10.52
Mechanics of Fluids
Spring 2006
Problem Set 6

Problem 1

Consider the differential equation:

$$\varepsilon^2 \frac{d^2f}{dx^2} - f = -e^{-2x^2}$$

subject to $f = o$ at $x = o$
and $f \rightarrow o$ as $x \rightarrow \infty$

You may assume that $\varepsilon^2 \ll 1$.

Please obtain a solution which is uniformly valid to order ε^2.

Problem 2

Consider a shallow $\left(\lambda / H = \dfrac{2\pi}{kH} \ll 1 \right)$ water wave of small amplitude (h/H<<1), as shown in the figure. If the water is treated as an ideal ($\mu=o$) fluid, the velocity in the x direction is independent of depth and is given by

$$u = u_o \cos \{k\,(x\text{-}ct)\}$$

(a) What is the corresponding equation for "h"?

(b) Find an expression for c in terms of g, H, etc.

(c) It is speculated that most of the viscous dissipation will occur in a boundary layer located near y = o. In other words, it is expected that the following will be true:

$$\frac{\displaystyle\int_{o}^{H} \Phi_{BULK}\, dy}{\displaystyle\int_{o}^{\delta} \Phi_{B/L}\, dy} \cong \frac{k^2 u_0^2 H}{\left(\dfrac{u_o}{\delta}\right)^2 \delta} = k^2 H\delta \ll 1$$

Thus, there is an incentive to obtain the velocity profile in the boundary layer (because it is largely responsible for the viscous dissipation).

(i)   Assume that the flow near $y = 0$ is of the boundary layer type, and obtain an order-of magnitude expression for the boundary layer thickness.

(ii)   What criterion must be satisfied if the flow is to be of the boundary layer type?

(iii) Obtain an expression for u(y,t) within the boundary layer.

Hints:

(i)   For (a) and (b), it may be helpful to use a differential control volume of the sort shown in the figure.

(ii)   Remember that products of small quantities may be neglected.

Figure for Problem 2 (Not to scale)
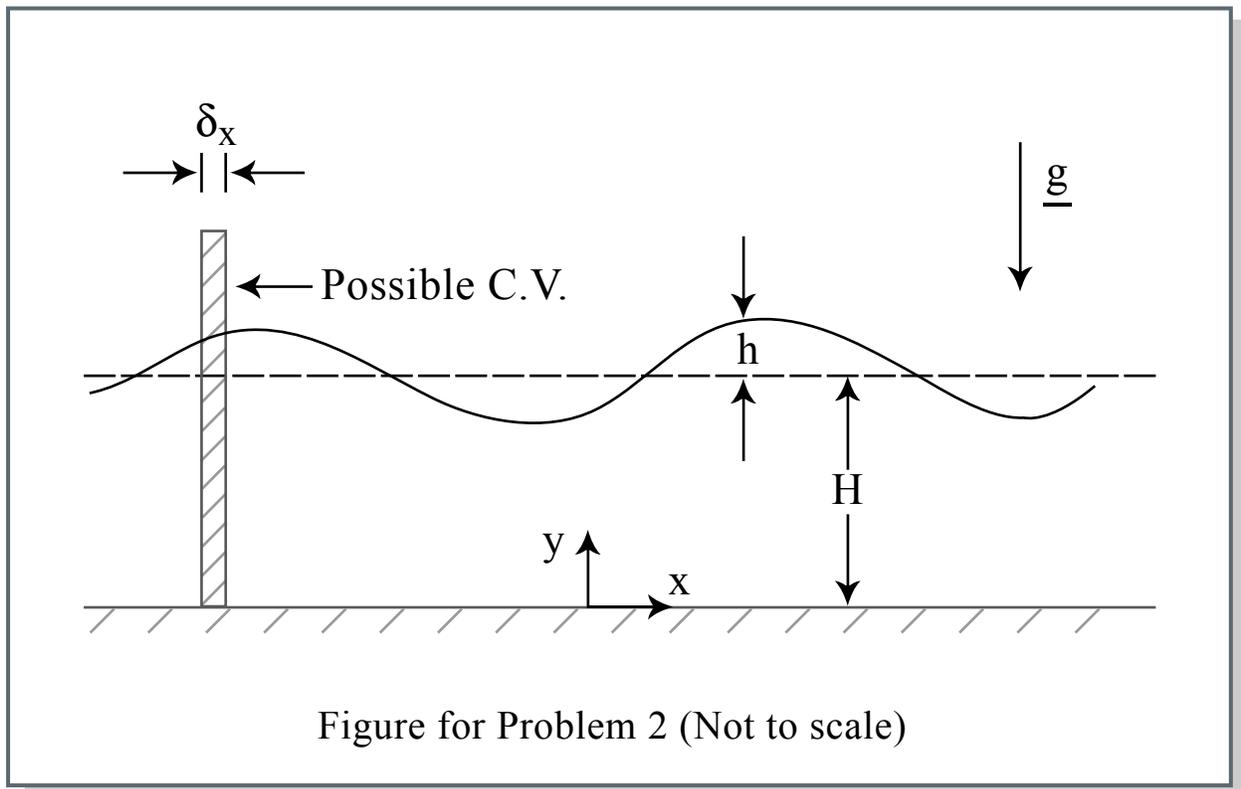
Problem 3

Problem 8-6(a) of Deen reads as follows:

> "Consider a jet of fluid which enters a large, stagnant volume of the same fluid via a small, circular hole in a wall. This problem is the axisymmetric analog of Example 8.5-2, and it too has an exact, analytical solution. The boundary layer momentum equation in this case is
>
> $$v_r \frac{\partial v_z}{\partial r} + v_z \frac{\partial v_z}{\partial z} = \frac{\nu}{r} \frac{\partial}{\partial r}\left(r \frac{\partial v_z}{\partial r} \right)$$
>
> and the kinematic momentum, which again is a known constant, is defined as
>
> $$K = 2\pi \int_0^\infty v_z^2 r \, dr$$
>
> (a) To obtain a similarity solution, assume that the stream function is of the form
>
> $$\psi(r,z) = \nu z^p F(\eta), \qquad \eta = \frac{r}{z^n}$$
>
> where the factor ν has been introduced for later convenience. Show that $p = n = 1$."

