Lesson 3: The Blending Tank

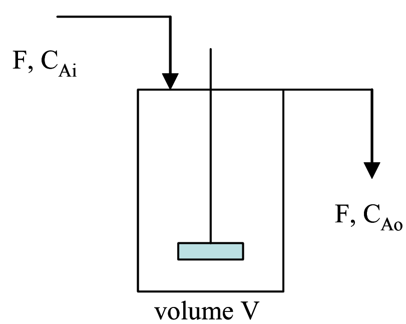
3.0 context and direction

A particularly simple process is a tank used for blending. Just as promised in Section 1.1, we will first represent the process as a dynamic system and explore its response to disturbances. Then we will pose a feedback control scheme. We will briefly consider the equipment required to realize this control. Finally we will explore its behavior under control.

DYNAMIC SYSTEM BEHAVIOR

3.1 math model of a simple continuous holding tank

Imagine a process stream comprising an important chemical species A in dilute liquid solution. It might be the effluent of some process, and we might wish to use it to feed another process. Suppose that the solution composition varies unacceptably with time. We might moderate these swings by holding up a volume in a stirred tank: intuitively we expect the changes in the outlet composition to be more moderate than those of the feed stream.



Our concern is the time-varying behavior of the process, so we should treat our process as a dynamic system. To describe the system, we begin by writing a component material balance over the solute.

$$\frac{d}{dt}VC_{Ao} = FC_{Ai} - FC_{Ao} \tag{3.1-1}$$

In writing (3.1-1) we have recognized that the tank operates in overflow: the volume is constant, so that changes in the inlet flow are quickly duplicated in the outlet flow. Hence both streams are written in terms of a single volumetric flow F. Furthermore, for now we will regard the flow as constant in time.

Balance (3.1-1) also represents the concentration of the outlet stream, C_{Ao}, as the same as the average concentration in the tank. That is, the tank is a perfect mixer: the inlet stream is quickly dispersed throughout the tank volume. Putting (3.1-1) into standard form,

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$$\frac{V}{F}\frac{dC_{Ao}}{dt} + C_{Ao} = C_{Ai} \tag{3.1-2}$$

we identify a first-order dynamic system describing the response of the outlet concentration C_{Ao} to disturbances in the inlet concentration C_{Ai}. The speed of response depends on the time constant, which is equal to the ratio of tank volume and volumetric flow. Although both of these quantities influence the dynamic behavior of the system, they do so as a ratio. Hence a small tank and large tank may respond at the same rate, if their flow rates are suitably scaled.

System (3.1-2) has a gain equal to 1. This means that a sustained disturbance in the inlet concentration is ultimately communicated fully to the outlet.

Before solving (3.1-2) we specify a reference condition: we prefer that C_{Ao} be at a particular value $C_{Ao,r}$. For steady operation in the desired state, there is no accumulation of solute in the tank.

$$\left.\frac{V}{F}\frac{dC_{Ao}}{dt}\right|_{r} = 0 = C_{Ai,r} - C_{Ao,r} \tag{3.1-3}$$

Thus, as expected, steady outlet conditions require a steady inlet at the same concentration; call it $C_{A,r}$. Let us take this reference condition as an initial condition in solving (3.1-2). The solution is

$$C_{Ao}(t) = C_{A,r}e^{-t/\tau} + \frac{e^{-t/\tau}}{\tau}\int_0^t e^{t/\tau}C_{Ai}(t)dt \tag{3.1-4}$$

where the time constant is

$$\tau = \frac{V}{F} \tag{3.1-5}$$

Equation (3.1-4) describes how outlet concentration C_{Ao} varies as C_{Ai} changes in time. In the next few sections we explore the transient behavior predicted by (3.1-4).

3.2 response of system to steady input

Suppose inlet concentration remains steady at $C_{A,r}$. Then from (3.1-4)

$$C_{Ao} = C_{A,r}e^{-t/\tau} + \frac{e^{-t/\tau}}{\tau}C_{A,r}\,\tau e^{t/\tau}\Big|_0^t$$
$$= C_{A,r}e^{-t/\tau} + C_{A,r}e^{-t/\tau}\left(e^{t/\tau} - 1\right) = C_{A,r} \qquad (3.2\text{-}1)$$

Equation (3.2-1) merely confirms that the system remains steady if not disturbed.

3.3 leaning on the system - response to step disturbance

Step functions typify disturbances in which an input variable moves relatively rapidly to some new value and remains there. Suppose that input C_{Ai} is initially at the reference value $C_{A,r}$ and changes at time t_1 to value C_{A1}. Until t_1 the outlet concentration is given by (3.2-1). From the step at t_1, the outlet concentration begins to respond.

$$C_{Ao} = C_{A,r}e^{-(t-t_1)/\tau} + \frac{e^{-t/\tau}}{\tau}C_{A1}\,\tau e^{t/\tau}\Big|_{t_1}^t \qquad t > t_1$$
$$= C_{A,r}e^{-(t-t_1)/\tau} + C_{A1}e^{-t/\tau}\left(e^{t/\tau} - e^{t_1/\tau}\right) \qquad (3.3\text{-}1)$$
$$= C_{A,r}e^{-(t-t_1)/\tau} + C_{A1}\left(1 - e^{-(t-t_1)/\tau}\right)$$

In Figure 3.3-1, $C_{A,r} = 1$ and $C_{A1} = 0.8$ in arbitrary units; t_1 has been set equal to τ. At sufficiently long time, the initial condition has no influence and the outlet concentration becomes equal to the new inlet concentration. After time equal to three time constants has elapsed, the response is about 95% complete – this is typical of first-order systems.

In Section 3.1, we suggested that the tank would mitigate the effect of changes in the inlet composition. Here we see that the tank will not eliminate a step disturbance, but it does soften its arrival.

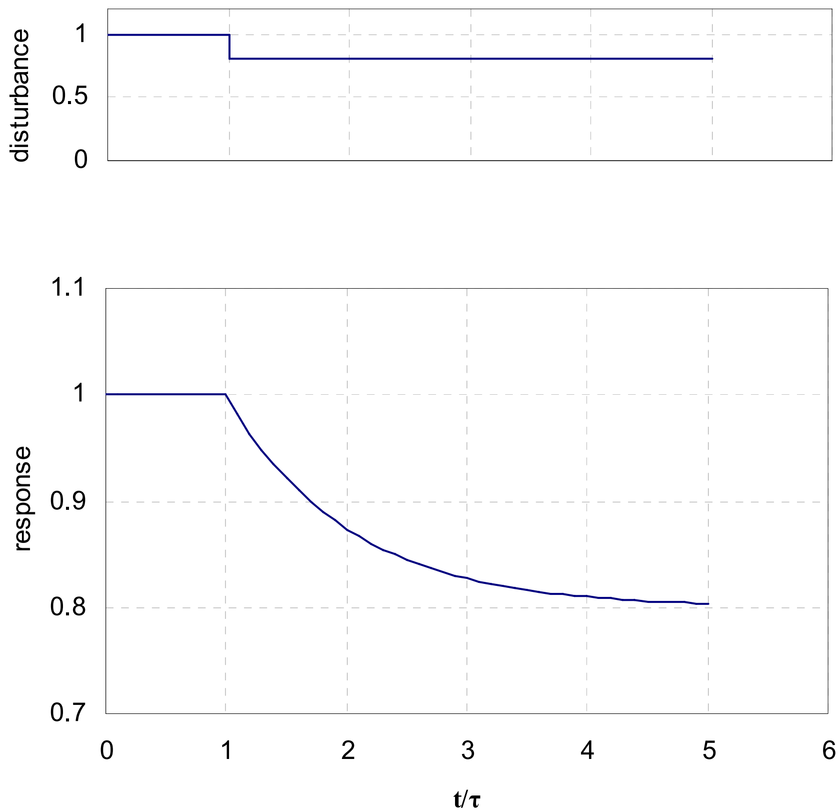

Figure 3.3-1 first-order response to step disturbance

3.4 kicking the system - response to pulse disturbance

Pulse functions typify disturbances in which an input variable moves relatively rapidly to some new value and subsequently returns to normal. Suppose that C_{Ai} changes to C_{A1} at time t_1 and returns to $C_{A,r}$ at t_2. Then, drawing on (3.2-1) and (3.3-1),

