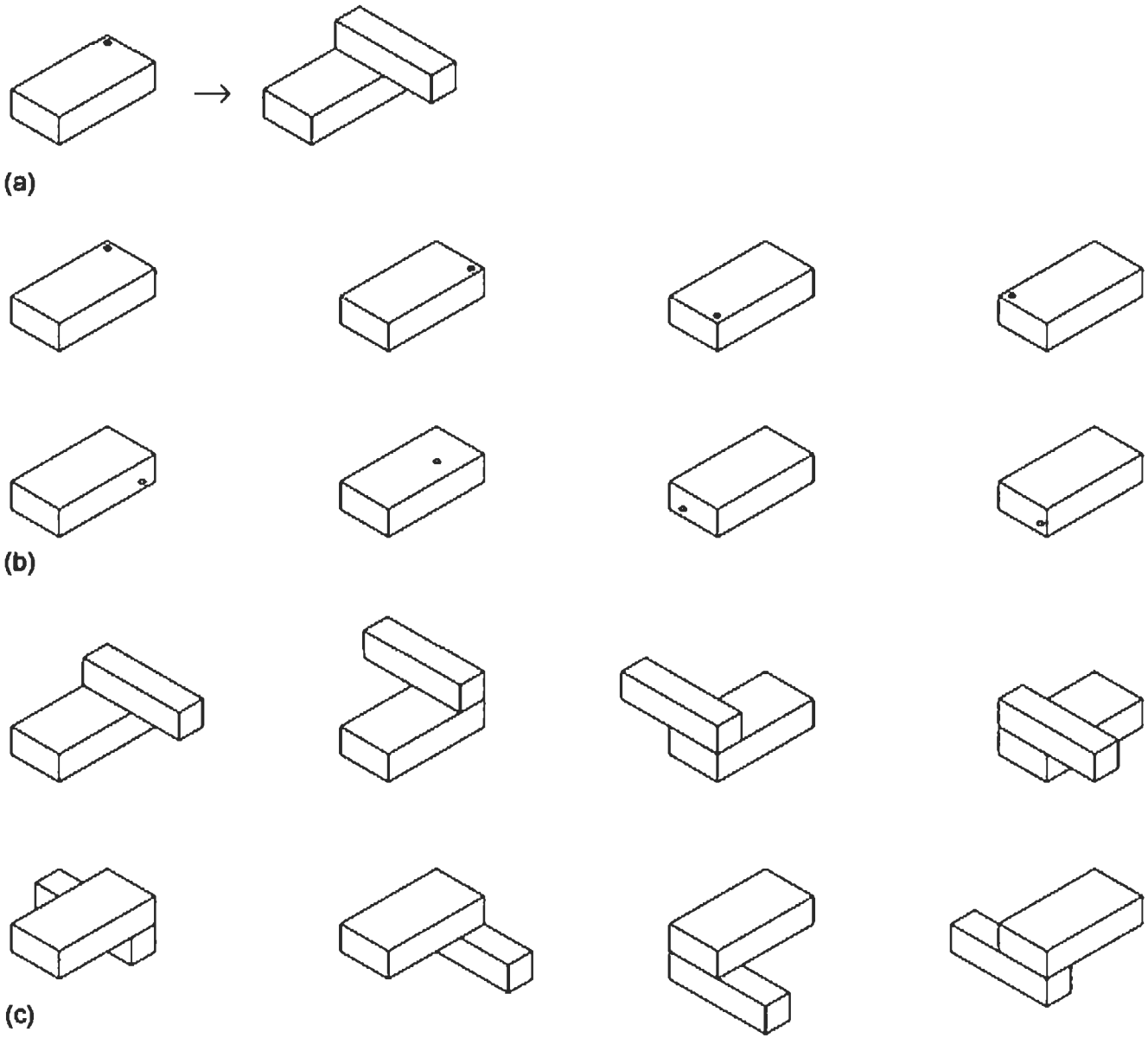


quarter-cube (4)

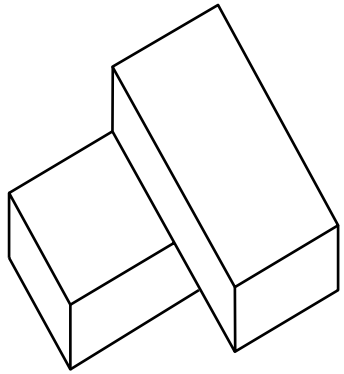


square (16)

