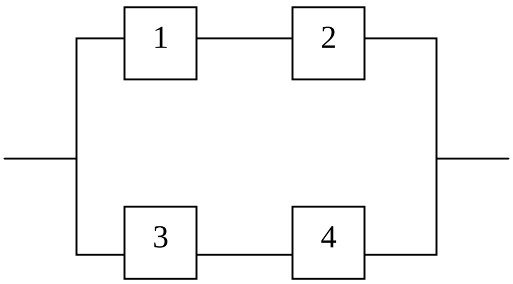
1.010 Uncertainty in Engineering
Fall 2008

# 1.010 – Final Exam

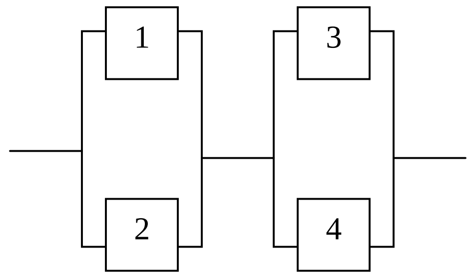
(3 hours – open books and notes)

***Problem 1 (10 Points)***

Christmas lights sold at Wall-Mart come in sets of 4 bulbs of different colors connected in two alternative configurations as shown below:




**Configuration 1**




**Configuration 2**

If both configurations cost the same, which configuration would you buy for a lower probability of failure (dark Christmas tree). Assume that each light bulb fails with probability 0.03 and bulbs fail independently.

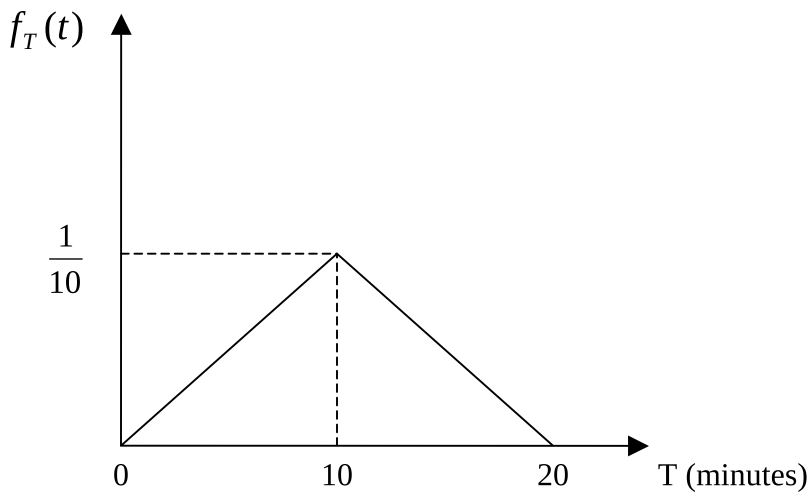
***Problem 2 (10 Points)***

A computer can generate random variables $U$ with uniform distribution in the interval [0, 1]. Suppose that you generate 10 such variables, $U_1$, $U_2$, … , $U_{10}$. Let N be the number of variables $U_i$ with value in the interval [0.4, 0.5]. Find:

(a) The probability that N = 0.
(b) The probability mass function of N.
(c) The mean value and variance of N.

***Problem 3 (10 Points)***

Suppose that the interarrival times T of busses has triangular distribution as shown below:




If it has been 10 minutes since the last bus left, find the probability that a bus will arrive:

(a) Within the next 5 minutes.
(b) Within the next 10 minutes.

