Musculoskeletal Tissues

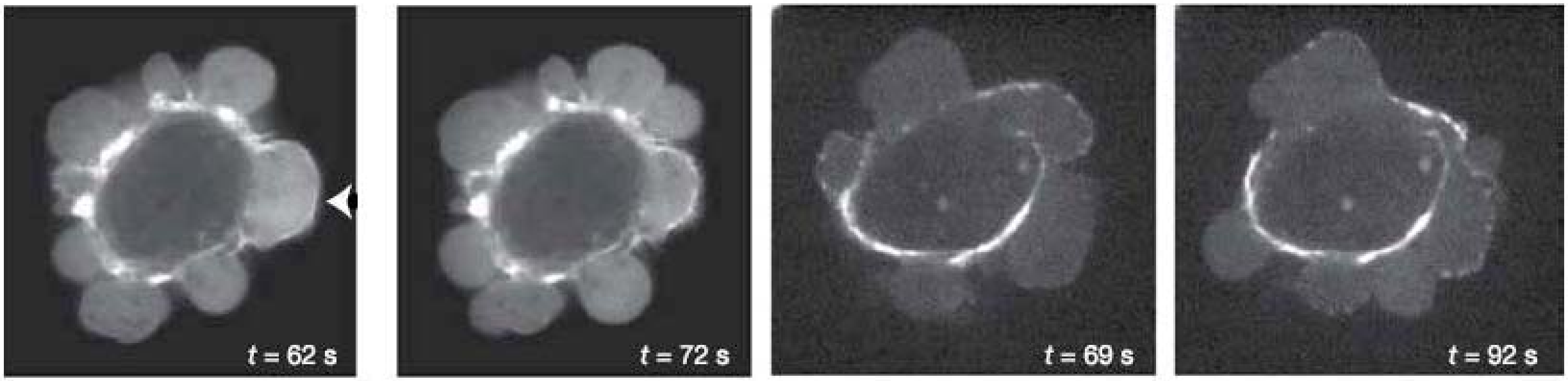
Cardiovascular Tissues

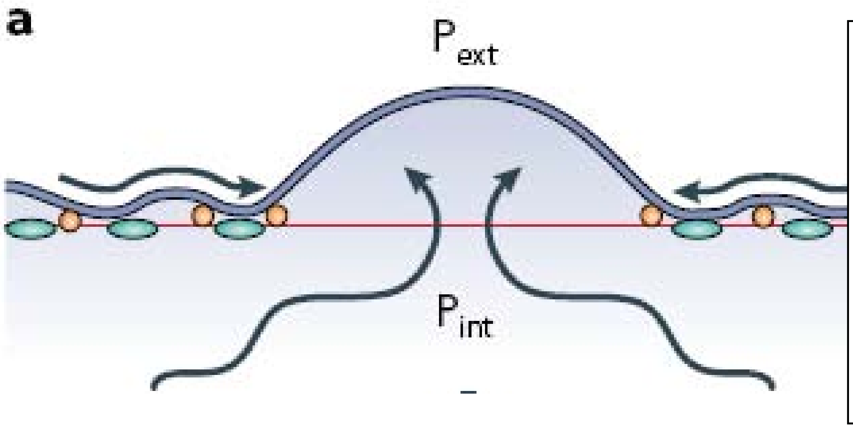
Courtesy of Ernst B. Hunziker. Used with permission.

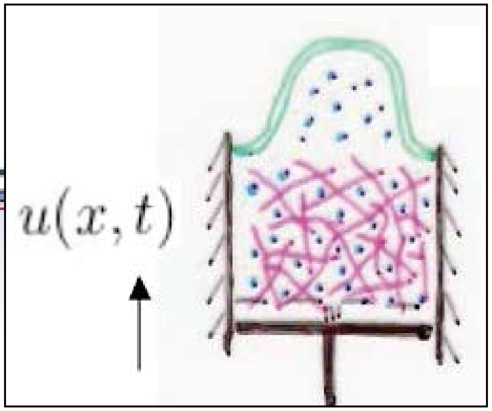
(Nature, 2005)

Non-equilibration of hydrostatic pressure in blebbing cells

Guillaume T. Charras[1], Justin C. Yarrow[1], Mike A. Horton[2], L. Mahadevan[1,3,4] & T. J. Mitchison[1]





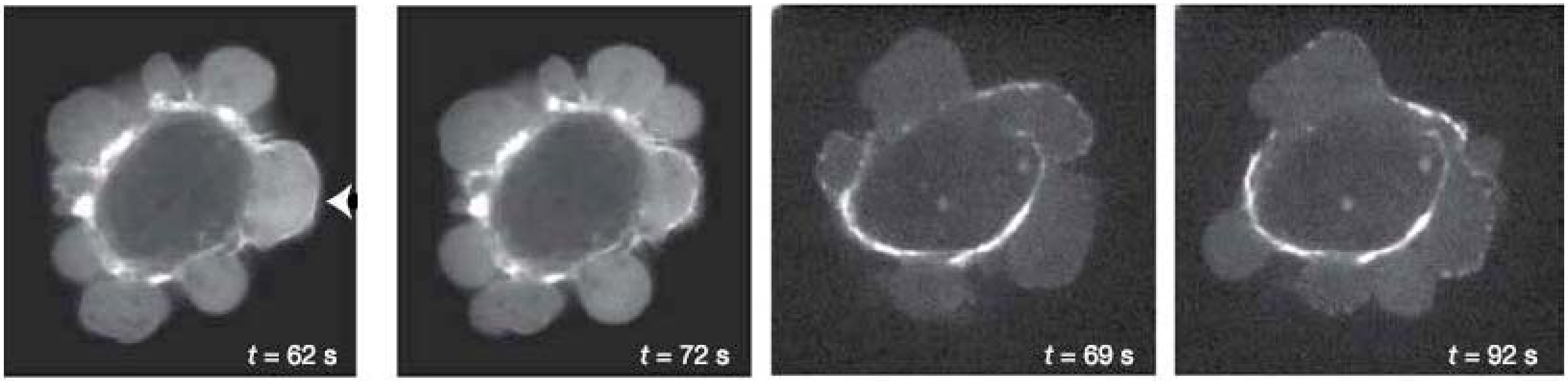


$$\frac{\partial u}{\partial t} = \frac{Ek}{\phi}\frac{\partial^2 u}{\partial x^2}$$

diffusion eqn:
“D” = “E•k”

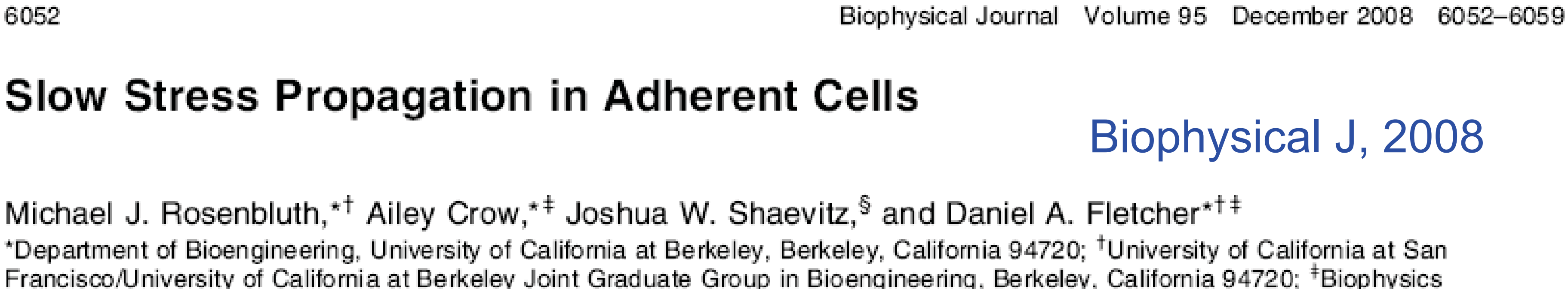
(Text: P 7.13)

Courtesy of Macmillan Publishers Limited. Used with permission.
Source: Charras, Guillaume T., et al. "Non-equilibration of Hydrostatic Pressure in Blebbing Cells." *Nature* 435, no. 7040 (2005): 365-9.



Courtesy of Macmillan Publishers Limited. Used with permission.
Source: Charras, Guillaume T., et al. "Non-equilibration of Hydrostatic Pressure in Blebbing Cells." *Nature* 435, no. 7040 (2005): 365-9.