$$C_{Ao} = \begin{cases} C_{A,r} & 0 < t < t_1 \\ C_{A,r} e^{-(t-t_1)/\tau} + C_{A1}\left(1 - e^{-(t-t_1)/\tau}\right) & t_1 < t < t_2 \\ \left[C_{A,r} e^{-(t_2-t_1)/\tau} + C_{A1}\left(1 - e^{-(t_2-t_1)/\tau}\right)\right] e^{-(t-t_2)/\tau} + C_{A,r}\left(1 - e^{-(t-t_2)/\tau}\right) & t_2 < t \end{cases} \quad (3.4\text{-}1)$$

In Figure 3.4-1, $C_{A,r} = 0.6$ and $C_{A1} = 1$ in arbitrary units; t_1 has been set equal to τ and t_2 to 2.5τ. We see that the tank has softened the pulse and reduced its peak value. A pulse is a sequence of two counteracting step changes. If the pulse duration is long (compared to the time constant τ), the system can complete the first step response before being disturbed by the second.

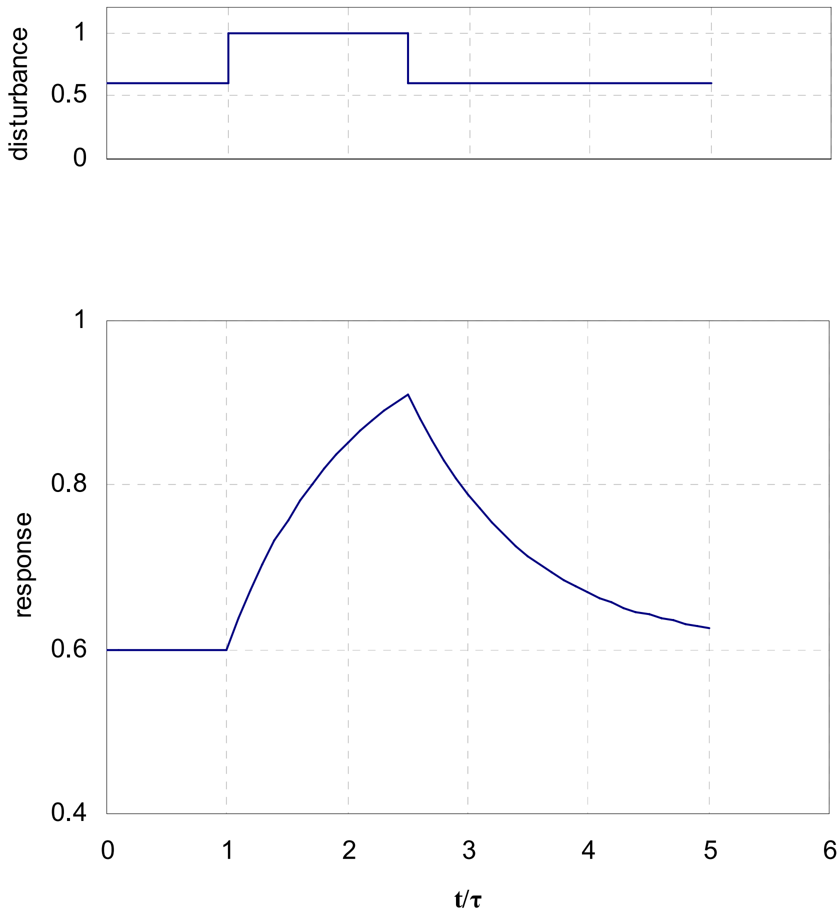

Figure 3.4-1 first-order response to pulse disturbance

3.5 shaking the system - response to sine disturbance

Sine functions typify disturbances that oscillate. Suppose the inlet concentration varies around the reference value with amplitude A and frequency ω, which has dimensions of radians per time.

$$C_{Ai} = C_{A,r} + A\sin(\omega t) \tag{3.5-1}$$

From (3.1-4),

$$C_{Ao} = C_{A,r} - \frac{A\omega\tau}{1+\omega^2\tau^2} e^{-t/\tau} + \frac{A}{\sqrt{1+\omega^2\tau^2}} \sin\left(\omega t + \tan^{-1}(-\omega\tau)\right) \tag{3.5-2}$$

Solution (3.5-2) comprises the mean value $C_{A,r}$, a term that decays with time, and a continuing oscillation term. Thus, the long-term system response to the sine input is to oscillate at the same frequency ω. Notice, however, that the amplitude of the output oscillation is diminished by the square-root term in the denominator. Notice further that the outlet oscillation lags the input by a phase angle $\tan^{-1}(-\omega\tau)$.

In Figure 3.5-1, $C_{A,r} = 0.8$ and $A = 0.5$ in arbitrary units; $\omega\tau$ has been set equal to 2.5 radians, and τ to 1 in arbitrary units. The decaying portion of the solution makes a negligible contribution after the first cycle. The phase lag and reduced amplitude of the solution are evident; our tank has mitigated the inlet disturbance.

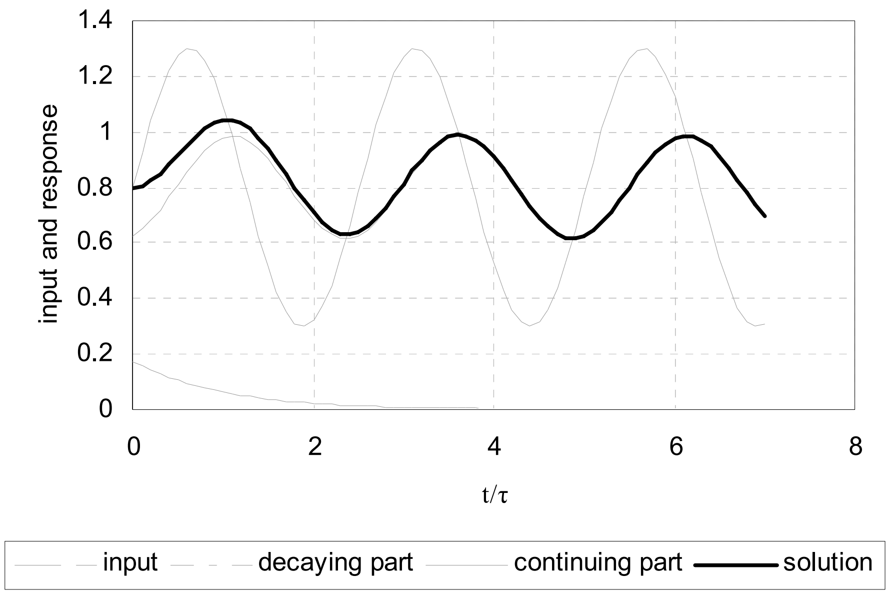

Figure 3.5-1 first-order response to sine disturbance

3.6 frequency response and the Bode plot

The long-term response to a sine input is the most important part of the solution; we call it the frequency response of the system. We will examine the frequency response for an abstract first order system. (Because we wish to focus on the oscillatory response, we will write (3.6-1) so that x and y vary about zero. The effect of a non-zero bias term can be seen in (3.5-1) and (3.5-2).)