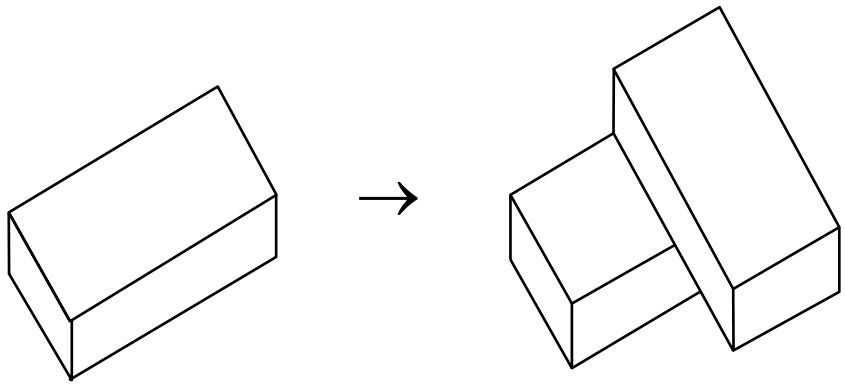
The Froebel blocks labeled according to their symmetries



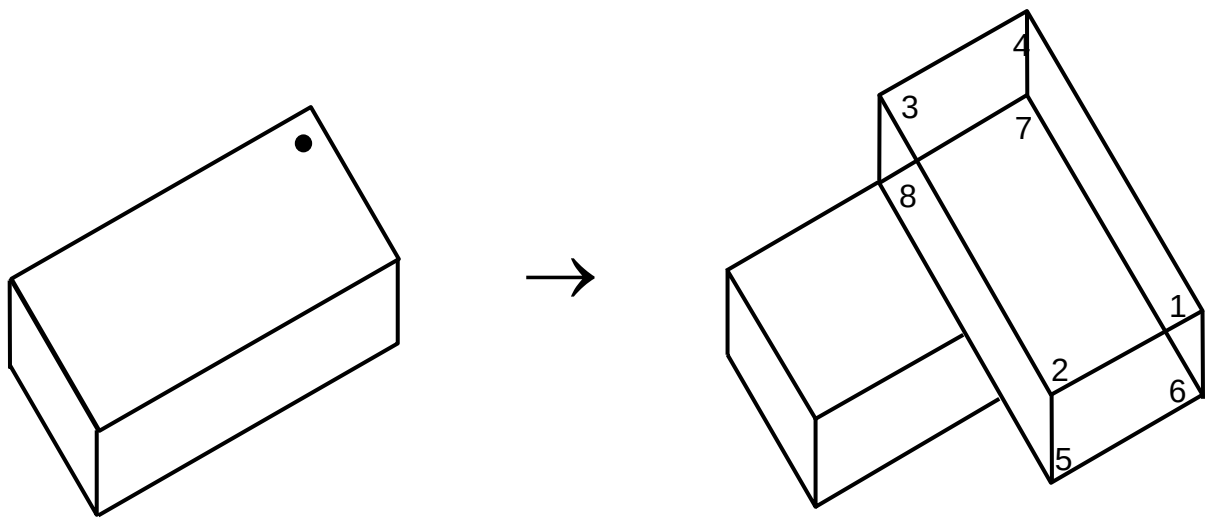
Applications of an addition rule labeled according to the symmetry group of the shape on the left-side of the rule. The labeled rule in (a) applies to each of the labeled oblongs in (b) to generate the different designs in (c)