Current models of the cytoplasm cannot account for spatio-temporal variations in hydrostatic pressure. **We propose a new description of the cytoplasm based on poroelasticity**. We consider cytoplasm to be composed of a porous, actively contractile, elastic network (cytoskeletal filaments, organelles, ribosomes), infiltrated with an interstitial fluid (…water, ions, soluble proteins), **similar to a fluid-filled sponge**. Contraction of the acto-myosin cortex creates a compressive stress on the cytoskeletal network, leading to localized increase in hydrostatic pressure & … cytosol flow out of the network..….

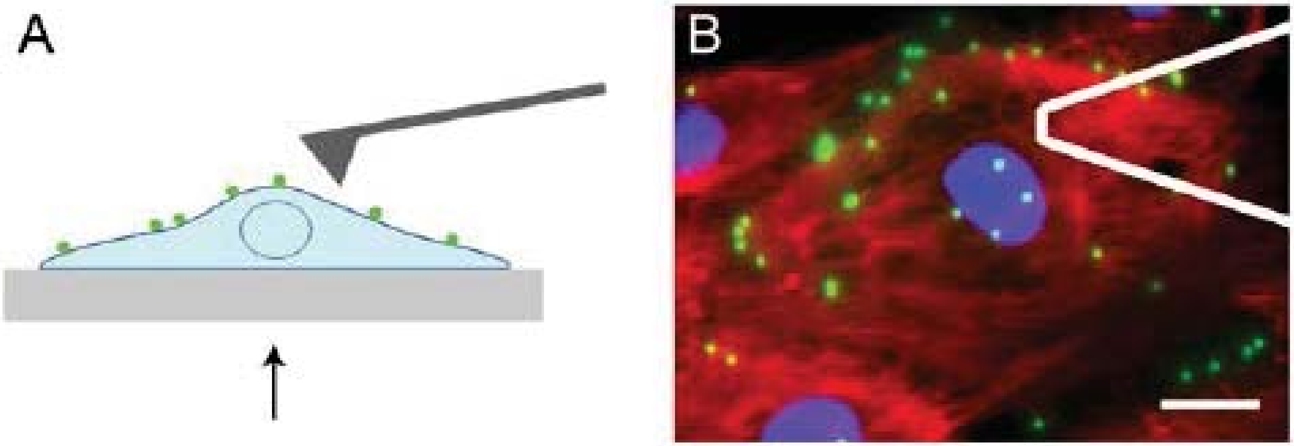


6052 Biophysical Journal Volume 95 December 2008 6052–6059

Slow Stress Propagation in Adherent Cells

Biophysical J, 2008

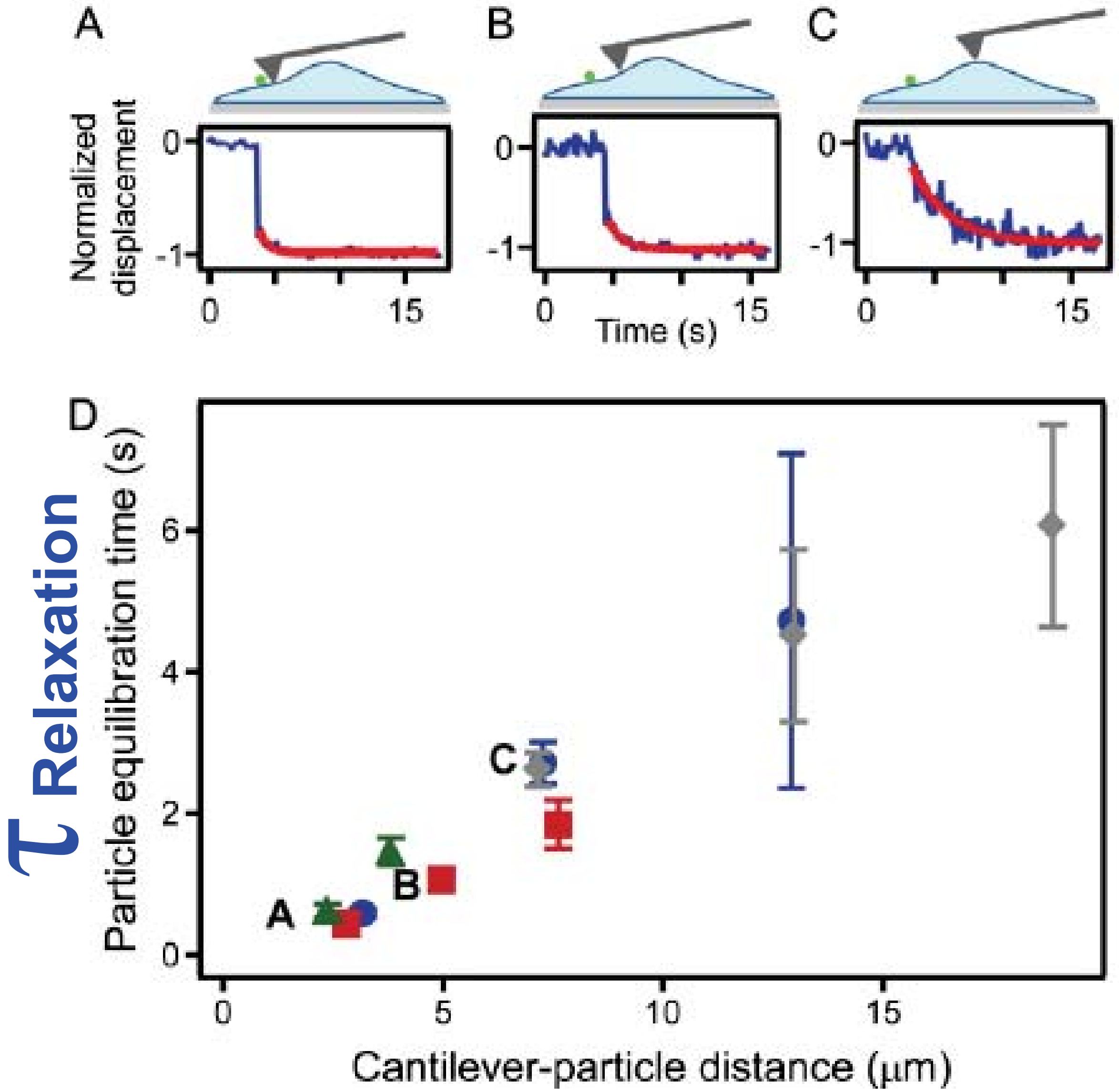
Michael J. Rosenbluth,*† Ailey Crow,*‡ Joshua W. Shaevitz,§ and Daniel A. Fletcher*†‡
*Department of Bioengineering, University of California at Berkeley, Berkeley, California 94720; †University of California at San Francisco/University of California at Berkeley Joint Graduate Group in Bioengineering, Berkeley, California 94720; ‡Biophysics



Courtesy of Elsevier, Inc., http://www.sciencedirect.com. Used with permission.
Source: Rosenbluth, Michael J., et al. "Slow Stress Propagation in Adherent Cells."
Biophysical Journal 95, no. 12 (2008): 6052-059.

ABSTRACT: Mechanical cues influence...motility, differentiation, tumorigenesis.… study of how mechanical perturbations propagate across the cell is necessary to understand spatial coordination of cellular processes.

- Here we quantify magnitude & timing of *intracellular stress propagation*, using AFM and particle tracking by defocused fluorescence microscopy.
- The apical cell surface is locally perturbed by AFM cantilever indentation, and distal displacements are measured in 3 dimensions by tracking integrin-bound fluorescent particles.

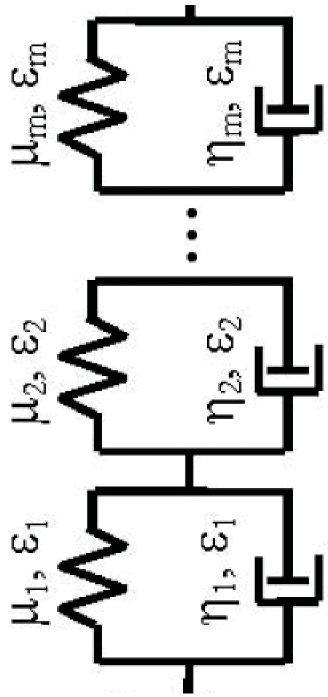


We observe an immediate response and slower equilibration, occurring over relaxation times that increase with distance from perturbation.