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$$
\begin{aligned}
&\text{system:} && \tau \frac{dy}{dt} + y = Kx \\
&\text{input:} && x = A\sin(\omega t) \\
&\text{freq resp:} && y_{fr} = |y_{fr}|\sin(\omega t + \phi) \\
&\text{amplitude:} && |y_{fr}| = \frac{KA}{\sqrt{1+\omega^2\tau^2}} \\
&\text{phase angle:} && \phi = \tan^{-1}(-\omega\tau) \\
&\text{amplitude ratio:} && R_A = \frac{|y_{fr}|}{|x|} = \frac{K}{\sqrt{1+\omega^2\tau^2}}
\end{aligned}
\qquad (3.6\text{-}1)
$$

The frequency response is a sine function, characterized by an amplitude, frequency, and phase angle. The amplitude and phase angle depend on system properties (τ and K) and characteristics of the disturbance input (ω and A). It is convenient to show the frequency dependence on a Bode plot, Figure 3.6-1.

The Bode plot abscissa is ω in radians per time unit; the scale is logarithmic. The frequency may be normalized by multiplying by the system time constant. Thus plotting ω is good for a particular system; plotting $\omega\tau$ is good for systems in general.

The upper ordinate is the amplitude ratio, also on logarithmic scale. R_A is often normalized by dividing by the system gain K. The lower ordinate is the phase angle, in degrees on a linear scale.

In Figure 3.6-1, the coordinates have been normalized to depict first-order systems in general; the particular point represents conditions in the example of Section 3.5.

For a first order system, the normalized amplitude ratio decreases from 1 to 0 as frequency increases. Similarly, the phase lag decreases from 0 to -90º. Both these measures indicate that the system can follow slow inputs faithfully, but cannot keep up at high frequencies.

Another way to think about it is to view the system as a low-pass filter: variations in the input signal are softened in the output, particularly for high frequencies.

The slope of the amplitude ratio plot approaches zero at low frequency; the high frequency slope approaches -1. These two asymptotes intersect at the corner frequency, the reciprocal of the system time constant. At the corner frequency, the phase lag is -45º.

$$\omega_{corner} = \frac{1}{\tau} \tag{3.6-2}$$

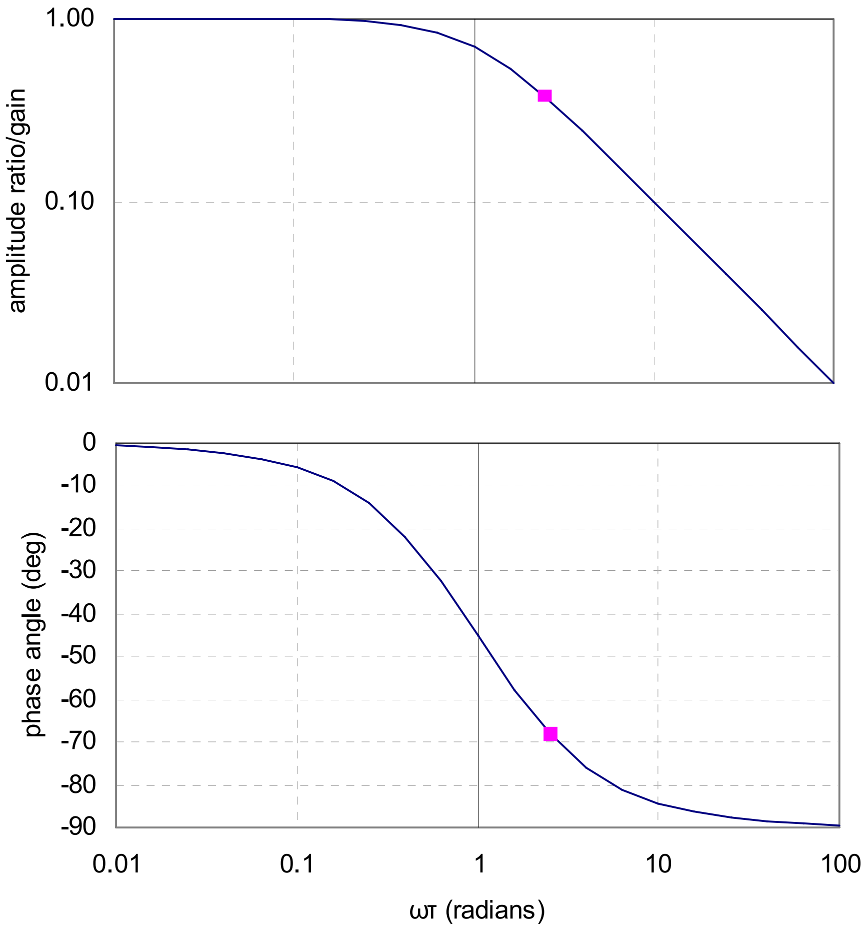


Figure 3.6-1: Bode plot for first-order system

3.7 stability of a system

If we disturb our system, will it return to good operation, or will it get out of hand? This is asking whether the system is stable. We define stability as "bounded output for a bounded input". That means that

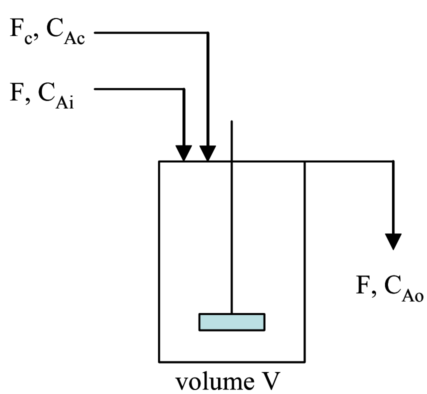
- a ramp disturbance is not fair – even stable systems can get into trouble if the input keeps rising.
- a stable system should handle a step change in input, ultimately coming to some new steady state. (We must be realistic, however. If the system is so sensitive that a small input step leads to an unacceptably high, though steady, output, we might declare it unstable for practical purposes.)

- it should also handle a sine input; here the result is in general not steady state, because the output may oscillate. (Thus we distinguish between 'steady state' and 'long-term stability'.)

The solutions for the typical bounded step, pulse, and sine disturbances, given in Sections 3.3 through 3.5, show no terms that grow with time, so long as the time constant τ is a positive value. For these categories of bounded input, at least, a first-order system appears to be stable. We will need to examine stability again when we introduce automatic control to our process.

3.8 concentration control in a blending tank

In Section 3.1 we described how variations in stream composition could be moderated by passing the stream through a larger volume - a holding tank. Let us be more ambitious and seek to *control* the outlet composition: we add a small inlet stream F_c of concentrated solution to the tank. This will allow us to adjust the composition in response to disturbances.