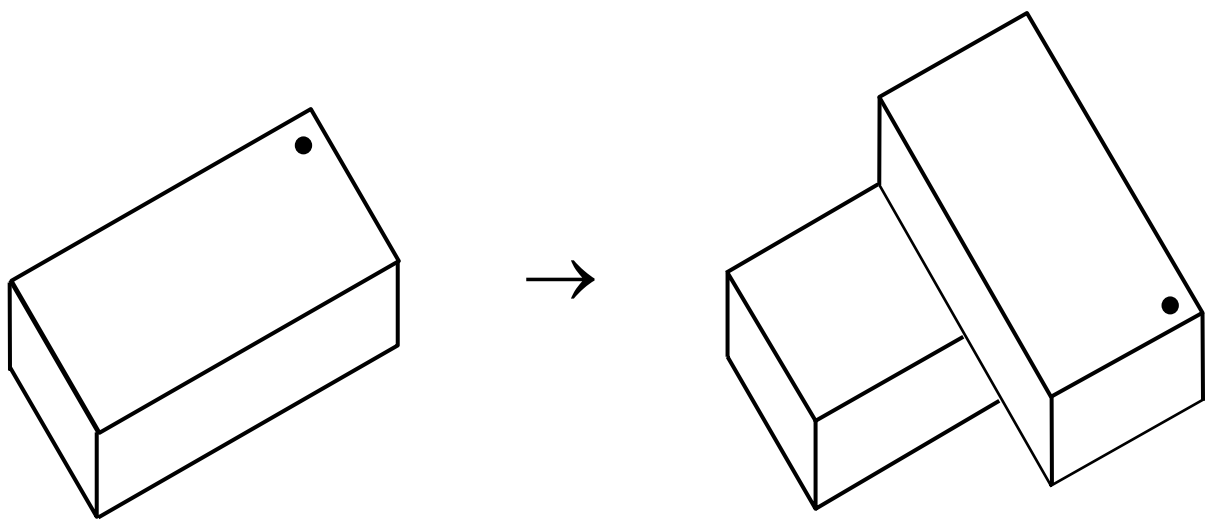
spatial relation A + B between two oblongs