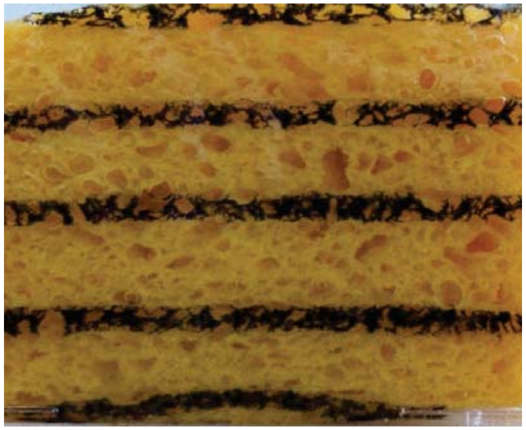
DISCUSSION

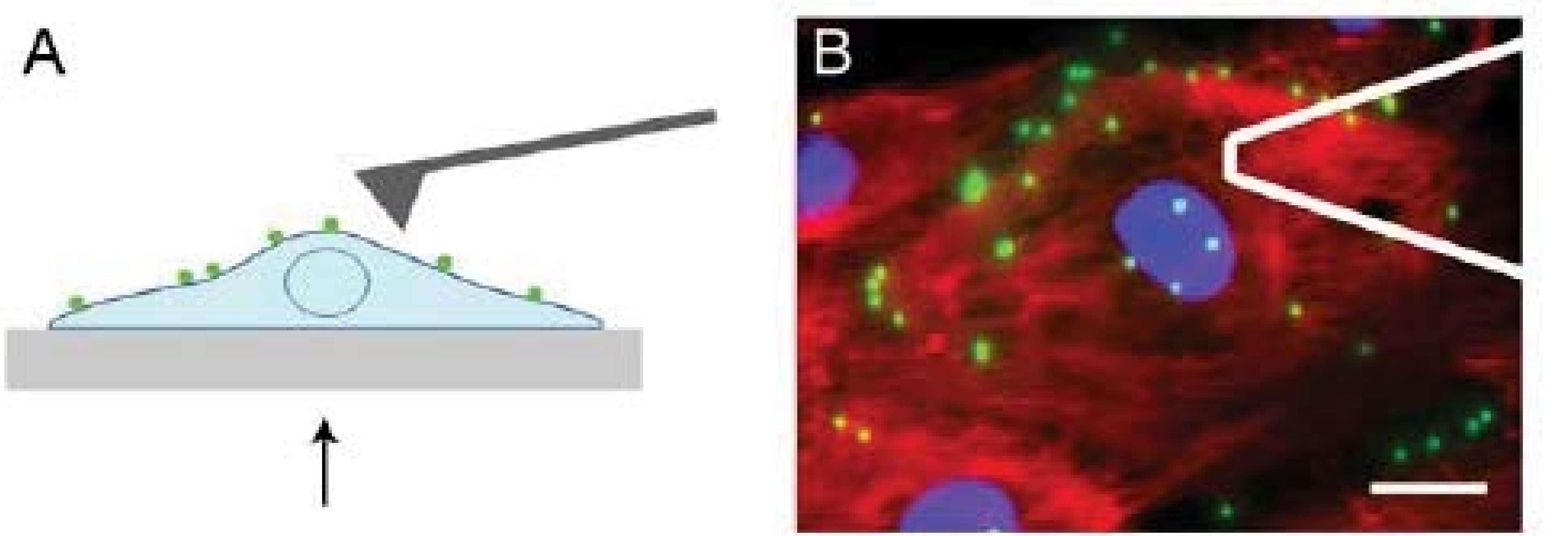
We compared our results to two material models: viscoelasticity and poroelasticity. A single-phase homogeneous viscoelastic material, such as the traditional spring-and-dashpot standard linear solid model, cannot explain the observed behavior, because it assumes that the material will simultaneously relax in response to a local perturbation with a single time-constant. To determine if a heterogeneous viscoelastic model could explain this behavior, we modeled the experiment as a step-strain of a series of parallel spring-dashpot pairs (Voigt-Kelvin material (29,30)

$$\boxed{T + \alpha \frac{dT}{dt} = E_1 e + \beta \frac{de}{dt}}$$



The poroelastic model can account for the observed slow distance-dependent equilibration across the cell. The biphasic nature of a poroelastic material results in both a fast propagation of stress through the solid phase (cytoskeleton), and a much slower diffusive equilibration of hydrostatic pressure of the fluid phase (cytosol), resulting in increasing equilibration time with distance (20).



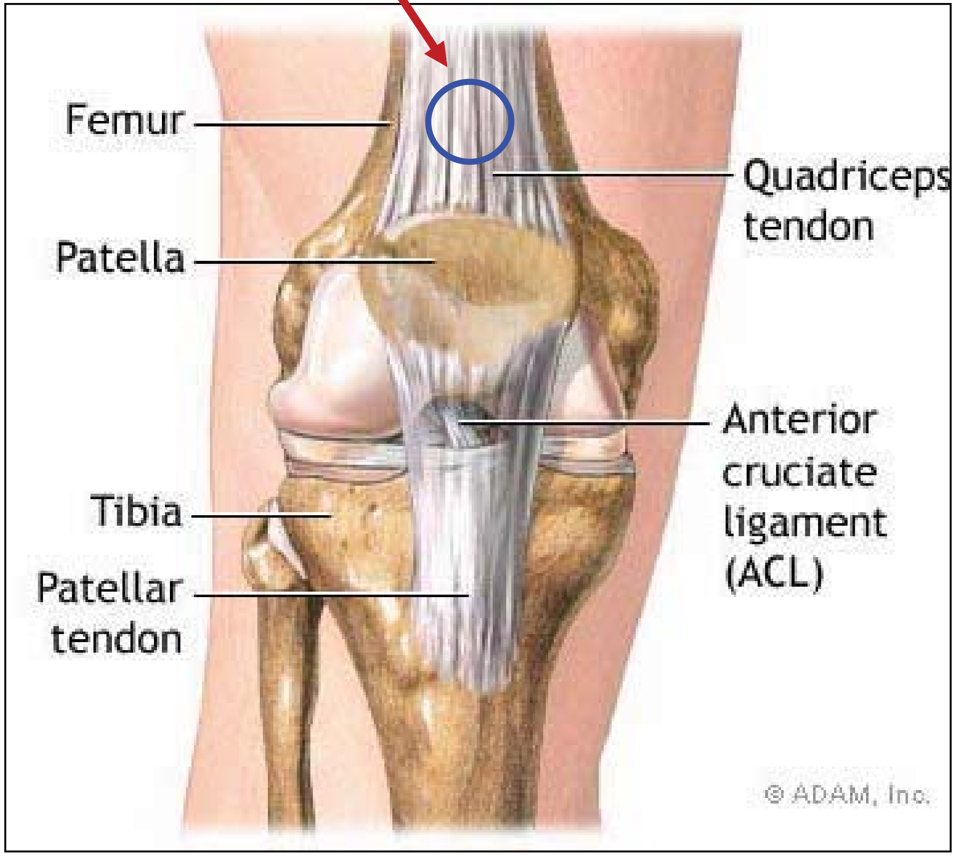


Courtesy of Elsevier, Inc., http://www.sciencedirect.com. Used with permission.
Source: Rosenbluth, Michael J., et al. "Slow Stress Propagation in Adherent Cells."
Biophysical Journal 95, no. 12 (2008): 6052-059.

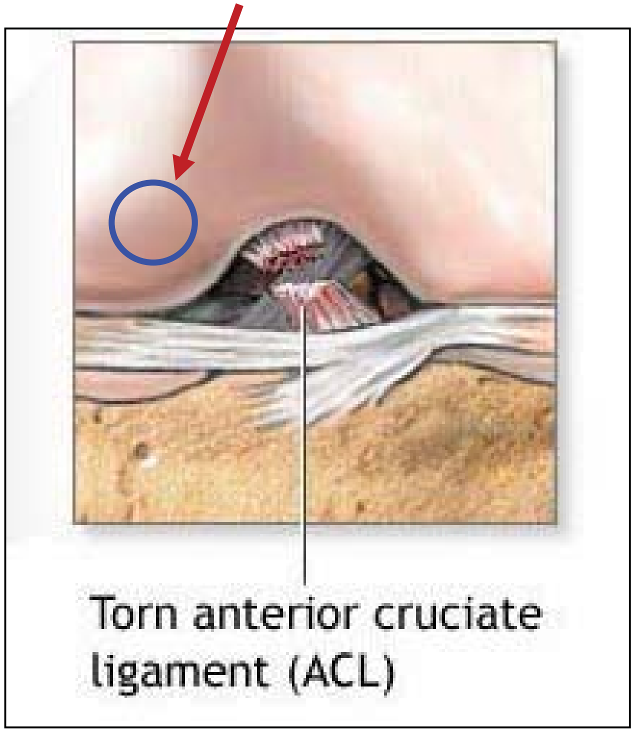
- Our experimental results are not explained by traditional viscoelastic models of cell mechanics, but they are consistent with predictions from poroelastic models that include both cytoskeletal deformation and flow of the cytoplasm….