Our analysis begins as in Section 3.1 with a component material balance.

$$\frac{d}{dt}VC_{Ao} = FC_{Ai} + F_c C_{Ac} - (F + F_c)C_{Ao} \tag{3.8-1}$$

As before, we place (3.8-1) in standard form (response variable on the left with a coefficient of +1).

$$\frac{\frac{V}{F}}{1+\frac{F_c}{F}}\frac{dC_{Ao}}{dt} + C_{Ao} = \frac{1}{1+\frac{F_c}{F}}C_{Ai} + \frac{\frac{C_{Ac}}{F}}{1+\frac{F_c}{F}}F_c \tag{3.8-2}$$

Notice that our equation coefficients each contain the input variable F_c. Notice, as well, that for dilute C_{Ao} and concentrated C_{Ac} stream F_c

(however it may vary) will not be very large in comparison to the main flow F. If this is the case, we may be justified in making an engineering approximation: neglecting the ratio F_c/F in comparison to 1. Thus

$$\frac{V}{F}\frac{dC_{Ao}}{dt} + C_{Ao} = C_{Ai} + \frac{C_{Ac}}{F}F_c \tag{3.8-3}$$

Now we have a linear first-order system. Comparison with (3.1-2) shows the same time constant V/F and the same unity gain for inlet concentration disturbances. There is a new input F_c, whose influence on C_{Ao} (i.e., gain) increases with high concentration C_{Ac} and decreases with large throughflow F.

3.9 use of deviation variables in solving equations

In process control applications, we usually have some desired operating condition. We now write system model (3.8-3) at the target steady state. All variables are at reference values, denoted by subscript r.

$$C_{Ao,r} = C_{Ai,r} + \frac{C_{Ac}}{F}F_{c,r} \tag{3.9-1}$$

We recognize that deviations from these reference conditions represent errors to be corrected. Hence we recast our system description (3.8-3) in terms of deviation variables; we do this by subtracting (3.9-1) from (3.8-3).

$$\begin{gathered}\frac{V}{F}\frac{d\left(C_{Ao} - C_{Ao,r}\right)}{dt} + \left(C_{Ao} - C_{Ao,r}\right) = \left(C_{Ai} - C_{Ai,r}\right) + \frac{C_{Ac}}{F}\left(F_c - F_{c,r}\right) \\ \frac{V}{F}\frac{dC'_{Ao}}{dt} + C'_{Ao} = C'_{Ai} + \frac{C_{Ac}}{F}F'_c\end{gathered} \tag{3.9-2}$$

where we indicate a deviation variable by a prime superscript. The target condition of a deviation variable is zero, indicating that the process is operating at desired conditions. Using deviation variables

- makes conceptual sense for process control because they indicate deviations from desired states
- makes the mathematical descriptions simpler

Thus we shall use deviation variables for derivations and modeling. For doing process control (computing valve positions, e.g.) we will return to the physical variables. We can recover the physical variable by adding its deviation variable to its reference value. For example,

$$C_{Ao}(t) = C_{Ao,r} + C'_{Ao}(t) \tag{3.9-3}$$

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where we emphasize the variables that are time-varying.

3.10 integration from zero initial conditions

As a rule, we will presume that our systems are initially at the reference condition. That is, the initial conditions for our differential equations are zero. Integrating (3.9-2) we find

$$C'_{Ao} = \frac{e^{-t/\tau}}{\tau}\int_0^t e^{t/\tau} C'_{Ai}(t)dt + \frac{e^{-t/\tau}}{\tau}\frac{C_{Ac}}{F}\int_0^t e^{t/\tau} F'_c(t)dt \quad (3.10\text{-}1)$$

Equation (3.10-1) shows how the outlet composition deviates from its desired value $C_{Ao,r}$ under disturbances to inlet composition C_{Ai} and the flow rate of the concentrated makeup stream F_c, where both of these are also expressed as deviations from reference values. Equation (3.10-1) is analogous to (3.1-4) for the simpler holding tank.

3.11 response to step changes

Proceeding as in Section 3.3, we presume a step in inlet composition of ΔC_{Ai} at time t_1 and of ΔF_c in makeup flow at time t_2.

$$\begin{aligned} C'_{Ao} &= \frac{e^{-t/\tau}}{\tau} U(t-t_1)\Delta C_{Ai}\int_{t_1}^t e^{t/\tau}dt + \frac{e^{-t/\tau}}{\tau} U(t-t_2)\Delta F_c \frac{C_{Ac}}{F}\int_{t_2}^t e^{t/\tau}dt \\ &= U(t-t_1)\Delta C_{Ai}\left(1-e^{-(t-t_1)/\tau}\right) + U(t-t_2)\Delta F_c\frac{C_{Ac}}{F}\left(1-e^{-(t-t_2)/\tau}\right) \end{aligned} \quad (3.11\text{-}1)$$

C_{Ao}' exhibits a first-order response to each of these step inputs.

Example: try these numbers:

$V = 6\ m^3$	$t_1 = 0\ s$
$F = 0.02\ m^3\ s^{-1}$	$\Delta C_{Ai} = 1\ kg\ m^{-3}$
$F_{cs} = 10^{-4}\ m^3\ s^{-1}$	$t_2 = 120\ s$
$C_{Ais} = 8\ kg\ m^{-3}$	$\Delta F_c = -5\times10^{-5}\ m^3\ s^{-1}$
$C_{Aos} = 10\ kg\ m^{-3}$	
$C_{Ac} = 400\ kg\ m^{-3}$	

First, verify the steady-state material balance (3.9-1) for the desired conditions:

$$(10)\frac{kg}{m^3} = (8)\frac{kg}{m^3} + (400)\frac{kg}{m^3}\frac{(0.0001)\frac{m^3}{s}}{(0.02)\frac{m^3}{s}} \quad (3.11\text{-}2)$$

(Notice that the exact steady-state balance, derived from (3.8-2), is satisfied to within 1%, so that our approximation in deriving (3.8-3) appears to be reasonable.) The time constant for our process is

$$\tau = \frac{V}{F} = \frac{(6)}{(0.02)} \frac{m^3}{\frac{m^3}{s}} = 300\,s \tag{3.11-3}$$

Substituting values into (3.11-1) we obtain

$$\begin{aligned} C'_{Ao} &= U(t-0)(1)\frac{kg}{m^3}\left(1-e^{-(t-0)/300}\right) + U(t-120)(400)\frac{kg}{m^3}\frac{(-5\times10^{-5})}{(0.02)}\left(1-e^{-(t-120)/300}\right) \\ &= (1)\frac{kg}{m^3}\left(1-e^{-t/300}\right) + U(t-120)(-1)\frac{kg}{m^3}\left(1-e^{-(t-120)/300}\right) \end{aligned} \tag{3.11-4}$$

where t must be computed with units of seconds. In Figure 3.11-1, we can see that the reduction in make-up flow at 120 s compensates for the earlier increase in inlet composition. Now we are ready to consider control.

CONTROL SCHEME

3.12 developing a control scheme for the blending tank

A control scheme is the plan by which we intend to control a process. A control scheme requires:

1) specifying control objectives, consistent with the overall objectives of safety for people and equipment, environmental protection, product quality, and economy
2) specifying the control architecture, in which various of the system variables are assigned to roles of controlled, disturbance, and manipulated variables, and their relationships specified
3) choosing a controller algorithm
4) specifying set points and limits

3.13 step 1 - specify a control objective for the process

Our control objective is to maintain the outlet composition at a constant value. Insofar as the process has been described, this seems consistent with the overall objectives.