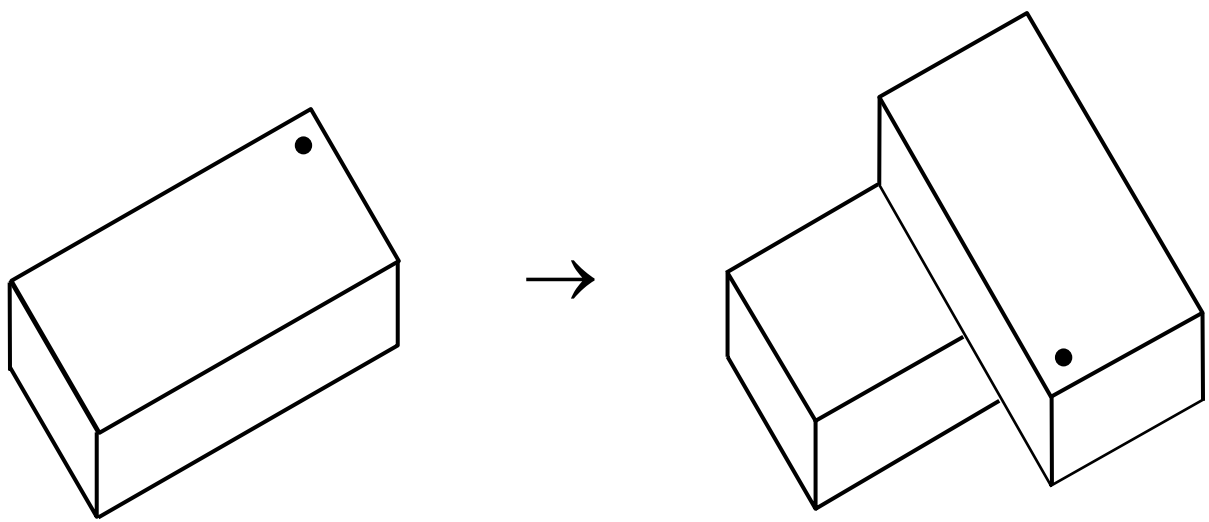


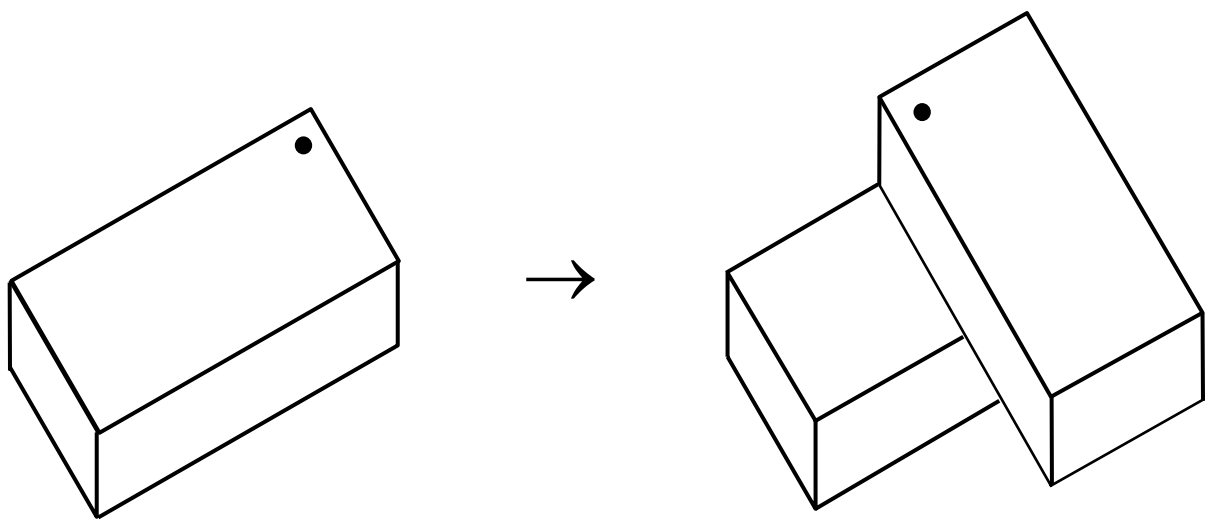
addition rule $A \rightarrow A + B$ based on the spatial relation