Please use order-of-magnitude methods to obtain the same similarity transformation.

Problem 4 (Problem by Ain Sonin and Ascher Shapiro. Used with permission.)

The kinetics of very fast chemical reactions are sometimes studied in shock tubes. The great advantage of this device is the extreme rapidity with which the reactants are brought to temperature.

For simplicity, consider the case in which no chemical reaction occurs and the fluid is simply air. The shock tube consists of a reservoir connected to a pipe which is separated into two portions by a diaphragm (see sketch on page 5). Air is admitted under pressure to the reservoir. Then the diaphragm is burst, and the high pressure gas rushes into the pipe, the useful fact being that the disturbance propagates down the pipe as essentially a step function. Moreover, the velocity of propagation is independent of time (see sketch).

If, in terms of the sketches:

$P_2 = 8$ atm

$P_1 = 1$ atm $T_1 = 20°C$ $V_1 = 0$

$L = 50$ m $D = 25$ cm

determine the time for the discontinuity to travel from the diaphragm to the end of the pipe if air is the only fluid. That is, estimate the duration of a kinetic experiment.

Data: $R = 8.315$ kJ/kmol·K $= 8.205 \times 10^{-2}$ m³·atm/kmol·K

       $C_p = 29.3$ kJ/kmol·K

ILLUSTRATION

Diaphragm

Reservoir

D

L
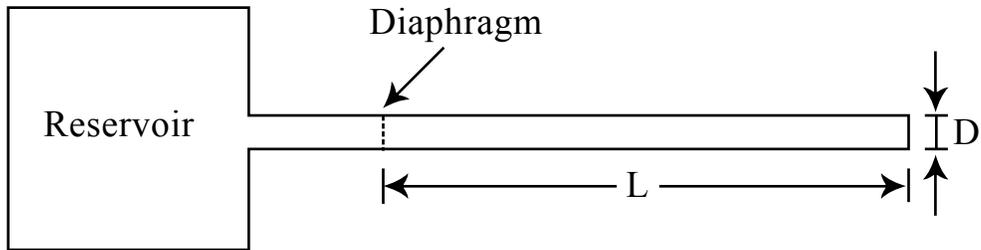
Sketch of Equipment

Temperature
Profile

T_2

T_1

Velocity
Profile

V_2

V_1
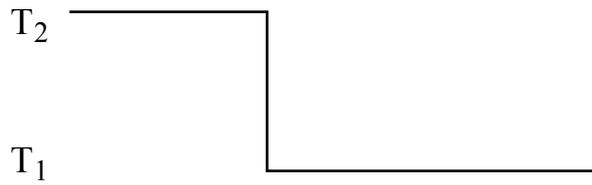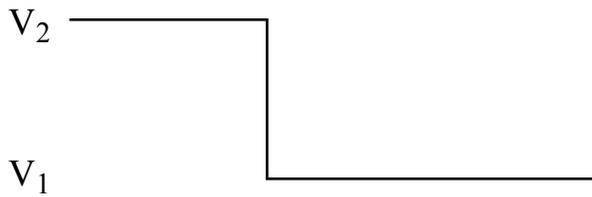
Pressure
Profile

P_2

P_1
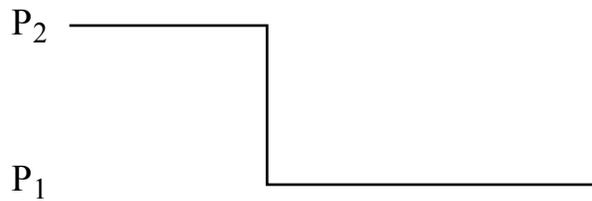
Problem 5

Consider fully developed flow in a parallel-plate channel in which there is a constant heat flux q_w at both surfaces for x>0. The plate spacing is 2H. Show that the Nusselt number for large x is given by

$$Nu = \frac{2hH}{k} = \frac{70}{17} = 4.118$$

Problem 6

Problem 5 addresses the issue of heat transfer in a two dimensional duct for the case $x \to \infty$. Please now treat the case for $x \to o$, for which the thermal boundary layer will be thin compared to the separation between the two plates. Continue to assume that the flow is hydrodynamically fully developed. Do not attempt to get an exact result for Nu, but do obtain an order-of-magnitude result for Nu as a function of H, k, x, etc. Is this different than you would obtain for an isothermal wall? Also, obtain the ODE that you would solve in order to get an exact result.

Problem 7

The potential function for the flow of an inviscid, incompressible fluid past a right circular cylinder is:

$$\phi_{CYL} = u_\infty \left(r + \frac{a^2}{r} \right) \cos\theta$$

Develop a relationship for the heat transfer coefficient in the neighborhood of the stagnation point. You may assume that the fluid is Newtonian, that the Reynolds Number is very high, and that the Prandtl Number is very much greater than one. Your relationship should be functionally correct, but it may contain a multiplicative constant of order one. What equations would you solve in order to determine the exact value of the multiplicative constant? Can a simple solution be obtained for Pr << 1? If so, what are the equations that would be solved to obtain an exact result? Do any additional criterian need to be satisfied?