3.14 step 2 - assign variables in the dynamic system

The controlled variable is clearly the outlet composition. The inlet composition is a disturbance variable: we have no influence over it, but must react to its effects on the controlled variable. We do have available a variable that we can manipulate: the make-up flow rate.

We specify feedback control as our control architecture: departure of the controlled variable from the set point will trigger corrective action in the manipulated variable. Said another way, we manipulate make-up flow to control outlet composition.

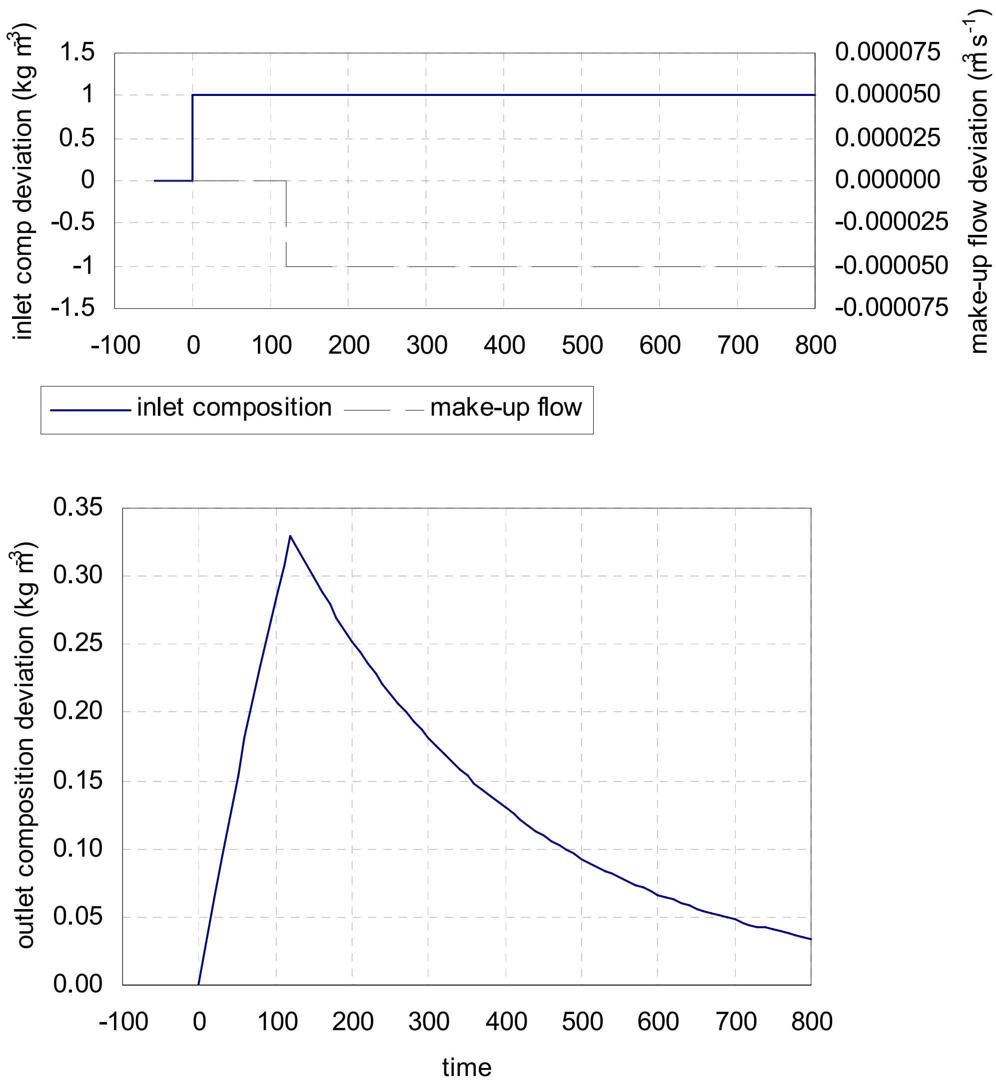

Figure 3.11-1 outlet composition response to opposing step inputs

3.15 step 3 - introduce proportional control for our process

The controller algorithm dictates how the manipulated variable is to be adjusted in response to deviations between the controlled variable and the set point. We will introduce a simple and plausible algorithm, called proportional control. This algorithm specifies that the magnitude of the manipulation is directly proportional to the magnitude of the deviation.

$$F_c - F_{bias} = K_{gain}\left(C_{Ao,setpt} - C_{Ao}\right) \tag{3.15-1}$$

In algorithm (3.15-1) the controlled variable C_{Ao} is subtracted from the set point. (Subtracting from the set point, rather than the reverse, is a convention.) Any non-zero result is an error. The error is multiplied by the controller gain K_{gain}. Their product determines the degree to which manipulated variable F_c differs from F_{bias}, its value when there is no error. The gain may be adjusted in magnitude to vary the aggressiveness of the controller. Large errors and high gain lead to large changes in F_c.

We must consider the direction of the controller, as well as its strength: should the outlet composition exceed the set point, the make-up flow must be reduced. Algorithm (3.15-1) satisfies this requirement if controller gain K_c is positive.

3.16 step 4 - choose set points and limits

The set point is the target operating value. For many continuous processes this target rarely varies. In our blending tank example, we may always desire a particular outlet concentration. In other cases, such as a process that makes several grades of product, the set point might be varied from time to time. In batch processes, moreover, the set point can show frequent variation because it provides the desired trajectory for the time-varying process conditions.

Several sorts of limits must be considered in control engineering:

<u>safety limits</u>: if a variable exceeds these limits, a hazard exists. Examples are explosive composition limits on mixtures, bursting pressure in a vessel, temperatures that trigger runaway reactions.

These limits are determined by the process, and the control scheme must be designed to abide by them.

<u>expected variation</u>: it is necessary to estimate how much variation might be expected in a disturbance variable. This estimate is the basis for specifying the strength of the manipulated variable response. In Section 3.11, our system model (based on the material balance) showed us how much variation in make-up flow, at specified make-up composition, was required to compensate for a particular change in the inlet composition.

These limits are determined by the process and its environment. No amount of controller design can compensate for a manipulated variable that is unequal to the disturbance task.

tolerable variation: ideally the controlled variable would never deviate from the set point. This, of course, is unrealistic; in practice some variation must be tolerated, because

- obtaining enough information on the process and disturbance is usually impossible, and in any case too expensive.
- exerting sufficient manipulative strength to suppress variation in the control variable might be expected to require large variations in the manipulated variable, which can cause problems elsewhere in the process.

Tolerable limits are determined by the safety limits, above, and then an economic analysis that considers the cost of variation and the cost of control. We do not expect to achieve perfect control, but good control is usually worth spending some money.

For the blending tank example, then, we select:

- set point: $C_{Ao,setpt} = 10$ kg m^{-3}. This would be determined by the user of the stream.
- safety limits: none apparent from problem statement
- expected variation: ±1 kg m^{-3}; such a specification might come from historical data or engineering calculations. The steady-state material balance (e.g., (3.11-1) applied at long times) shows that the make-up flow must vary at least $\pm 5\times 10^{-5}$ m^3 s^{-1} to compensate such disturbances. However, might we need more capability during the course of a transient??
- tolerable variation: ±0.1 kg m^{-3}. This specification depends on the user of the stream.

