1.022 Introduction to Network Models

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Lecture 1

Contributors/Acknowledgement

- Four other instructors contributing in putting together this course:

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  Santiago Segarra

  Milad Siami

Prerequisites

- Courses: 1.00 or 1.000; 18.03; 1.010 (or permission of instructor)
- Mathematical maturity
  - ⇒ Proofs and deductive reasoning
  - ⇒ Basic probability and statistics
  - ⇒ Basic calculus and linear algebra
- Programming
  - ⇒ We will use python for homework and final project
  - ⇒ Recitation will go over basics
  - ⇒ No need to know python in advance but ...
  - ⇒ Need to know basic programming logic (conditional, loops, ...)
- No knowledge on networks/graphs is required

Homework, midterm, and final project

- Homework ⇒8 sets ⇒9 days f or each
- 7 best out of 8 submissions will be considered
- Late homework will not be accepted
- We encourage collaboration but final version must be yours

- Midterm ⇒ in class
- Based on the first 12 lectures

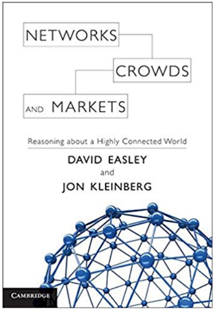
- Final Project ⇒ Teams of 2-3 students
- How to control epidemics on graphs?
- Applied project plus competition module
- More information during week 4 ⇒ start early!

- Grading: Homework 30%, Midterm 30%, Final project 40%

- Come to the lectures and take notes

  ⇒ Slides act as guide for blackboard discussions

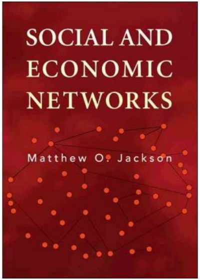
- *Networks, Crowds, and Markets: Reasoning about a Highly Connected World* by David Easley and Jon Kleinberg



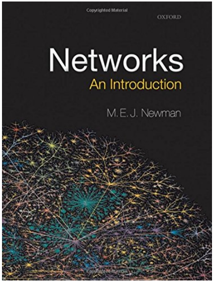
- Can be downloaded at
  https://www.cs.cornell.edu/home/kleinber/networks-book/

Additional bibliography

- Material complemented with chapters from two additional books
  - ⇒ *Social and Economic Networks* by Matthew O. Jackson
  - ⇒ *Networks: An Introduction* by Mark Newman



Jackson, Matthew O. *Social and Economic Networks*. Princeton University Press, 2010.

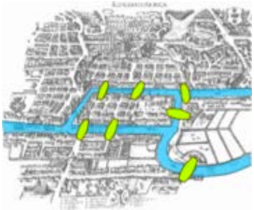


Newman, M.E.J. *Networks: An Introduction*. Oxford University Press, 2010.

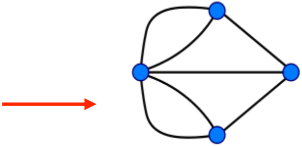
- Occasionally, we will reference published journal papers

Networks

- Pattern of interconnections among a set of "things"
- Very general definition $\Rightarrow$ enormous range of topics
- Growing public fascination with connectedness of modern society
- Started with Euler's 1735 solution of the Königsberg bridge problem.
- Can you cross each bridge exactly once in a walk?



The source map is public domain, modified by Bogdan Giuşcă.

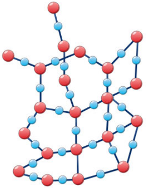


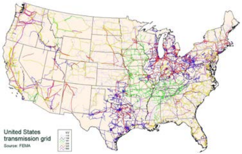
- Complex system modeled as a mathematical network (graph)
- Understanding systems at this level of abstraction

  $\Rightarrow$ Powerful tool across disciplines

Increasing interest

- Network-based analysis has a long-standing history
  - ⇒ Study of Königsberg bridges (Euler, 1736)
  - = Laws of electrical circuitry (Kirchoff, 1845)
  - = Molecular structure in chemistry (Cayley, 1874)
  - = Power grids (1910), telecommunications and the Internet (1960)))

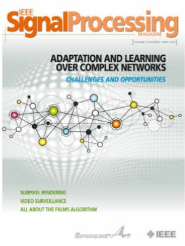


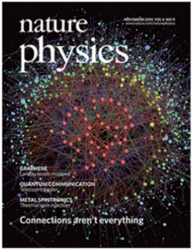


- Explosion of interest in the last two decades
  - ⇒ Systems-level perspective in science
  - ⇒ High-throughput data collection and computational power
  - ⇒ Globalization and connected modern society

Network Science

- Study of complex systems through their network representations
  - ⇒ Brain, society, economy, ...
- Universal terminology for describing complex systems
  - ⇒ Consistent across areas of knowledge





- Ubiquity of networks redounds in high impact of Network Science

Networks and Sociology

- Early 20th century Hungarian poet and writer Frigyes Karinthy first came up with the idea that we live in a small world.