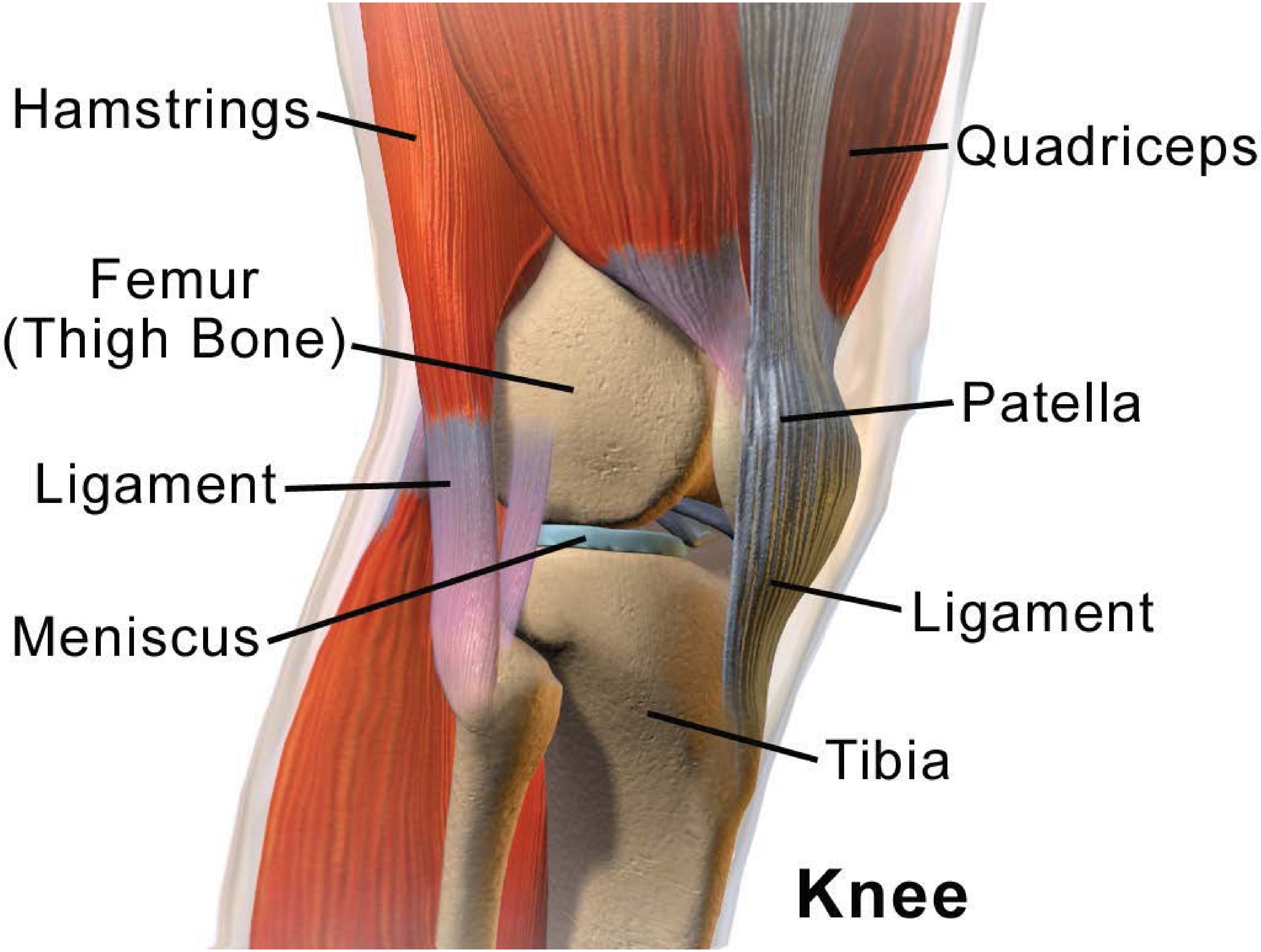
Musculoskeletal Tissues

Tension



Compression & Shear





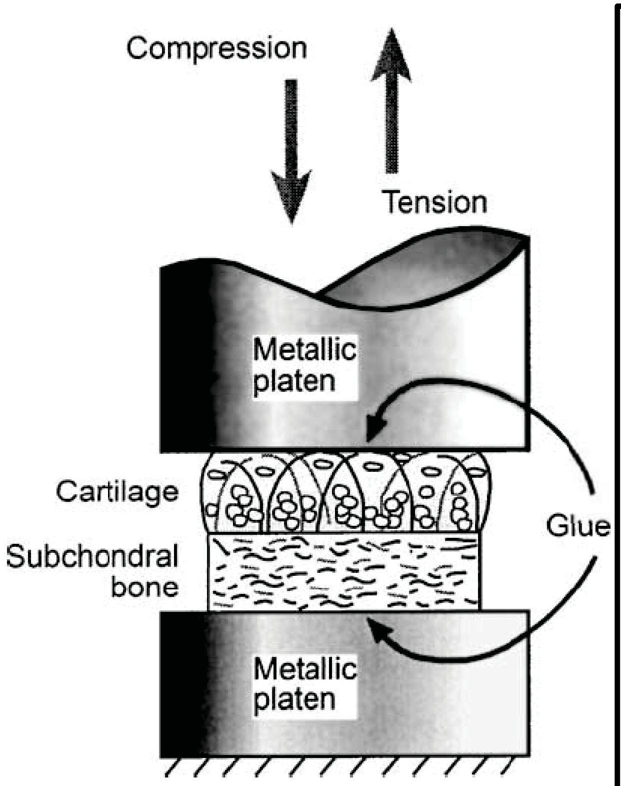
Courtesy of Blausen.com staff. License: CC BY.

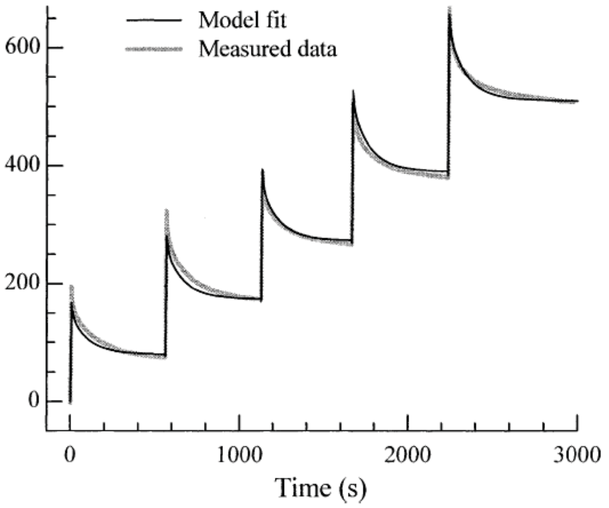
Image of knee anatomy removed due to copyright restrictions.

The role of viscoelasticity of collagen fibers in articular cartilage: axial tension versus compression

L.P. Li[a,*], W. Herzog[a], R.K. Korhonen[b], J.S. Jurvelin[b,c]

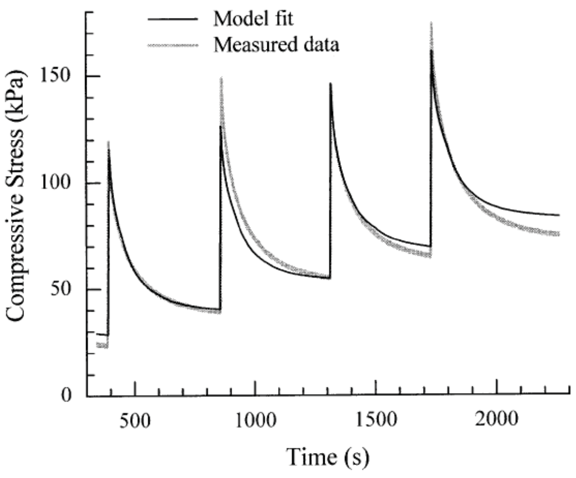
(Med Eng & Physics, 2005)





$$\sigma_r^f(t) = \sigma_r^f(0) + \int_0^t G_r(t-\tau)\, E_r^f(\varepsilon_r)\, \dot{\varepsilon}_r \, d\tau$$

Viscoelastic in Tension (many springs/dashpots)



Poroelastic in Compression

(compression-induced pressure & fluid flow)

Abstract: …..For axial tension, collagen viscoelasticity was found to account for most of the stress relaxation, while the effects of fluid pressurization on the tensile stress were negligible. In contrast, for axial compression, the dominant mechanism for stress relaxation arose from fluid pressurization and fluid flow…….