EQUIPMENT

3.17 type of equipment needed for process control

Figure 3.17-1 shows our process and control scheme as two communicating systems. The system representing the process has two inputs and one output. Of these only one is a material stream; however, we recall that systems communicate with their environment (and other systems) through signals, and in the blending process the outlet composition responds to the inlet composition and make-up flow rate.

The system representing feedback control describes the needed operations, but we have not described the nature of the equipment – could there be a single device that takes in a composition measurement and puts out a flow? Can we find a vendor to make such a device to execute controller algorithm (3.15-1)? Can we have the gain knob calibrated in units consistent with those we want to use for flow and composition?

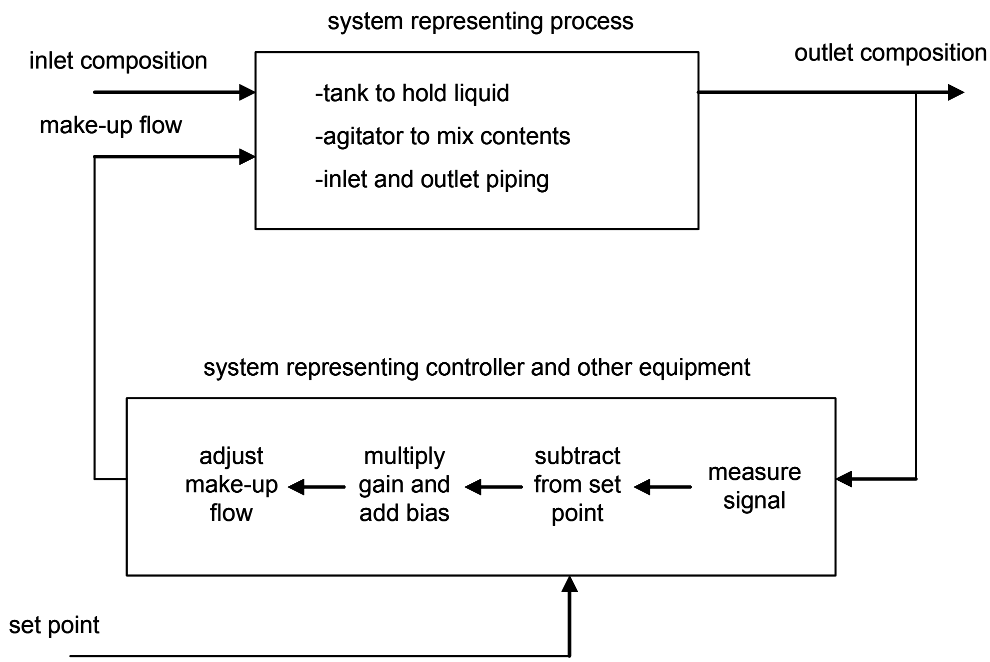

Figure 3.17-1: Closed loop feedback control of process

We will address these questions in later lessons. For now, we assume that there will be several distinct pieces of equipment involved, and that they work together so that

$$F_c - F_{bias} = K_c\left(C_{Ao,setpt} - C_{Ao}\right) \tag{3.17-1}$$

where we use the conventional symbol K_c for controller gain. In the case of (3.17-1), we notice that the dimensions of K_c are volume^2 mass^{-1} time^{-1}.

In good time we will improve our description of both equipment and controller algorithms. When we do, however, we will find that the overall concept of feedback control is the same as presented in Figure 3.17-1: the controlled variable is measured, decisions are made, and the manipulated variable is adjusted to improve the controlled variable.

CLOSED LOOP BEHAVIOR

3.18 closing the loop - feedback control of the blending process

Our next task will be to combine our controller algorithm with our system model to describe how the process behaves under control. We begin by expressing algorithm (3.17-1) in deviation variables. At the reference condition, all variables are at steady values, indicated by subscript r.

$$F_{c,r} - F_{bias} = K_c\left(C_{Ao,setpt,r} - C_{Ao,r}\right) \tag{3.18-1}$$

Presumably the reference condition has no error, so that the set point is simply the target outlet composition $C_{Ao,r}$. Thus we learn that F_{bias}, the zero-error manipulated variable value, is simply $F_{c,r}$. Subtracting (3.18-1) from (3.17-1), we find

$$\begin{aligned} F_c - F_{c,r} - F_{bias} + F_{bias} &= K_c\left(C_{Ao,setpt} - C_{Ao,setpt,r} - C_{Ao} + C_{Ao,r}\right) \\ F_c' &= K_c\left(C_{Ao,setpt}' - C_{Ao}'\right) \end{aligned} \tag{3.18-2}$$

If the set point remains at $C_{Ao,r}$, the deviation variable $C'_{Ao,setpt}$ will be identically zero.

We replace the manipulated variable in system model (3.9-2) with controller algorithm (3.18-2) to find

$$\tau\frac{dC_{Ao}'}{dt} + C_{Ao}' = C_{Ai}' + \frac{C_{Ac}}{F}K_c\left(C_{Ao,setpt}' - C_{Ao}'\right) \tag{3.18-3}$$

On expressing (3.18-3) in standard form, we arrive at a first-order dynamic system model representing the process under proportional-mode feedback control, as shown in Figure 3.17-1.

$$\frac{\tau}{1+\frac{C_{Ac}K_c}{F}}\frac{dC_{Ao}'}{dt} + C_{Ao}' = \frac{1}{1+\frac{C_{Ac}K_c}{F}}C_{Ai}' + \frac{\frac{C_{Ac}K_c}{F}}{1+\frac{C_{Ac}K_c}{F}}C_{Ao,setpt}' \tag{3.18-4}$$

Equation (3.18-4) describes a dynamic system (process and controller in closed loop) in which the outlet composition varies with two inputs: the inlet composition and the set point. Figure 3.18-1 compares (3.18-4) with the process model (3.9-2) alone; we see that

- the closed loop responds more quickly because the closed loop time constant is less than process time constant τ.
- the closed loop has a smaller dependence on disturbance C'_{Ai} because the gain is less than unity. Both time constant and gain are reduced by increasing the controller gain K_c.