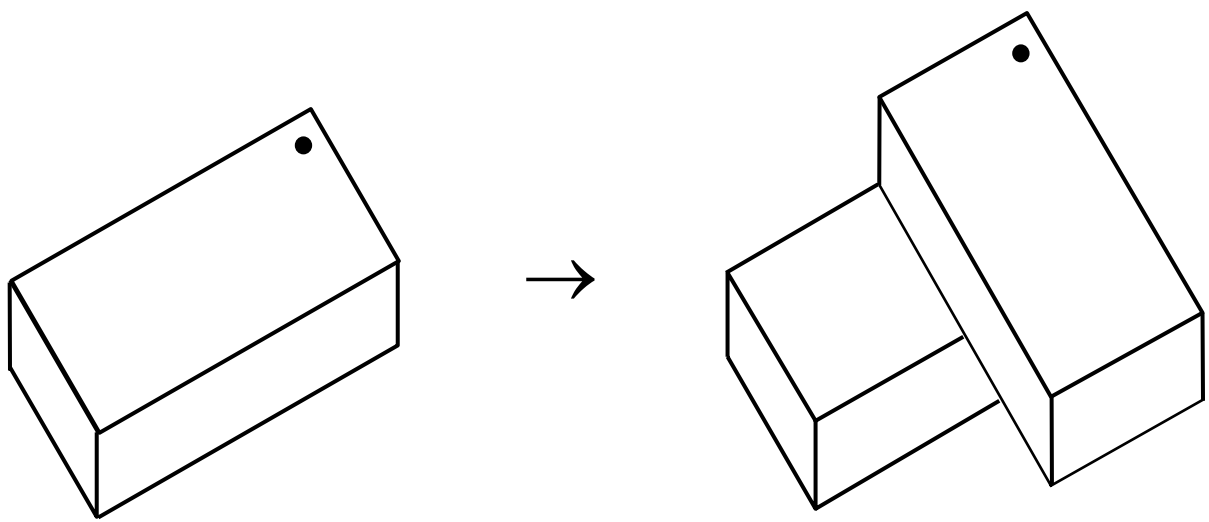


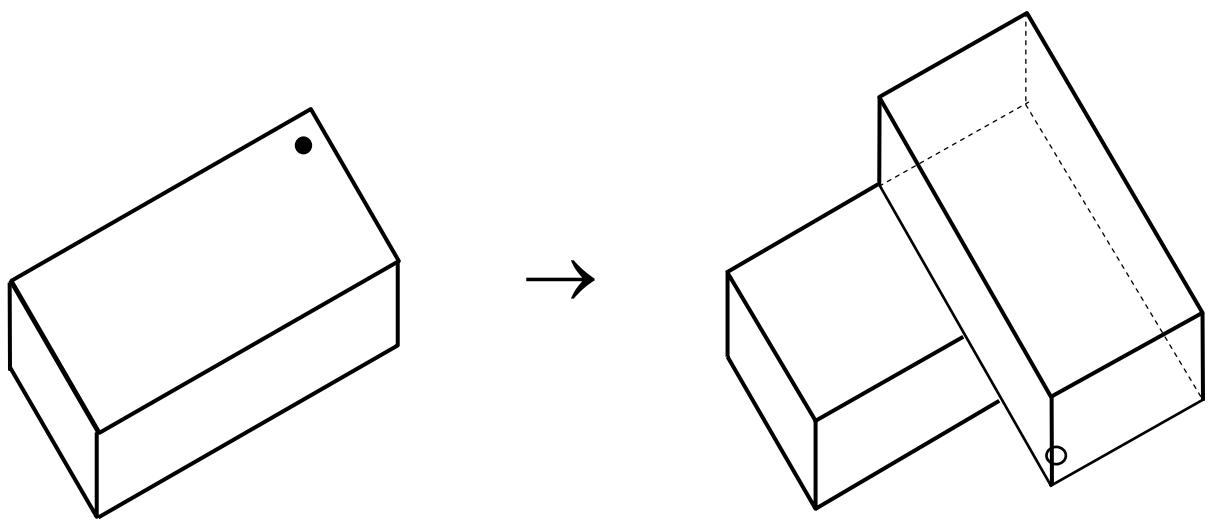
different labelings of the rule A $\rightarrow$ A + B

