1.033/1.57

Mechanics of Material Systems

(Mechanics and Durability of Solids I)

Franz-Josef Ulm

Lecture: MWF1 // Recitation: F3:00-4:30

Part II: Momentum Balance, Stresses and Stress States

3. Momentum Balance

Content 1.033/1.57

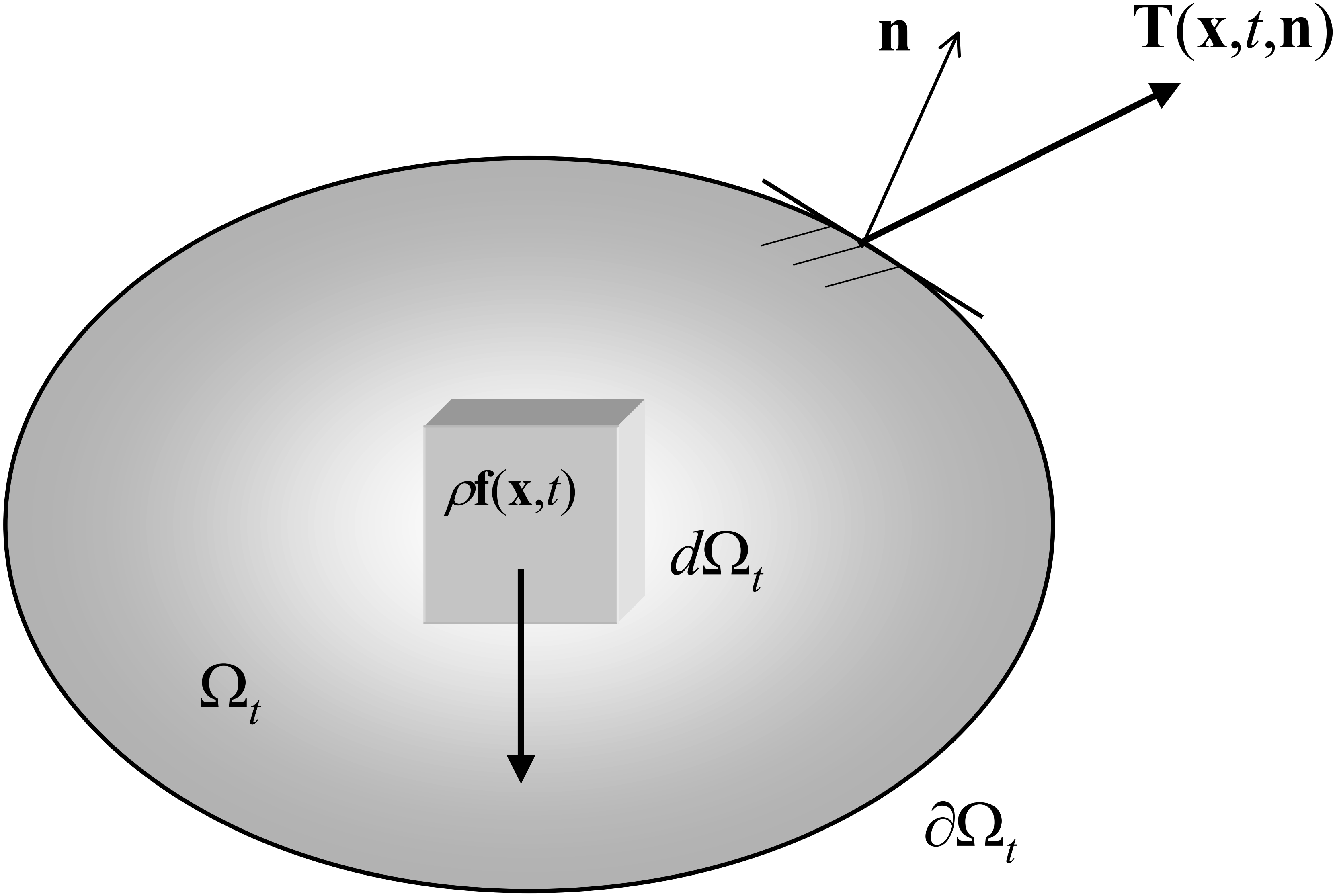
Part I. **Deformation and Strain**
- 1 Description of Finite Deformation
- 2 Infinitesimal Deformation