3.19 integration from zero initial conditions

In Section 3.10, we integrated our open-loop system model to find how C'_{Ao} responded to inputs C'_{Ai} and F'_c. Now we integrate closed-loop system model (3.18-4) in a similar manner.

$$C_{Ao}' = \frac{e^{-t/\tau_{CL}}}{\tau_{CL}}K_{CL}\int_0^t e^{t/\tau_{CL}}C_{Ai}'dt + \frac{e^{-t/\tau_{CL}}}{\tau_{CL}}K_{SP}\int_0^t e^{t/\tau_{CL}}C_{Ao,setpt}'dt \tag{3.19-1}$$

where

$$\tau_{CL} = \frac{\tau}{1+\dfrac{C_{Ac}K_c}{F}} \qquad K_{CL} = \frac{1}{1+\dfrac{C_{Ac}K_c}{F}} \qquad K_{SP} = \frac{\dfrac{C_{Ac}K_c}{F}}{1+\dfrac{C_{Ac}K_c}{F}} \tag{3.19-2}$$

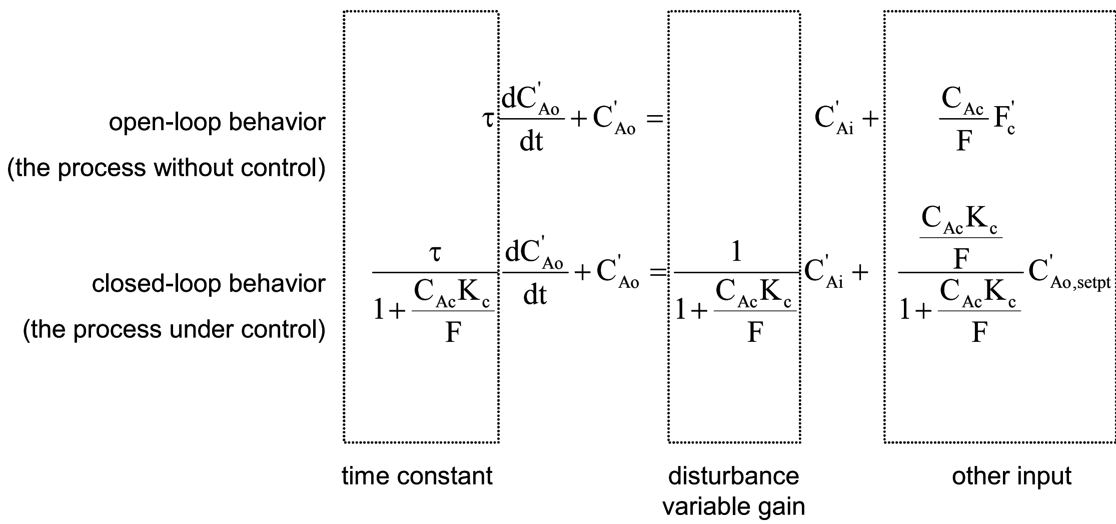

Figure 3.18-1: Comparing open- and closed-loop system descriptions

3.20 closed-loop response to pulse disturbance

We test our controlled process by a pulse ΔC in the inlet composition that begins at time t_1 and ends at t_2. We find

$$C^*_{Ao} = \begin{cases} 0 & 0 < t < t_1 \\ \Delta C_{Ai} K_{CL}\left(1 - e^{-(t-t_1)/\tau_{CL}}\right) & t_1 < t < t_2 \\ \left[\Delta C_{Ai} K_{CL}\left(1 - e^{-(t_2-t_1)/\tau_{CL}}\right)\right] e^{-(t-t_2)/\tau_{CL}} & t_2 < t \end{cases} \tag{3.20-1}$$

Figure 3.20-1 shows both uncontrolled (open-loop) and controlled (closed-loop) process responses for the same operating conditions used in Section 3.11. We see the faster response and smaller error that we expected when we examined (3.18-4) in Section 3.18. These characteristics improve as gain increases. Increasing gain also elicits stronger manipulated variable action. Thus automatic control appears to have improved matters.

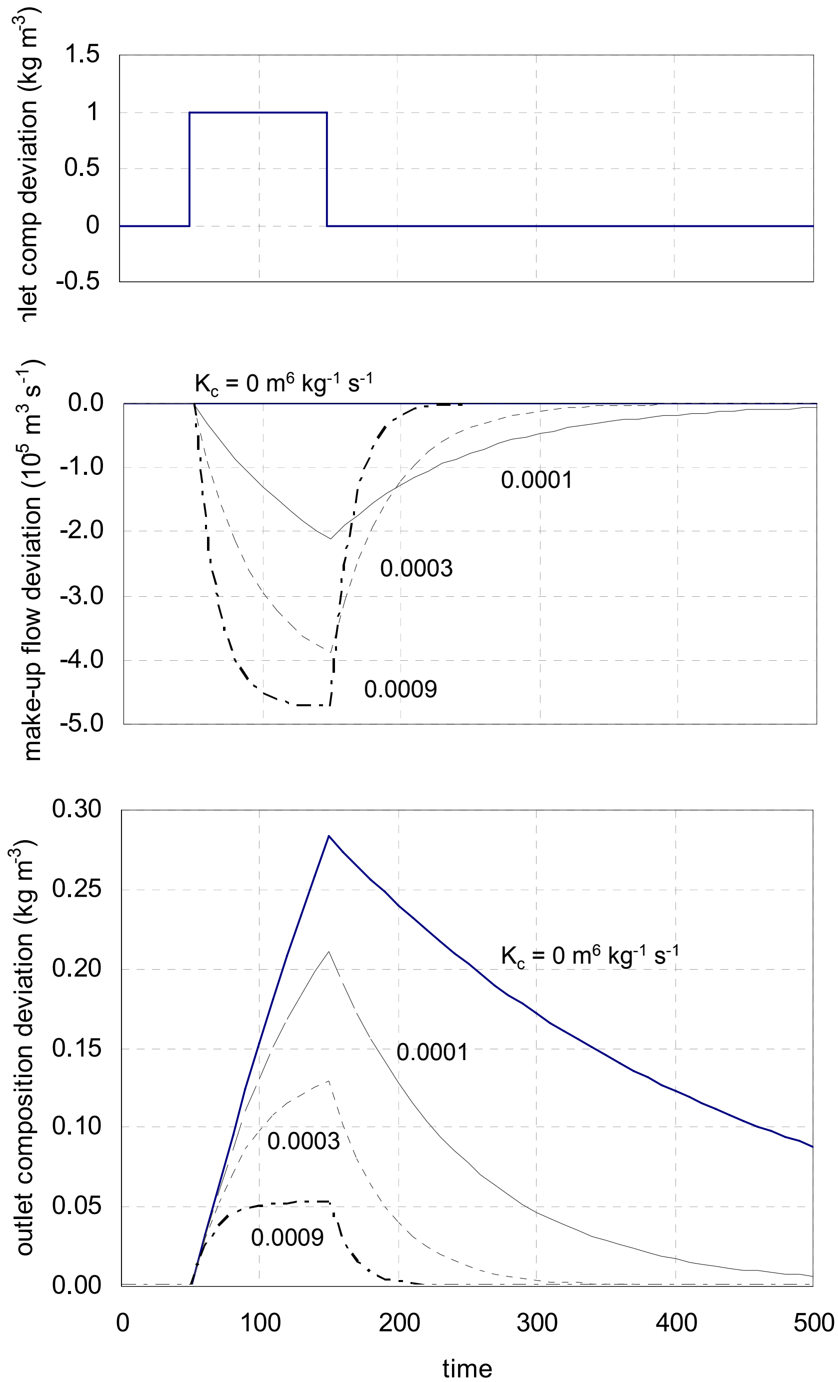

Figure 3.20-1: Response to pulse input under proportional control.

3.21 closed-loop response to step disturbance - the offset phenomenon

Integrating (3.19-1) for a step of ΔC, we obtain

$$C'_{Ao} = \Delta C\, K_{CL}\left(1 - e^{-t/\tau_{CL}}\right) \qquad (3.21\text{-}1)$$

Figure 3.21-1 shows open- and closed-loop step responses. Notice that for no case does the controlled variable return to the set point! This is the phenomenon of offset, which is a characteristic of the proportional control algorithm responding to step inputs.

$$\begin{aligned} \text{offset} &= \text{longterm response - set point} \\ &= C_{Ao}(\infty) - C_{Ao,setpt} \\ &= C'_{Ao}(\infty) \\ &= \Delta C K_{CL} \end{aligned} \tag{3.21-1}$$

Recalling (3.19-2), increasing the controller gain decreases the closed-loop disturbance gain K_{CL}, and thus decreases the offset.