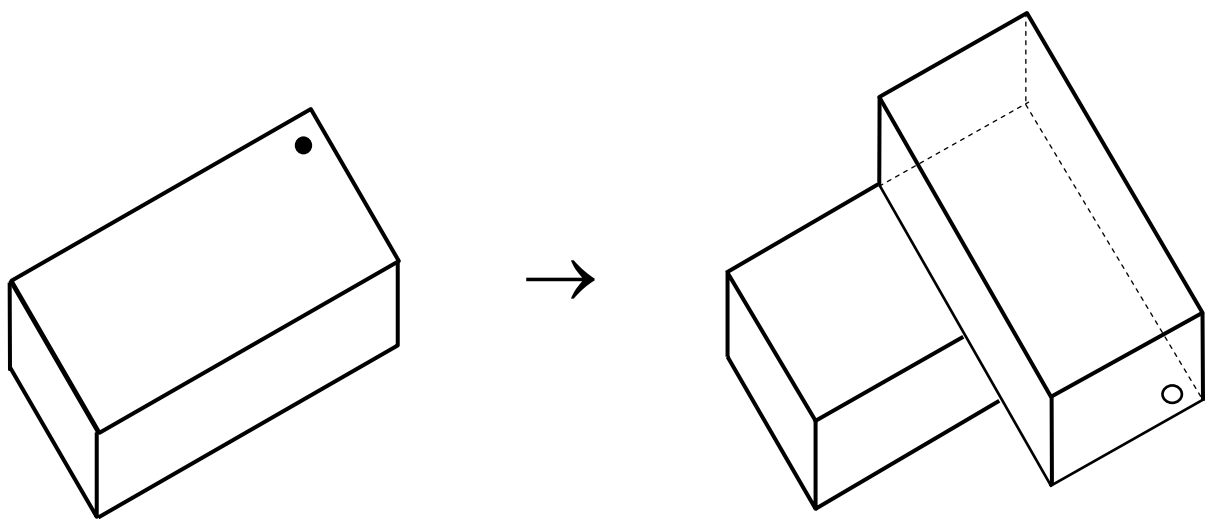


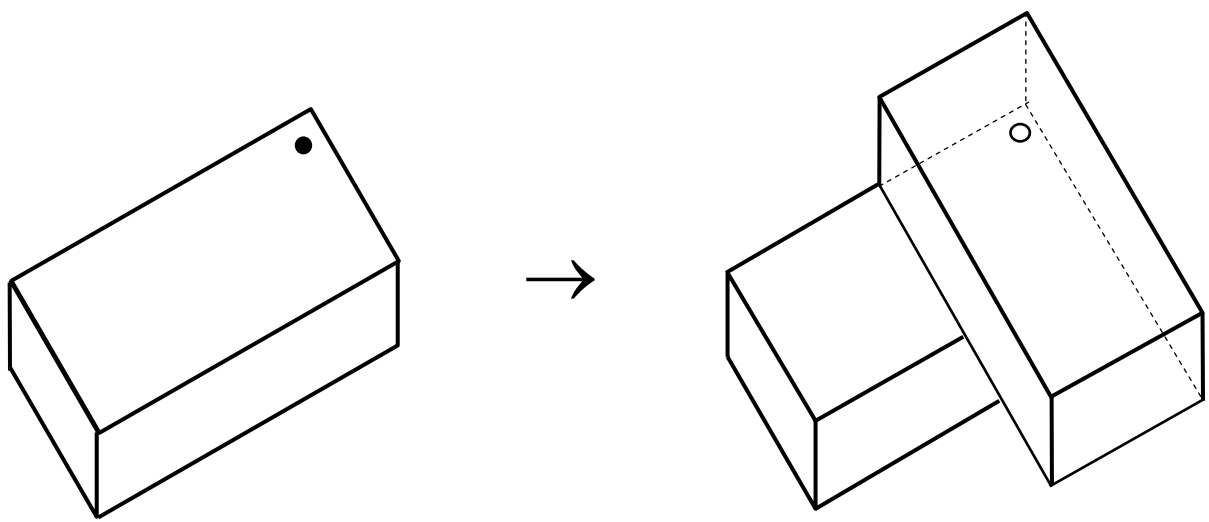


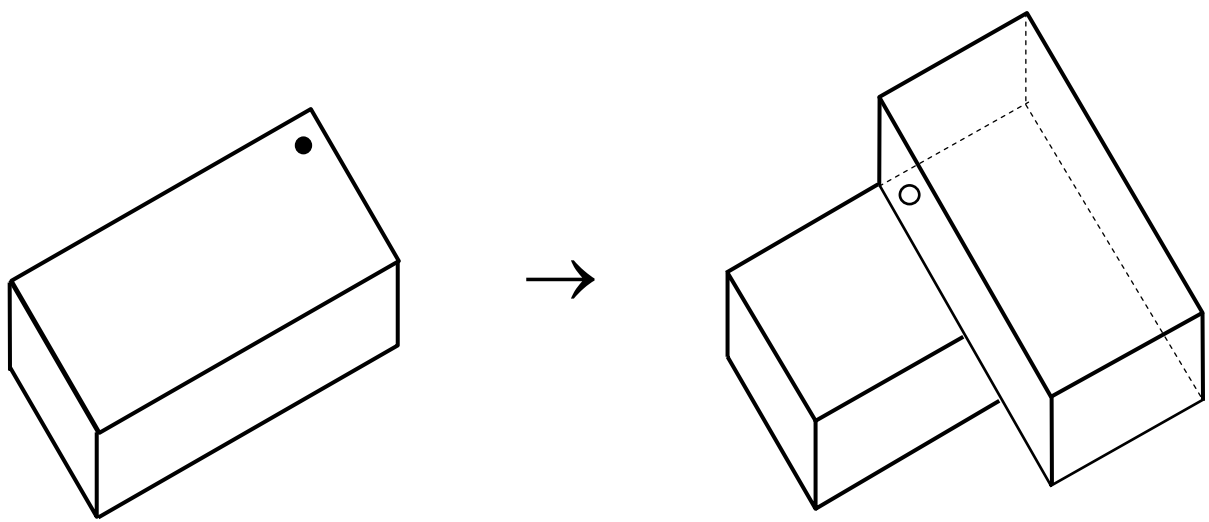


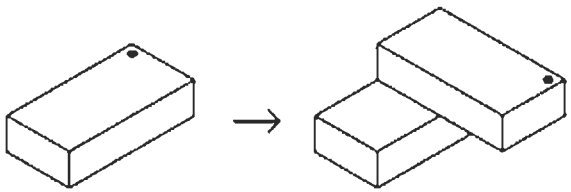




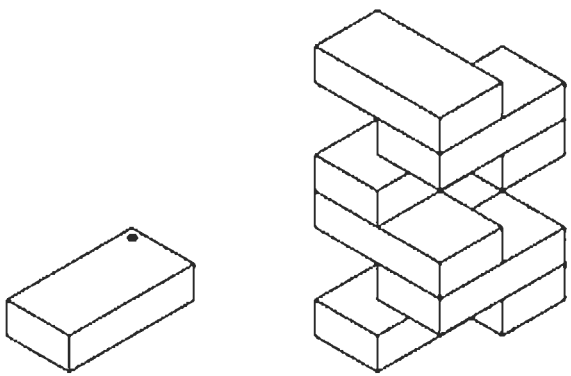






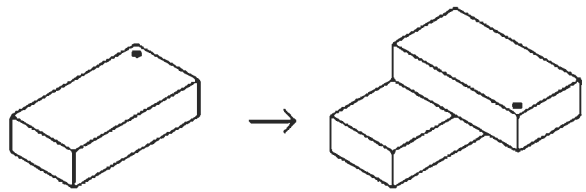


rule

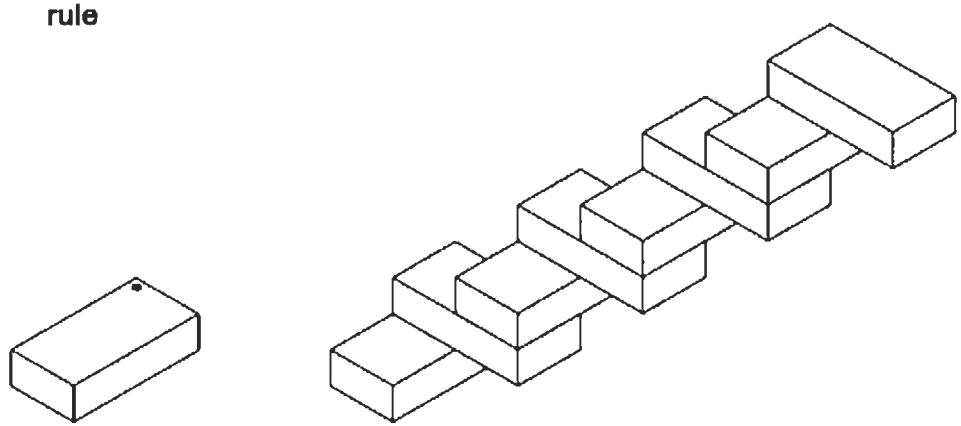


initial shape