  ⇒ In a play, he suggested all 1.5Billion people of the earth are at most 5 acquaintances away from each other

- The sociologist Stanley Milgram made this famous in his study The Small World Problem(1967) —though this study is now largely discredited

  ⇒ performed an experiment among residents of Wichita and Omaha to send a folder to a target, via acquaintances

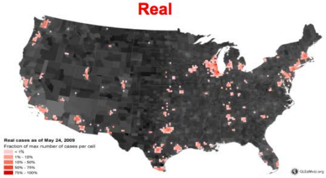
  ⇒ Consistent across areas of knowledge

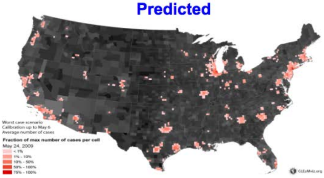
  ⇒ 42 of the 160 letters supposedly made it to their target, with a median number of intermediates equal to 5.5. Hence 6 degrees!

  ⇒ Yet his study was flawed ... why?

Huge potential impact

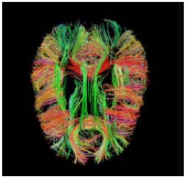
- Prediction of epidemics, e.g. the 2009 H1N1 pandemic



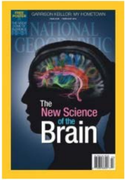


- Human Connectome Project to map the brain circuitry

HUMAN Connectome PROJECT



The Laboratory of Neuro Imaging and Martinos Center for Biomedical Imaging, Consortium of the Human Connectome Project – www.humanconnectomeproject.org.

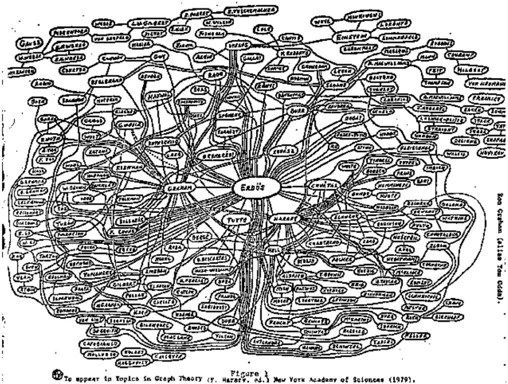


Examples of networks

- Increasing availability of large, detailed network datasets
- Different reasons for wanting to study a particular network dataset
  - ⇒ Care about the actual domain it comes from
  - ⇒ Dataset as a proxy for a related network that is impossible to measure
  - ⇒ Look for network properties that are common across different domains
- One particular way of dividing the network datasets
  - ⇒ Collaboration graphs
  - ⇒ Who-talks-to-whom graph
  - ⇒ Information linkage graph
  - ⇒ Technological networks
  - ⇒ Networks in the natural world

Collaboration graph

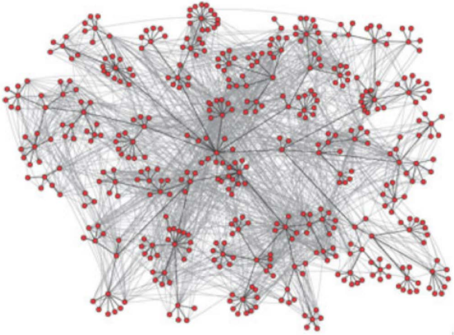
- Nodes are mathematicians, edges denote co-authorship
  - $\Rightarrow$ Paul Erdős ($\sim$ 1500 papers) $\Rightarrow$ central collaborative figure
  - $\Rightarrow$ Erdős number $\Rightarrow$ distance from Erdős in this graph
  - $\Rightarrow$ Most scientists have Erdős number ≤ 6 $\Rightarrow$ Small world phenomenon



Graham, Ron. "A Portion of the Erdős Collaboration Graph." In *Topics of Graph Theory*. Edited by Frank Harary. New York Academy of Sciences, 1979.

Who-talks-to-whom graph

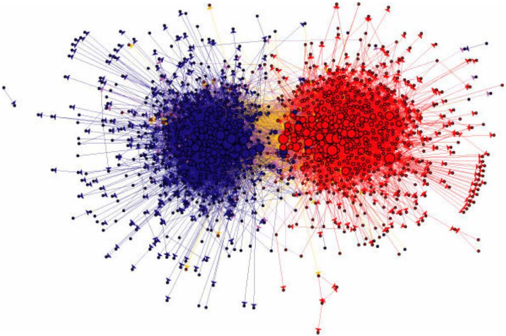
- Nodes are employees of HP, edges denote email exchanges
  - ⇒ Email interactions highly determined by hierarchy in company
  - ⇒ Can be used to determine inter-departamental collaborations
  - ⇒ What other insights can we get?