Courtesy of Elsevier, Inc., http://www.sciencedirect.com. Used with permission.
Source: Li, L. P., et al. "The Role of Viscoelasticity of Collagen Fibers in Articular Cartilage: Axial Tension Versus Compression." *Medical Engineering & Physics* 27, no. 1 (2005): 51-57.

Kinetics of swelling of gels

Toyoichi Tanaka and David J. Fillmore

(J Chem Phys, 1979)

Gel displacement *u* satisfies a poroelastic diffusion equation!

$$\frac{\partial u}{\partial t} = D_{\text{gel}} \frac{\partial}{\partial r}\left[\frac{1}{r^2}\left(\frac{\partial}{\partial r}(r^2 u)\right)\right]$$

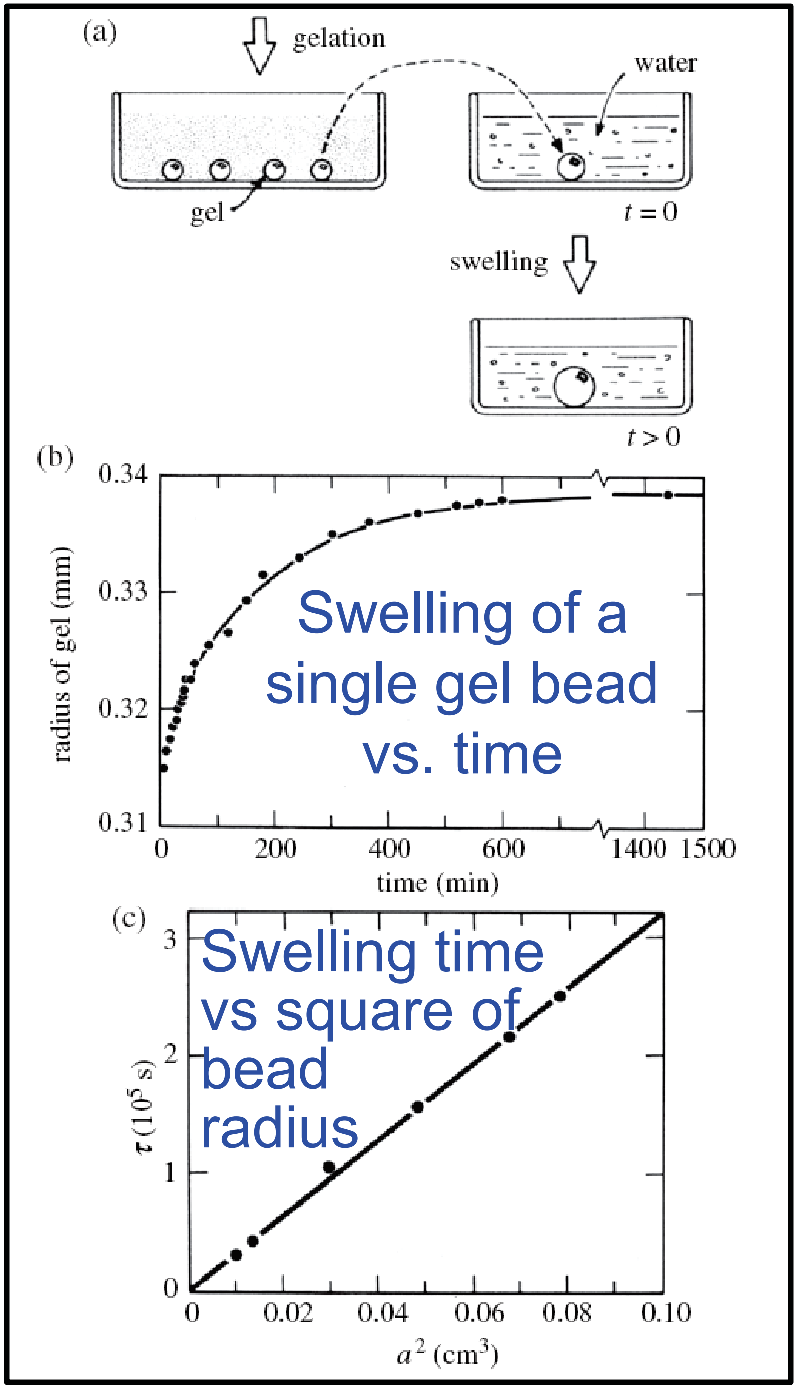
$$\tau_{\text{gel}} = \frac{L^2}{\pi^2 D_{\text{gel}}}$$

“D_{gel}” = “H” • “k”

H = (2G + λ) gel elasticity

k = gel hydraulic permeability

(see Section 7.5 of text, page 260-261)



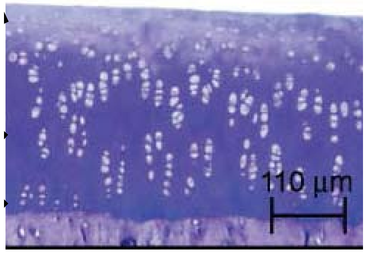
Swelling of a single gel bead vs. time

Swelling time vs square of bead radius

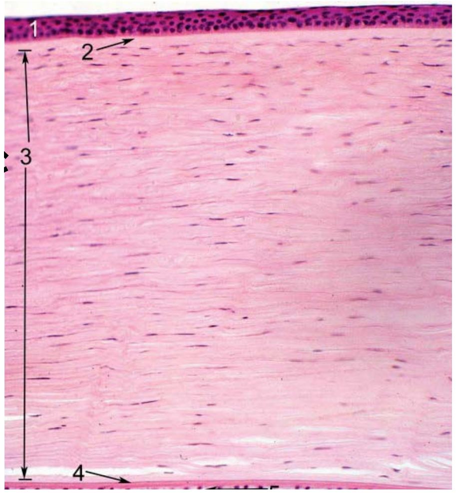
Source: Tanaka, Toyoichi, and David J. Fillmore. "Kinetics of Swelling of Gels." *The Journal of Chemical Physics* 70, no. 3 (1979): 1214-8.

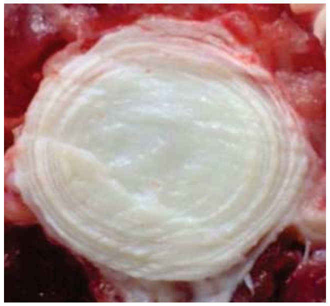
Problem 7.10 removed due to copyright restrictions. See the problem in the textbook.
Source: Grodzinsky, Alan. *Field, Forces and Flows in Biological Systems*. Garland Science, 2011.