design in language

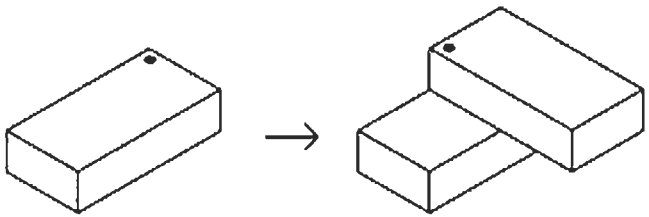


rule

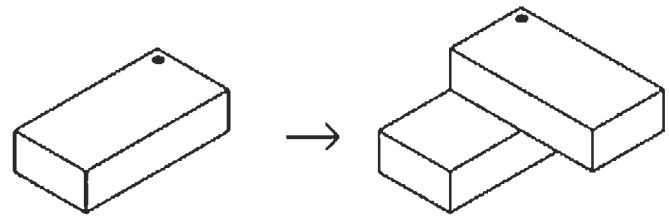


initial shape

design in language



rule



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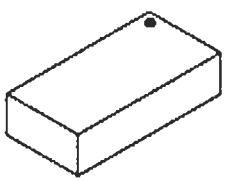
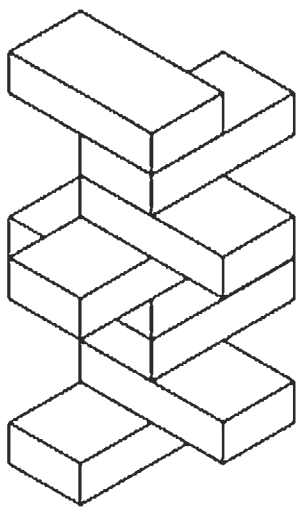
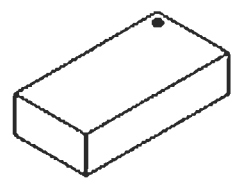