Source: Lada Adamic and Eytan Adar. "How to Search a Social Network." *Social Networks* 27 (2005): 187–203.

Information linkage graph

- Nodes are political blogs, edges denote hyperlinks
  - ⇒ Colors represent main political inclination of blog
  - ⇒ Two clear clusters arise ⇒ Community detection
  - ⇒ Can we recover these communities only from the network structure?



Adamic, Lada and Natalie Glance. "The Political Blogosphere and the 2004 U.S. Election: Divided They Blog." March 4, 2005.

Internet

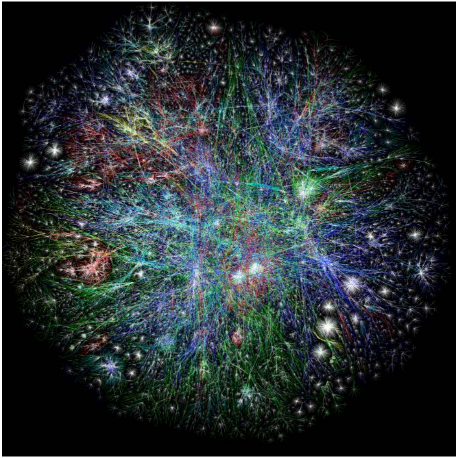


Figure: A depiction of the map of Internet.

World Wide Web

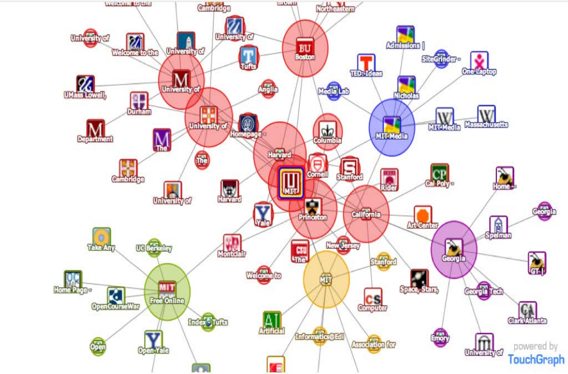


Figure: The web link structure centered at http://web.mit.edu (touchgraph)

Infection Network

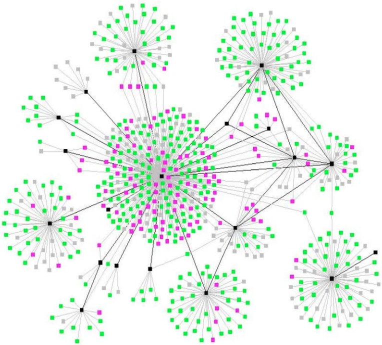


Figure: The spread of an epidemic disease (such as the tuberculosis outbreak shown here) is another form of cascading behavior in a network. The similarities and contrasts between biological and social contagion lead to interesting research questions. (Andre et al. 2007)

Financial Network

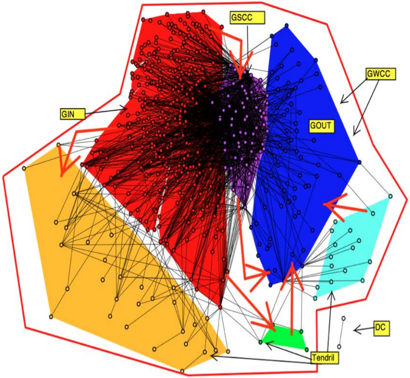


Figure: The network of loans among financial institutions can be used to analyze the roles that different participants play in the financial system, and how the interactions among these roles affect the health of individual participants and the system as a whole. (Bech and Atalay 2008)

Topics that we will cover

- Graph theory ⇒ Study the structure of networks
  - ⇒ Network models ⇒ How is network formation modeled?
- Social network analysis
  - ⇒ Centrality measures ⇒ Who are the most important agents?
  - ⇒ Community detection ⇒ Which are the main groups of agents?
  - ⇒ Spectral graph theory and graph matrices
- Dynamics and learning on networks
  - ⇒ Information spread and random walk on graphs
  - ⇒ Propagation of epidemics on networks
  - ⇒ Opinion formation from social interactions
- Game theory ⇒ Study the behavior on networks
  - ⇒ Group of people must simultaneously choose how to act
  - ⇒ Outcome depends on the actions of all the people
  - ⇒ Payoffs, strategies, and equilibria ⇒ Counter-intuitive results