Cartilage

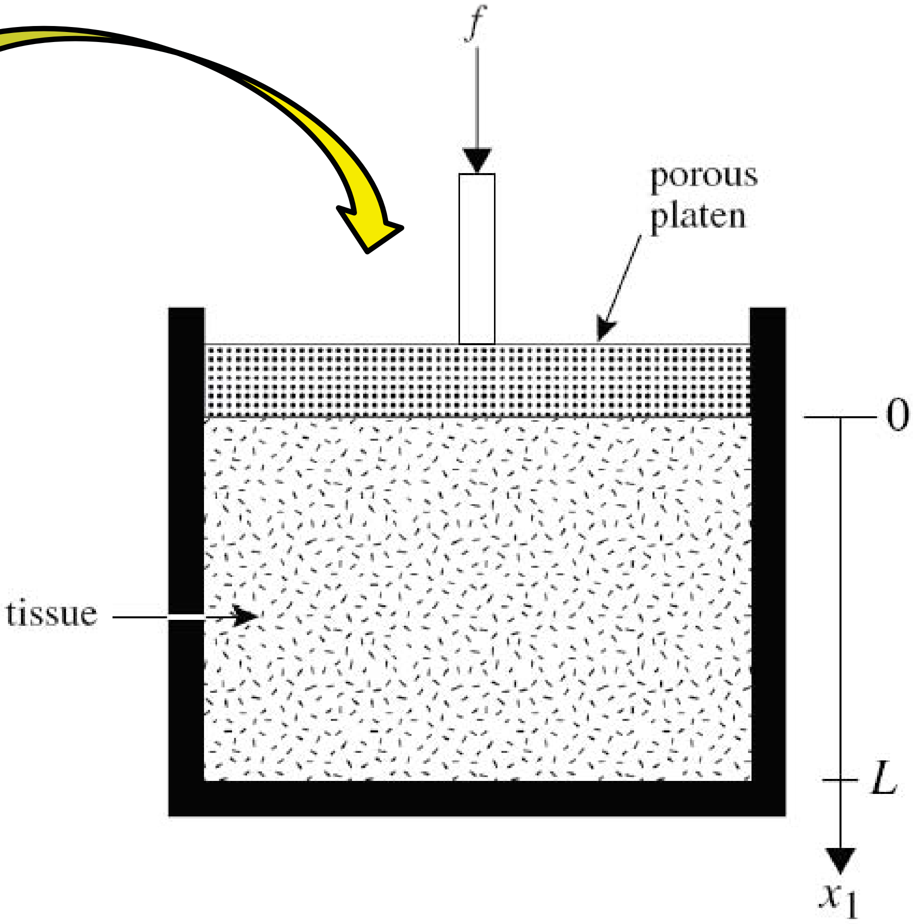


Human Cornea



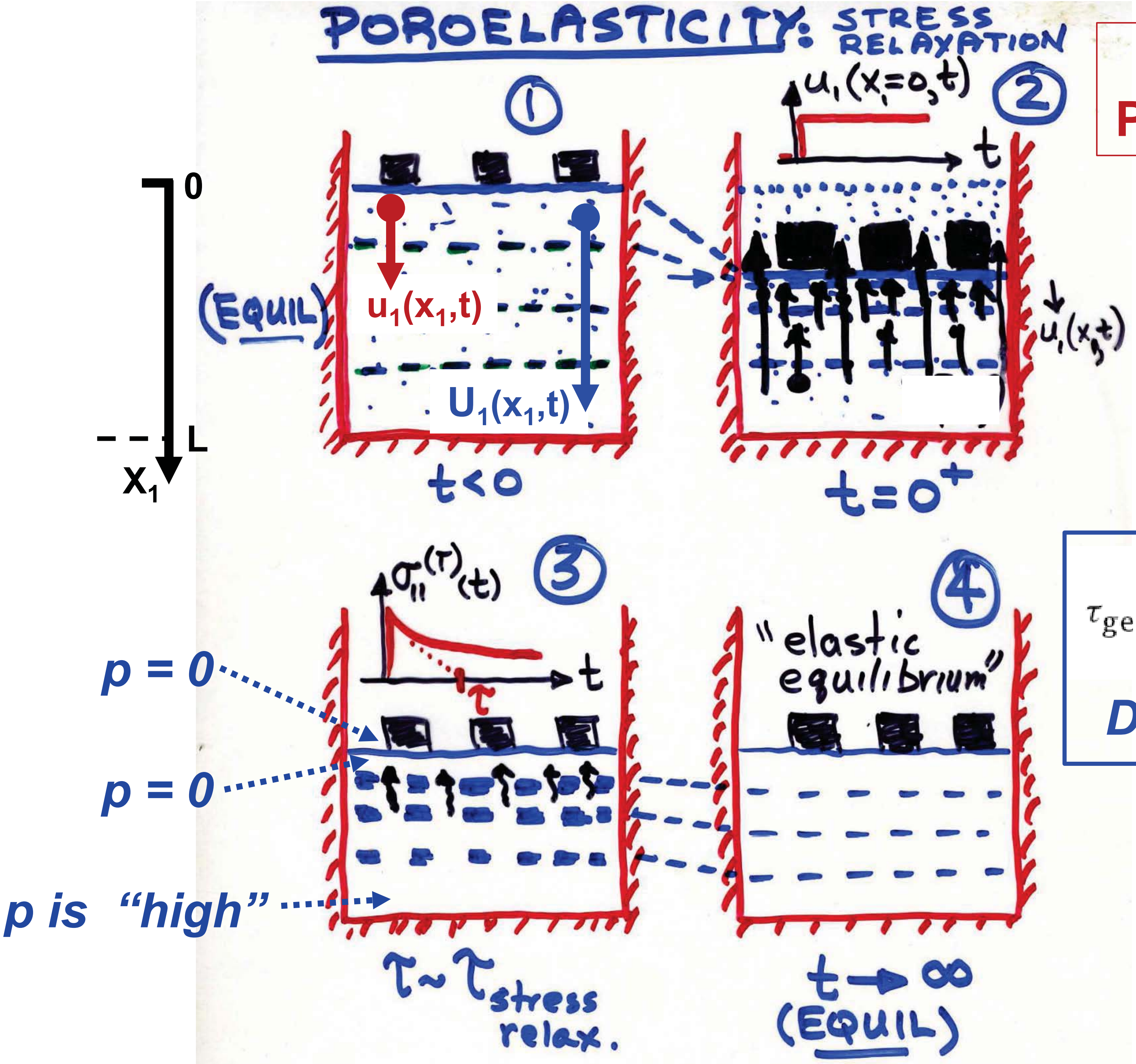


Human intervertebral disc



Problem 7.10 removed due to copyright restrictions. See the problem in the textbook.
Source: Grodzinsky, Alan. *Field, Forces and Flows in Biological Systems*. Garland Science, 2011.

PSet 5
Prob 7.10

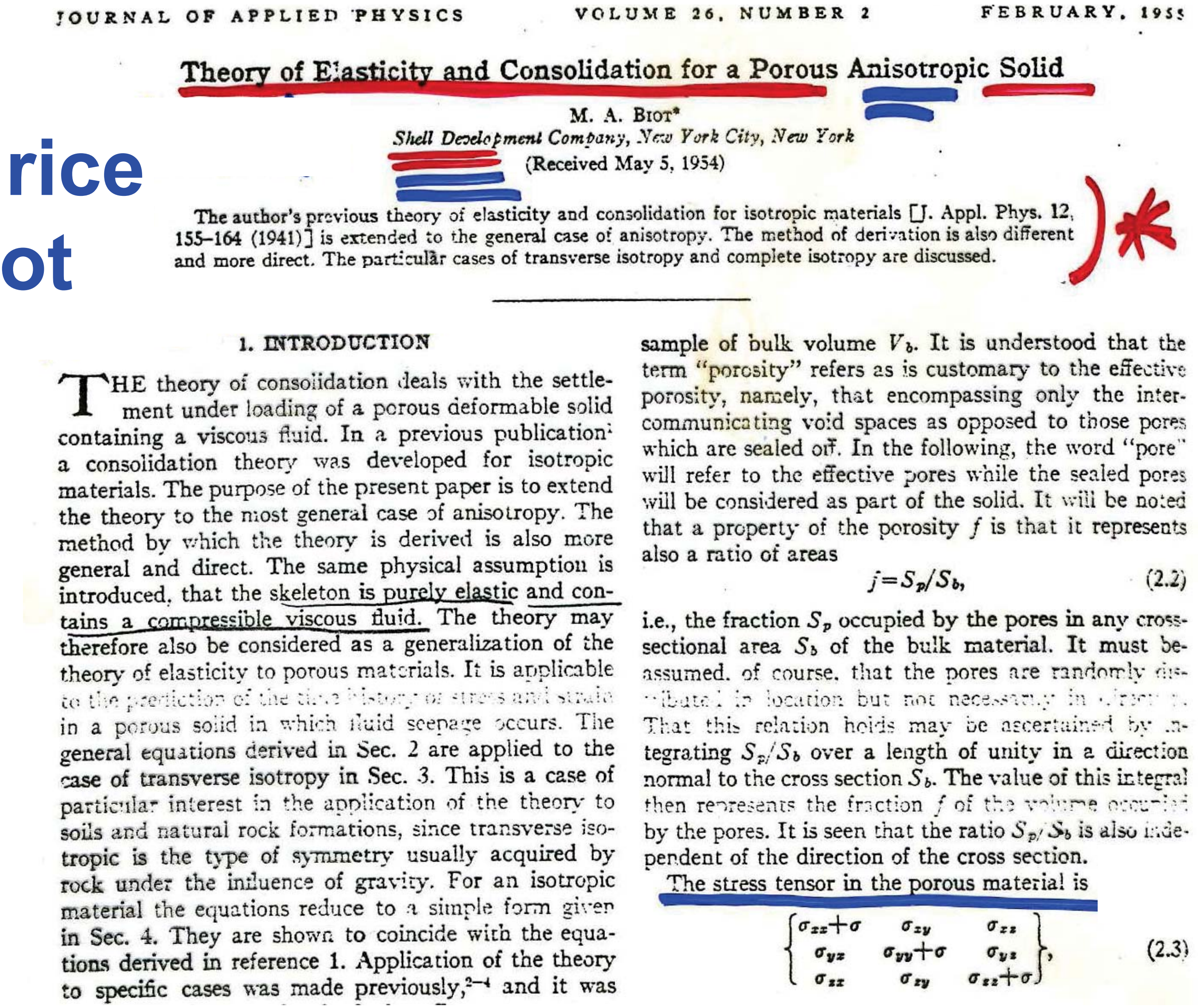


$$\tau_{\mathrm{gel}} = \frac{L^2}{\pi^2 D_{\mathrm{gel}}}$$

$$D_{gel} = H \cdot k$$

J Applied Physics, 1954

Maurice Biot



JOURNAL OF APPLIED PHYSICS VOLUME 26, NUMBER 2 FEBRUARY, 1955

Theory of Elasticity and Consolidation for a Porous Anisotropic Solid

M. A. Biot*
Shell Development Company, New York City, New York

(Received May 5, 1954)

The author's previous theory of elasticity and consolidation for isotropic materials [J. Appl. Phys. 12, 155–164 (1941)] is extended to the general case of anisotropy. The method of derivation is also different and more direct. The particular cases of transverse isotropy and complete isotropy are discussed.