Part II. **Momentum Balance and Stresses**
- ➡ 3 Momentum Balance
- 4 Stress States / Failure Criterion

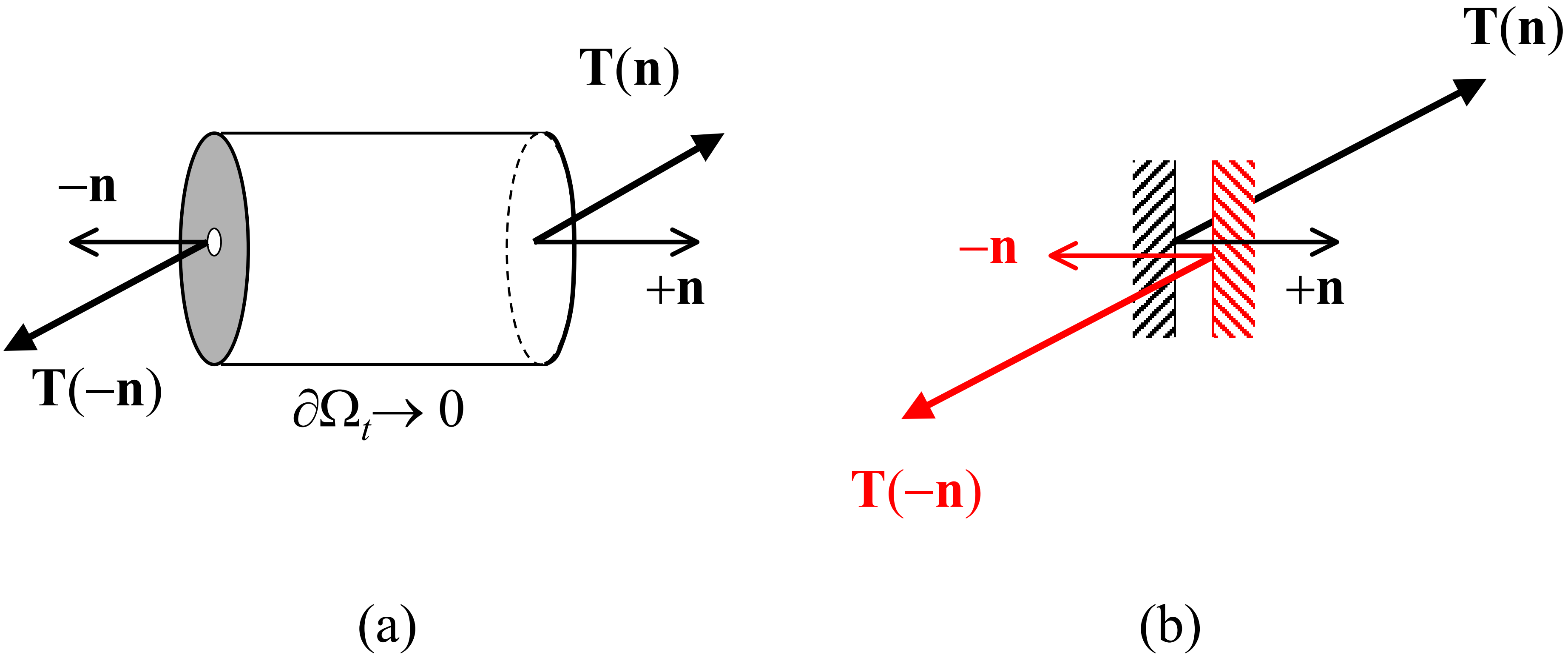
Part III. **Elasticity and Elasticity Bounds**
- 5 Thermoelasticity,
- 6 Variational Methods

Part IV. **Plasticity and Yield Design**
- 7 1D-Plasticity – An Energy Approac
- 8 Plasticity Models
- 9 Limit Analysis and Yield Design