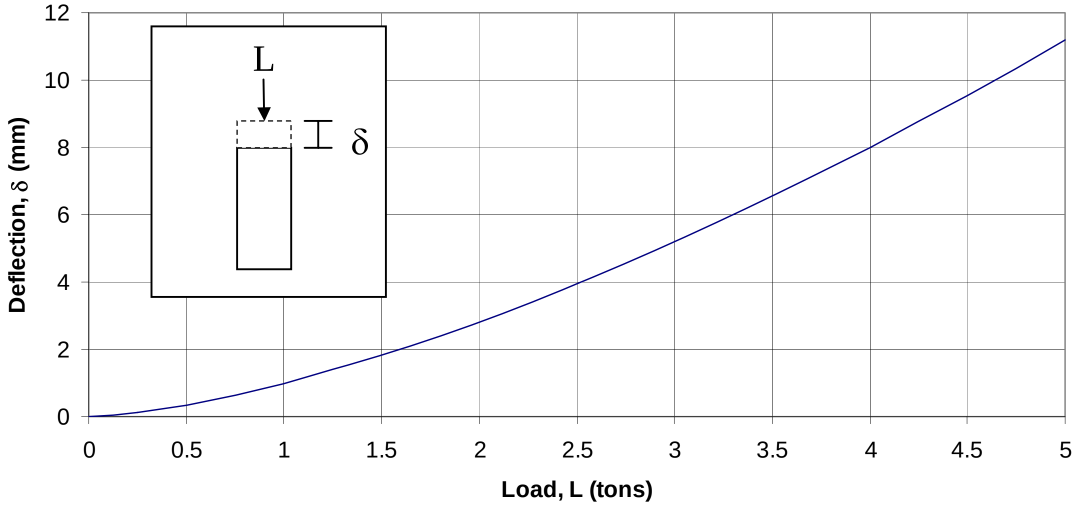
***Problem 4 (15 Points)***

Days may be sunny ($I_1 = 1$) or cloudy ($I_1 = 0$), and either dry ($I_2 = 1$) or humid ($I_2 = 0$). Suppose that 80% of the days are sunny and of these, 80% are dry. Of the cloudy days, 50% are dry. Find:

(a) The joint PMF of $I_1$ and $I_2$.
(b) The marginal PMF of $I_2$.
(c) The mean value and variance of $I_1$ and $I_2$.
(d) The covariance between $I_1$ and $I_2$.

***Problem 5 (10 Points)***

The deflection of a column, δ (in mm) depends on the applied load L (in tons) as $\delta = L^{1.5}$. This function is plotted in the figure below.




Suppose that L has exponential distribution with probability density function:

$$f_L(\ell) = \begin{cases} e^{-\ell}, & \ell \geq 0 \\ 0, & \ell < 0 \end{cases}$$

Find:

(a) The probability density function of δ.
(b) The probability that δ exceeds 3 mm.

***Problem 6 (10 Points)***

At a given location, the mean daily temperature in December (in degrees Celsius) is $10^o$, with a standard deviation of $5^o$. The correlation coefficient between temperatures in day *i* and *i* + *j* is $\rho_{i,i+j} = 0.8^{|j|}$. Find the mean value and variance of the average temperature in a three day period.

***Problem 7 (10 Points)***

A coin you found on a beach looks old. A friend, who is an antique dealer, says that it is probably 150 years old $\pm$ 40 years (meaning that the mean value of the age is 150 years and the standard deviation is 40 years). You decide to consult a coin expert who says that the coin may actually be 120 years old $\pm$ 20 years (consider this as a 'noisy' observation). How would you combine the two estimates to find the most likely age an its uncertainty?

[Hint: You could view 150 $\pm$ 40 years as your own uncertainty prior to consulting the coin expert, and the expert assessment of 120 years as a 'noisy measurement' of the true age, with an error standard deviation of 20 years. Notice that in this case, the coin expert acts like an imperfect measuring device.]

***Problem 8 (10 Points)***

The condition for failure of a column is given by $D + L > R$, where *D* is the dead load, *L* is the live load, and *R* is the resistance, all expressed in the same units.

Suppose that *D*, *L* and *R* are independent normally distributed variables with the following distributions:

$$D \sim N(100, 25^2), \quad L \sim N(150, 50^2) \text{ and } R \sim N(300, 20^2)$$

Find the probability of failure of the column.

***Problem 9 (15 Points)***

Hurricanes occur according to a Poisson Point Process with unknown parameter $\lambda$. Given that 5 hurricanes occurred during a two-month period, estimate $\lambda$ by:

(a) The Method of Moments
(b) The Method of Maximum Likelihood