We find that offset is implicit in the proportional control definition (3.15-1). An off-normal disturbance variable requires the manipulated variable to change to compensate. For the manipulated variable to differ from its bias value, (3.15-1) shows that the error must be non-zero. Hence some error must persist so that the manipulated variable can persist in compensating for a persistent disturbance.

3.22 response to set point changes

We apply (3.19-1) to a change in set point.

$$C'_{Ao} = \Delta C_{Ao,setpt} K_{SP} \left(1 - e^{-t/\tau_{CL}}\right) \tag{3.22-1}$$

We recall from (3.19-2) that K_{SP} is less than 1. Thus, the outlet composition follows the change, but cannot reach the new set point. This is again offset due to proportional-mode control. Increasing controller gain increases K_{SP} and reduces the offset.

3.23 tuning the controller

Choosing values of the adjustable controller parameters, such as gain, for good control is called tuning the controller. So far, our experience has been that increasing the gain decreases offset - then should we not set the gain as high as possible?

We should not jump to that conclusion. In general, tuning positions the closed-loop response between two extremes. At one extreme is no control at all, gain set at zero (open-loop). At the other is too much attempted control, driving the system to instability. In the former case, the controlled variable wanders where it will; in the latter case, over-aggressive manipulation produces severe variations in the controlled variable, worse than no control at all. Tuning seeks a middle ground in which control reduces variability in the controlled variable. This means both rejection of disturbances and fidelity to set point changes.

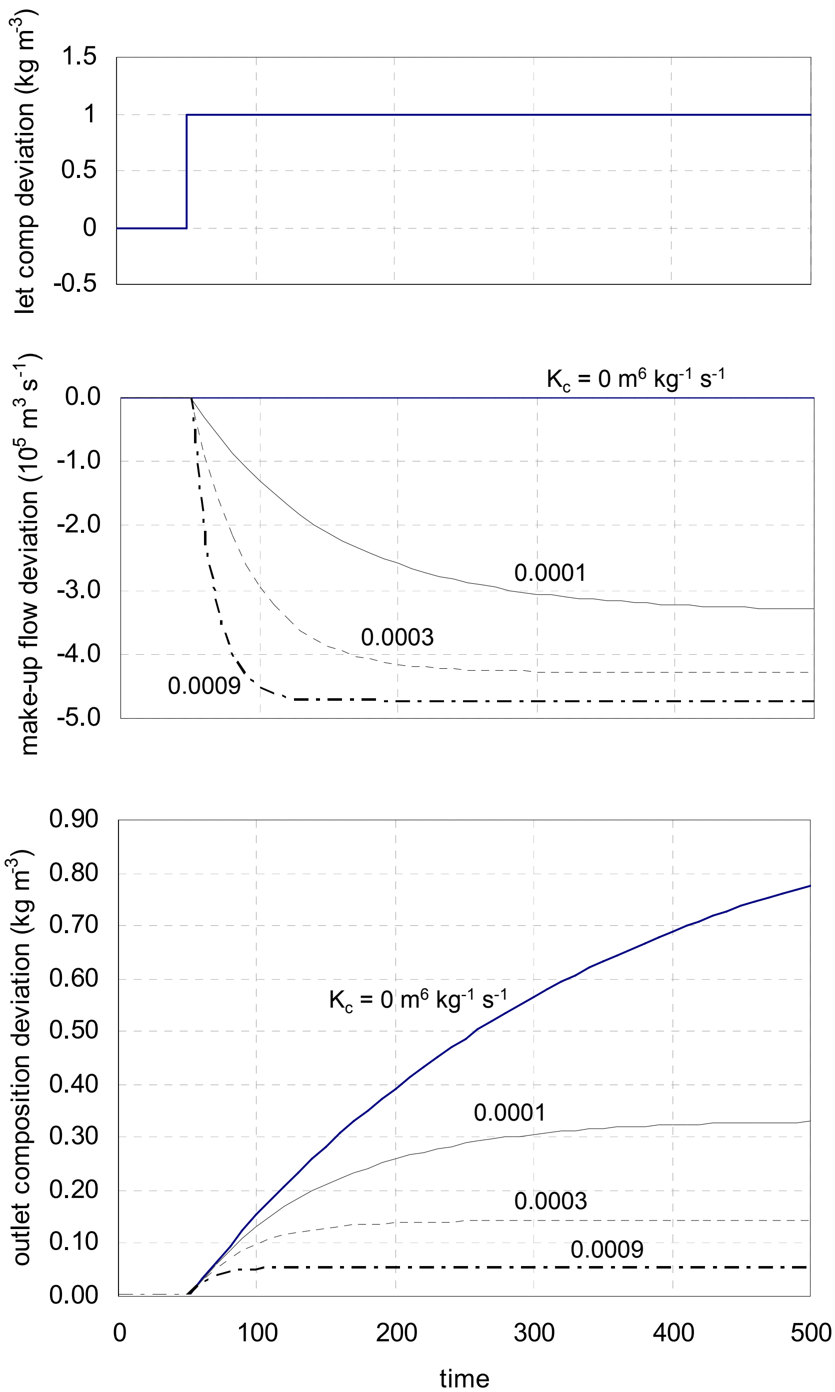

Figure 3.21-1: proportional control step response, showing offset

Lesson 3: The Blending Tank

Recall Figures 3.20-1 and 3.21-1. In these, we achieved our ±0.1 kg m^{-3} specification on outlet concentration at a gain between 0.0003 and 0.0009 m^6 kg^{-1} s^{-1}. Hence we will use our model, which predicts the response to disturbances, to guide us in tuning. In operation, we would use the predicted value as a starting point, and make further adjustments, if required, in the field.

3.24 stability of the closed-loop system

In Section 3.23, we said that tuning positions the controlled process between non-control and instability. We must therefore inquire into the stability limit. Because (3.19-1) describes the closed-loop system, we should be able to seek the conditions under which it becomes unstable. We invoke the notion of stability to bounded inputs that we introduced in Section 3.7, and we come to the same conclusion we reached there: a first-order system is stable to all bounded inputs, and we have not changed the order of the system by adding feedback control in the proportional mode.

So "theoretically" we can increase gain as much as we like with no possibility of reaching instability. Equation (3.19-2) shows that in the limit of infinite controller gain, the response will be instantaneous (τ_{CL} = 0), disturbances will be completely rejected (K_{CL} = 0) and set points will be faithfully tracked (K_{SP} = 1).

Practically, we will not be surprised to find that this is NOT true. No automatically-controlled chemical process will really be first order. Increasing the gain in real processes will ultimately lead to instability. We will explore this point further in future lessons.

3.25 conclusion

We have done quite a lot:

- used conservation equations to derive a dynamic system model of a process
- identified three characteristic disturbances to test system responses
- introduced the frequency response and Bode plots
- discussed how to formulate a control scheme
- introduced proportional-mode control and explored its behavior
- compared open- and closed-loop response
- learned how tuning fits between limits of no control and instability

We will elaborate each of these topics in later lessons.