1. INTRODUCTION

THE theory of consolidation deals with the settlement under loading of a porous deformable solid containing a viscous fluid. In a previous publication[1] a consolidation theory was developed for isotropic materials. The purpose of the present paper is to extend the theory to the most general case of anisotropy. The method by which the theory is derived is also more general and direct. The same physical assumption is introduced, that the skeleton is purely elastic and contains a compressible viscous fluid. The theory may therefore also be considered as a generalization of the theory of elasticity to porous materials. It is applicable to the prediction of the time history of stress and strain in a porous solid in which fluid seepage occurs. The general equations derived in Sec. 2 are applied to the case of transverse isotropy in Sec. 3. This is a case of particular interest in the application of the theory to soils and natural rock formations, since transverse isotropic is the type of symmetry usually acquired by rock under the influence of gravity. For an isotropic material the equations reduce to a simple form given in Sec. 4. They are shown to coincide with the equations derived in reference 1. Application of the theory to specific cases was made previously,[2–4] and it was

sample of bulk volume V_b. It is understood that the term "porosity" refers as is customary to the effective porosity, namely, that encompassing only the intercommunicating void spaces as opposed to those pores which are sealed off. In the following, the word "pore" will refer to the effective pores while the sealed pores will be considered as part of the solid. It will be noted that a property of the porosity f is that it represents also a ratio of areas

$$f=S_p/S_b, \tag{2.2}$$

i.e., the fraction S_p occupied by the pores in any cross-sectional area S_b of the bulk material. It must be assumed, of course, that the pores are randomly distributed in location but not necessarily in direction. That this relation holds may be ascertained by integrating S_p/S_b over a length of unity in a direction normal to the cross section S_b. The value of this integral then represents the fraction f of the volume occupied by the pores. It is seen that the ratio S_p/S_b is also independent of the direction of the cross section.

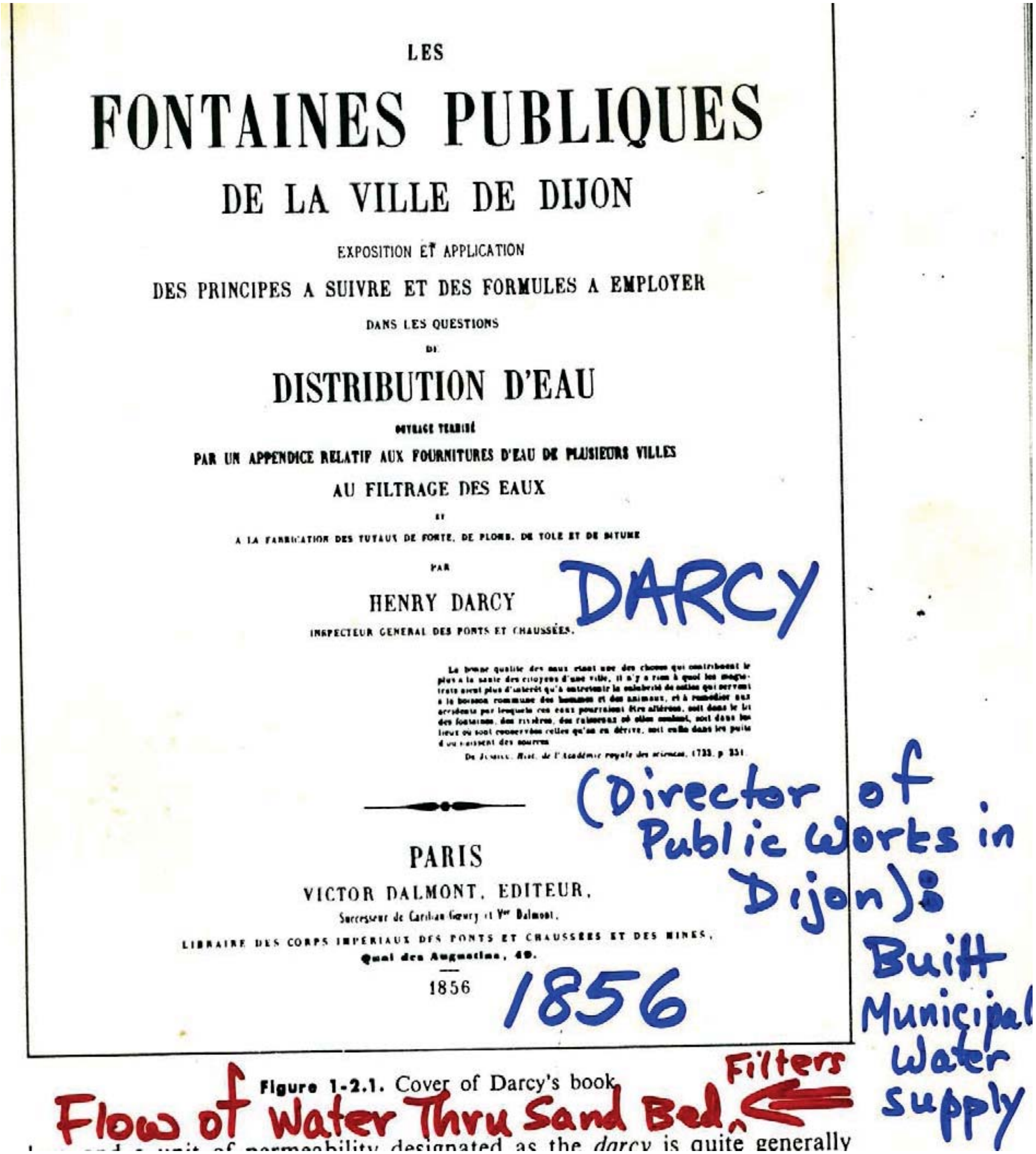
The stress tensor in the porous material is

$$\left\{\begin{matrix} \sigma_{xx}+\sigma & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy}+\sigma & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz}+\sigma \end{matrix}\right\}, \tag{2.3}$$

Source: Biot, M. Av. "Theory of Elasticity and Consolidation for a Porous Anisotropic Solid." *Journal of Applied Physics* 26, no. 2 (1955): 182-5.

Photograph of a bust of Henry Darcy removed due to copyright restrictions.

In the Jardin Darcy, Dijon, France



LES

FONTAINES PUBLIQUES

DE LA VILLE DE DIJON

EXPOSITION ET APPLICATION

DES PRINCIPES A SUIVRE ET DES FORMULES A EMPLOYER

DANS LES QUESTIONS

DE

DISTRIBUTION D'EAU

OUVRAGE TERMINÉ

PAR UN APPENDICE RELATIF AUX FOURNITURES D'EAU DE PLUSIEURS VILLES

AU FILTRAGE DES EAUX

ET

A LA FABRICATION DES TUYAUX DE FONTE, DE PLOMB, DE TOLE ET DE BITUME

PAR

HENRY DARCY

INSPECTEUR GÉNÉRAL DES PONTS ET CHAUSSÉES.

DARCY

> La bonne qualité des eaux étant une des choses qui contribuent le plus à la santé des citoyens d'une ville, il n'y a rien à quoi les magistrats aient plus d'intérêt qu'à entretenir la salubrité de celles qui servent à la boisson commune des hommes et des animaux, et à remédier aux accidents par lesquels ces eaux pourraient être altérées, soit dans le lit des fontaines, des rivières, des ruisseaux où elles coulent, soit dans les lieux où sont conservées celles qu'on en dérive, soit enfin dans les puits d'où naissent des sources.
>
> De Jussieu, *Hist. de l'Académie royale des sciences*, 1733, p. 351.

(Director of Public Works in Dijon):

PARIS

VICTOR DALMONT, EDITEUR,

Successeur de Carilian Gœury et Vve Dalmont,

LIBRAIRE DES CORPS IMPÉRIAUX DES PONTS ET CHAUSSÉES ET DES MINES,

Quai des Augustins, 49.