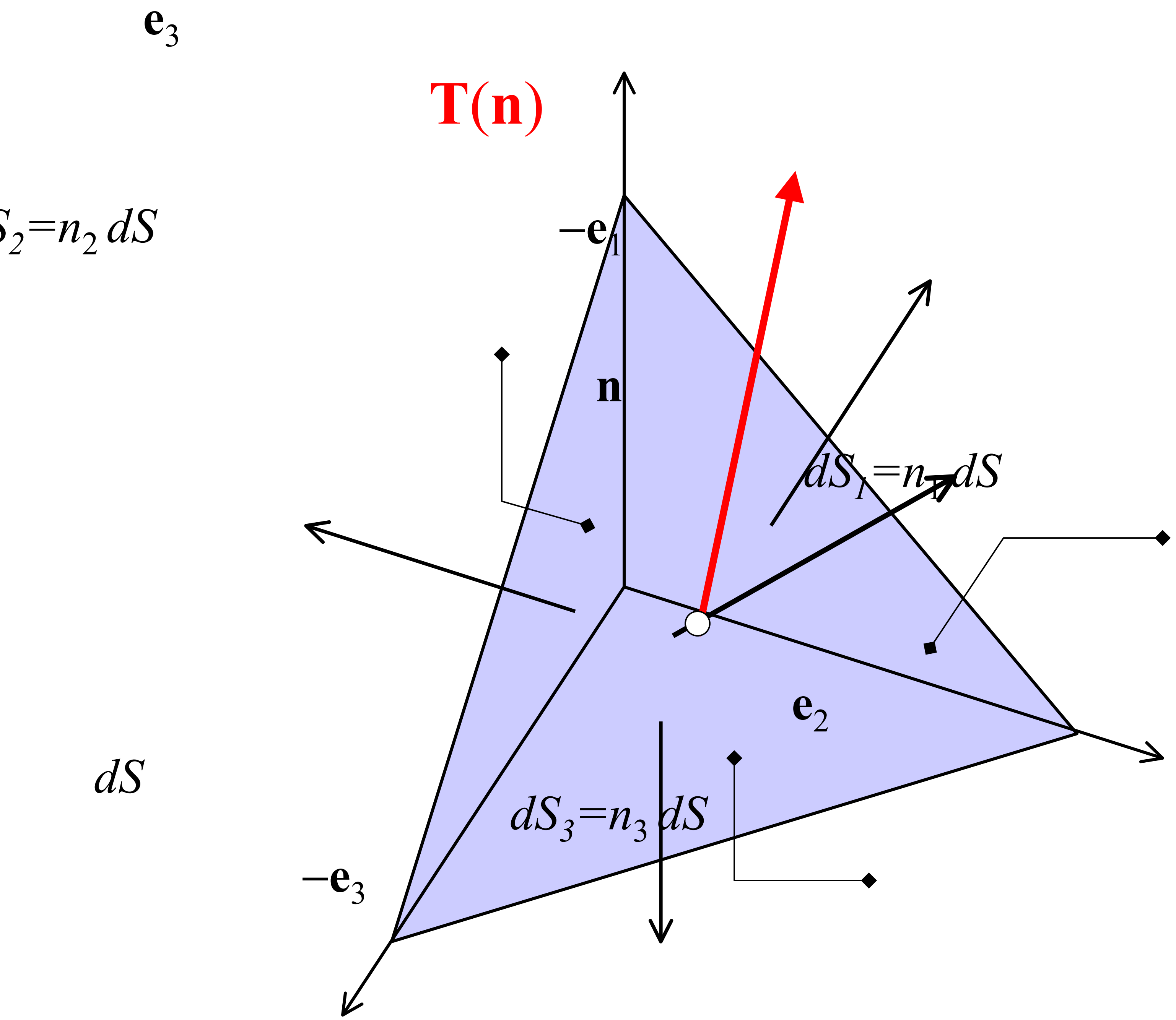
Body & Surface Forces



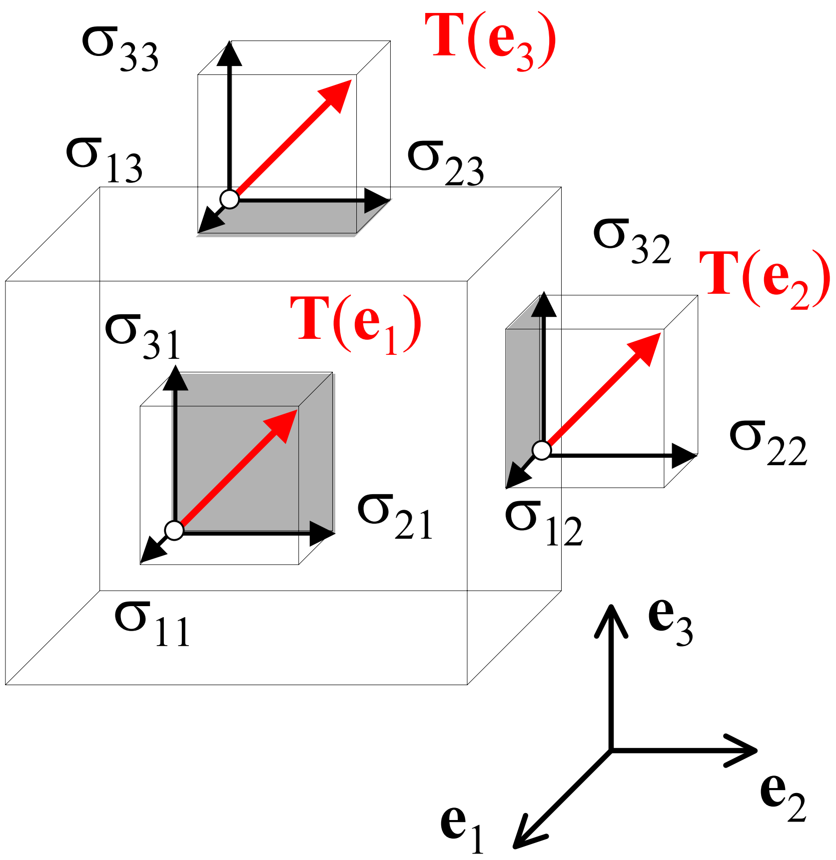
Hypothesis of local contact forces = Action-Reaction 'Law'



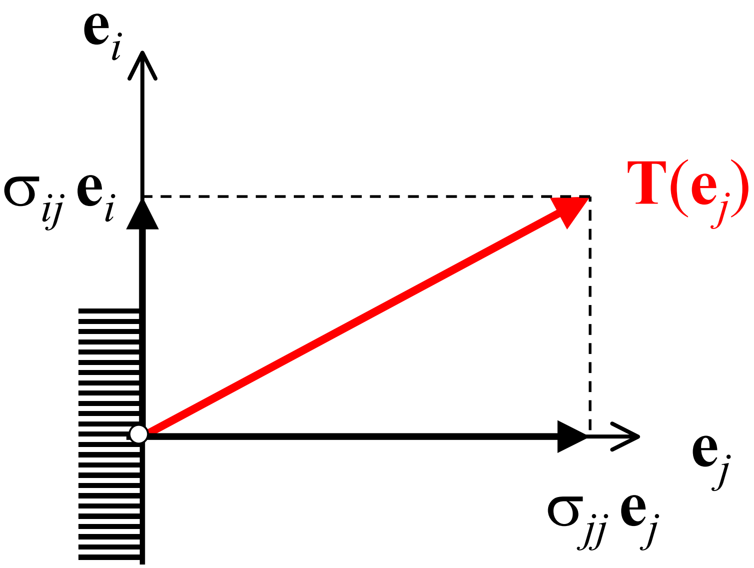
dron Lemma: Stress vector T(n)



Cauchy Stresses σ_{ij} and Stress Vector T(n)