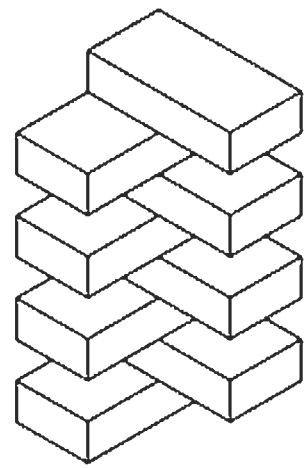
initial shape



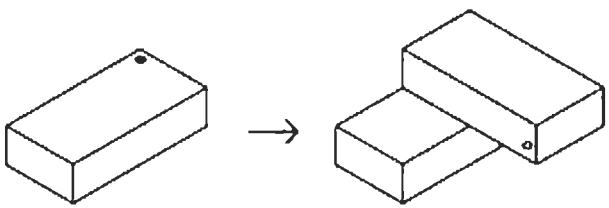
design in language



initial shape



design in language



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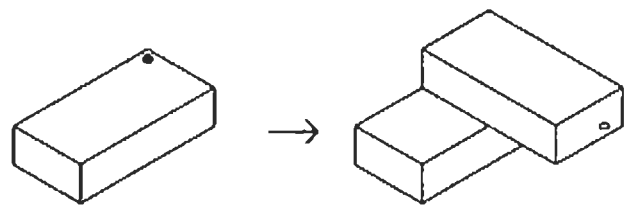

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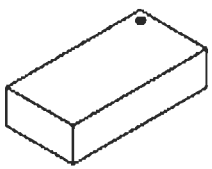
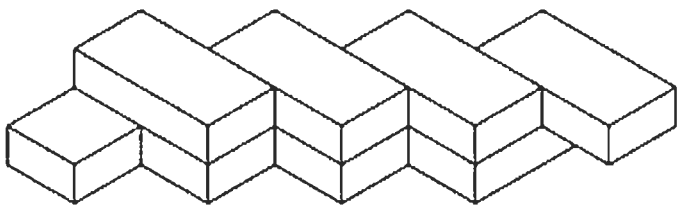
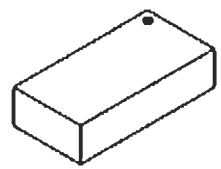


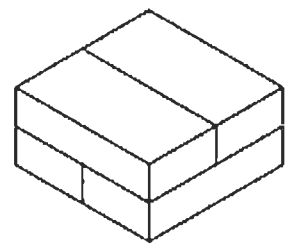
initial shape



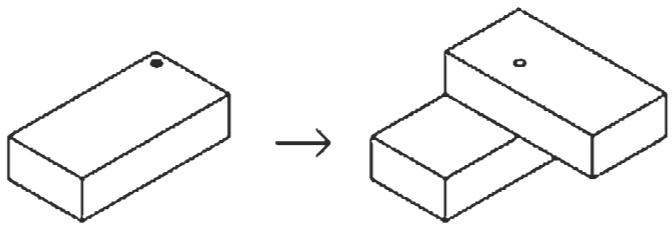
design in language



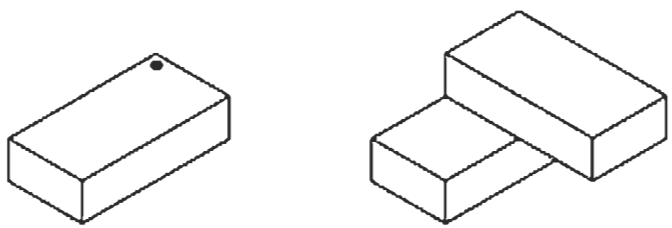
initial shape



design in language

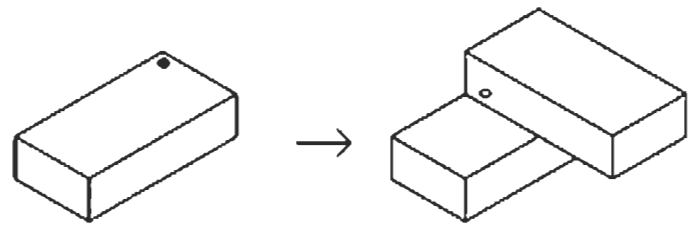


rule



initial shape

design in language



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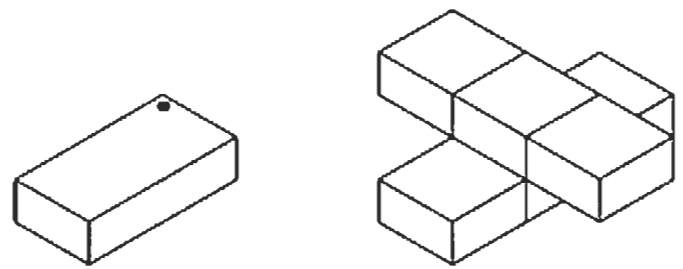

initial shape

design in language