

# 5.73

## Quiz 22

$$[\mathbf{L}_i, \mathbf{p}_j] = i\hbar \sum_k \varepsilon_{ijk} \mathbf{p}_k$$

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$\varepsilon_{ijk} =$  +1 if  $ijk$  are in cyclic order (i.e.  $xyz$ ,  $yzx$ , or  $zxy$ )  
-1 if  $ijk$  are in anti-cyclic order  
0 if any index is repeated.

$$\mathbf{L} = (\mathbf{q} \times \mathbf{p}) = \begin{pmatrix} \hat{i} & \hat{j} & \hat{k} \\ \mathbf{x} & \mathbf{y} & \mathbf{z} \\ \mathbf{p}_x & \mathbf{p}_y & \mathbf{p}_z \end{pmatrix}$$

- A. What are  $\mathbf{L}_y$  and  $\mathbf{L}_z$  in terms of  $(\mathbf{x}, \mathbf{y}, \mathbf{z})$  and  $(\mathbf{p}_x, \mathbf{p}_y, \mathbf{p}_z)$ ?
- B. Use  $\varepsilon_{ijk}$  notation to evaluate  $[\mathbf{L}_x, \mathbf{x}]$ ,  $[\mathbf{L}_x, \mathbf{z}]$ ,  $[\mathbf{L}_x, \mathbf{p}_x]$ , and  $[\mathbf{L}_x, \mathbf{p}_z]$ .
- $[\mathbf{L}_x, \mathbf{x}] =$
- $[\mathbf{L}_x, \mathbf{z}] =$
- $[\mathbf{L}_x, \mathbf{p}_x] =$
- $[\mathbf{L}_x, \mathbf{p}_z] =$
- C. Use the results of part B to show that  $[\mathbf{L}_x, \mathbf{L}_y] = i\hbar \mathbf{L}_z$ . Recall that  $[\mathbf{A}, \mathbf{BC}] = \mathbf{B}[\mathbf{A}, \mathbf{C}] + [\mathbf{A}, \mathbf{B}]\mathbf{C}$ .

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