1856

1856

Built Municipal Water Supply

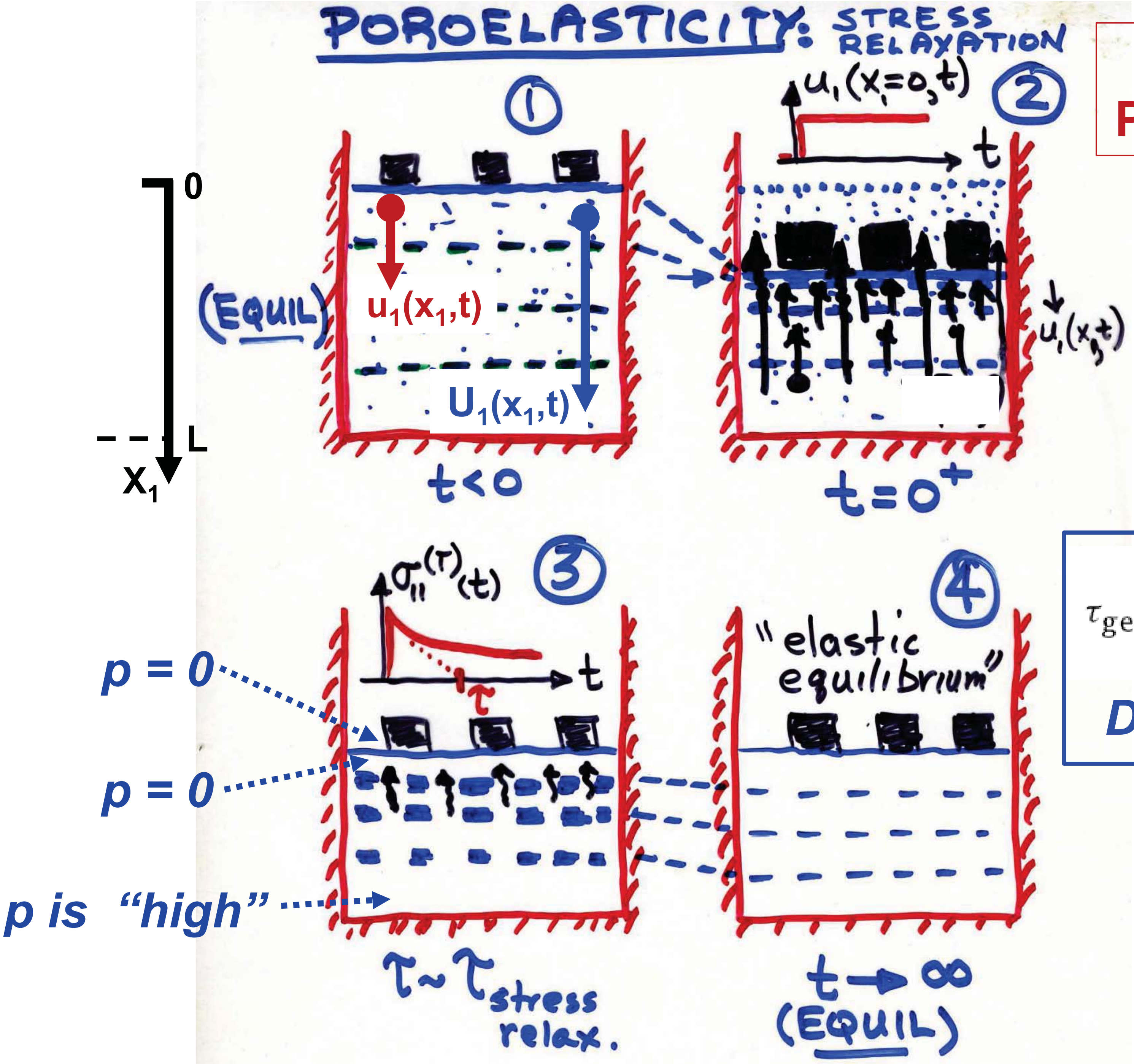
Figure 1-2.1. Cover of Darcy's book.

Flow of Water Thru Sand Bed ^ Filters ⟸

and a unit of permeability designated as the *darcy* is quite generally

POROELASTICITY: STRESS RELAXATION

PSet 5
Prob 7.10



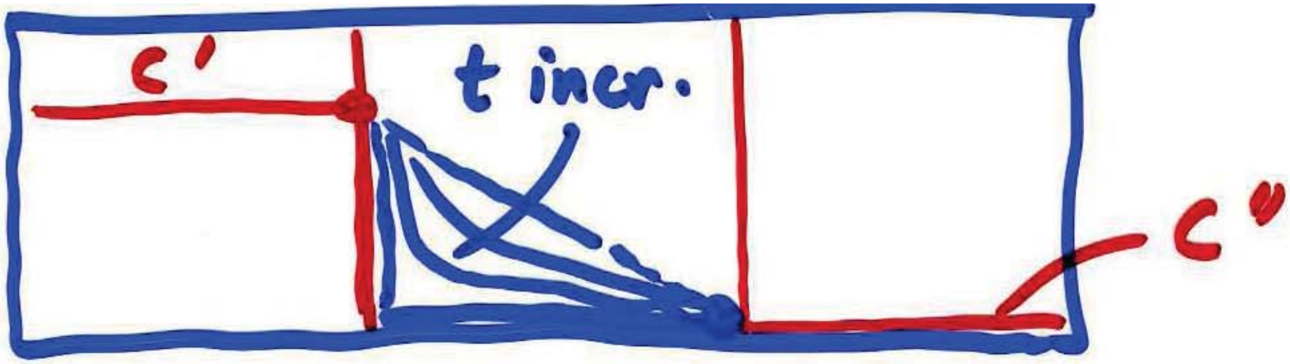
p = 0

p = 0

p is "high"

$$\tau_{gel} = \frac{L^2}{\pi^2 D_{gel}}$$

$$D_{gel} = H \cdot k$$



Like Chemical Diffusion:
Boundary conditions fix c', c"

$$\boxed{\frac{\partial c_i}{\partial t} = D_i \frac{\partial^2 c_i}{\partial x^2}}$$

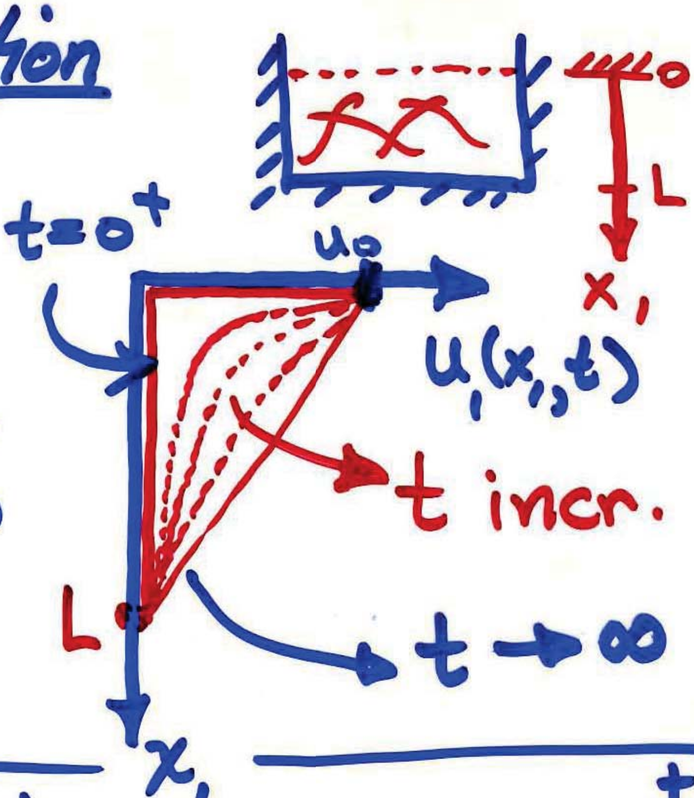
Stress Relaxation

$$\boxed{\frac{\partial u_1}{\partial t} = Hk\frac{\partial^2 u_1}{\partial x_1^2}}$$

B.C: $u_1 = 0$ @ $x_1 = L$

$u_1 = u_0$ @ $x_1 = 0$

I.C. $u_1 = 0$, $t < 0$



Text Eq. (7.55)

$$\boxed{u_1(x_1,t) = u_0\left(1 - \frac{x_1}{L}\right) - \sum A_n \sin\left(\frac{n\pi x_1}{L}\right) e^{-t/\tau_n}}$$

described by 2 material prop. for all x_1 and time

$$\tau_n = \frac{L^2}{n^2\pi^2(Hk)}$$

$$\epsilon_{11} = \frac{\partial u_1}{\partial x_1}$$

20.310J / 3.053J / 6.024J / 2.797J Molecular, Cellular, and Tissue Biomechanics
Spring 2015