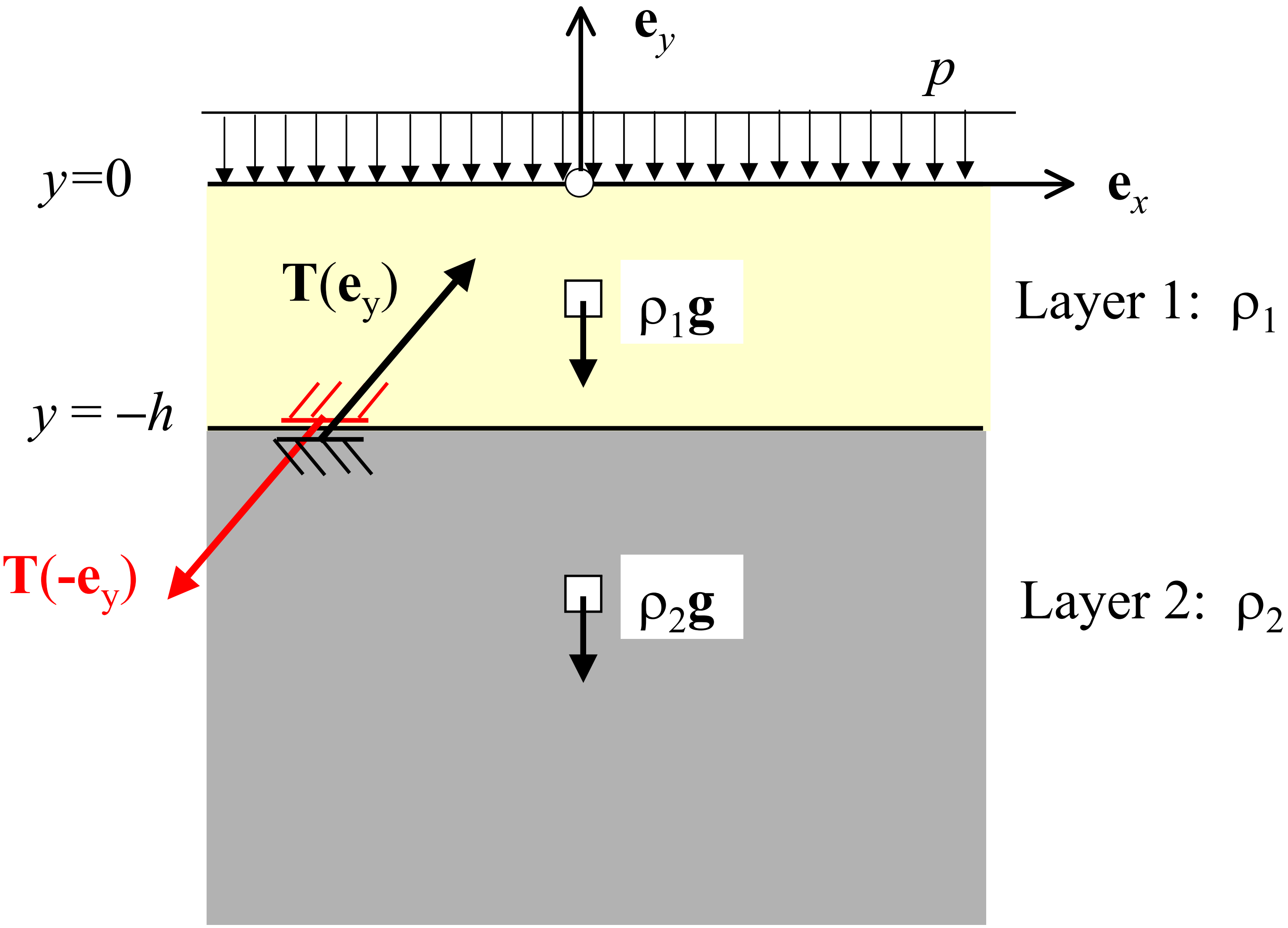


Components of σ_{ij}



Stress Vector $T_j = \sigma_{ij} n_j$

Exercise: Two-Layer Soil Substratum



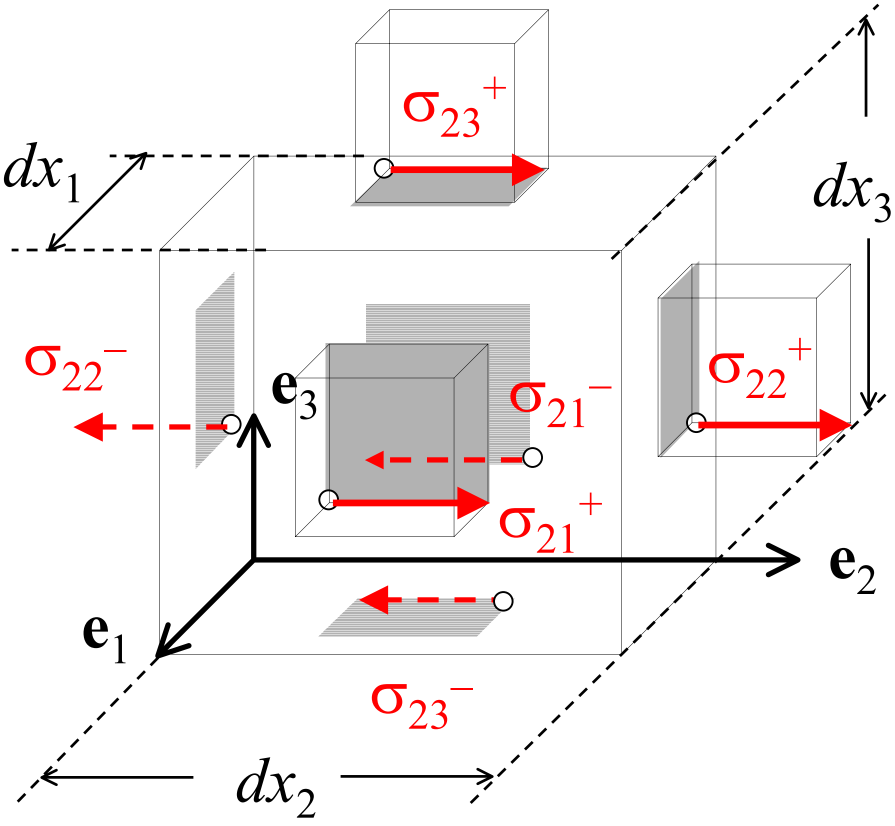
Reduction Formulae

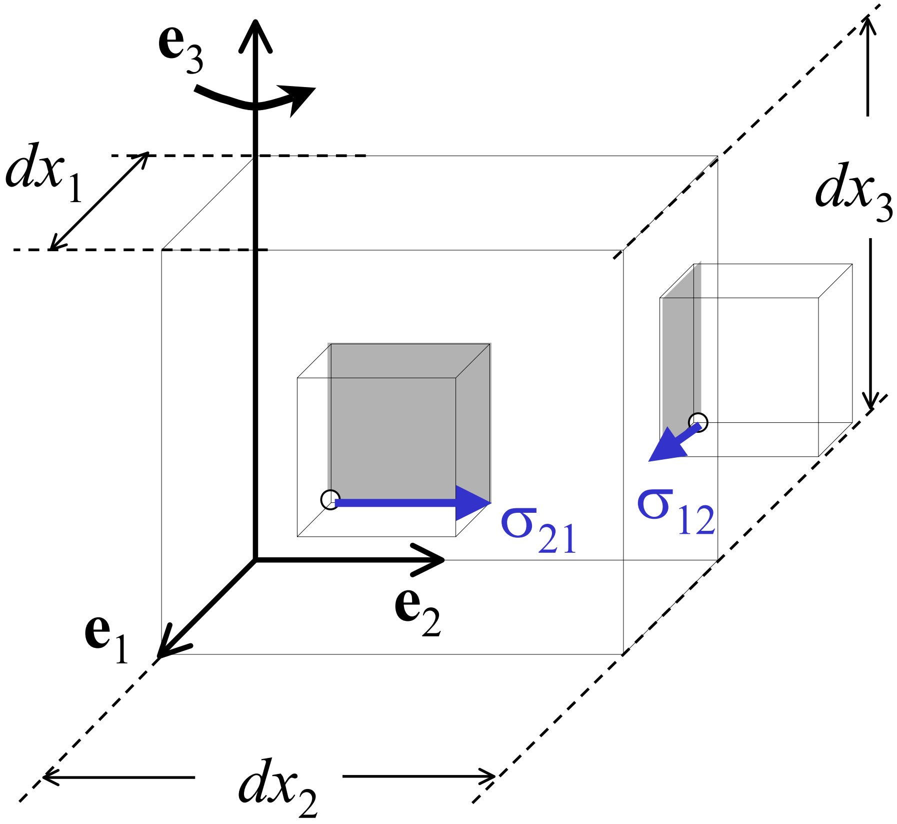
= *Relation between local stress and section forces and moments*

$$\mathbf{N}=\int_{\overset{\text{Section}}{\downarrow}\partial\Omega}\boldsymbol{\sigma}\cdot\overbrace{\mathbf{n}da}^{\text{Oriented Surface}}$$

$$\mathbf{M}=\int_{\partial\Omega}(\underbrace{\mathbf{OM}}_{\text{Position Vector in Section}}\times\underbrace{\boldsymbol{\sigma}\cdot\mathbf{n}}_{\text{Stress Vector}})da$$

Momentum Balance and Symmetry of Stress Tensor





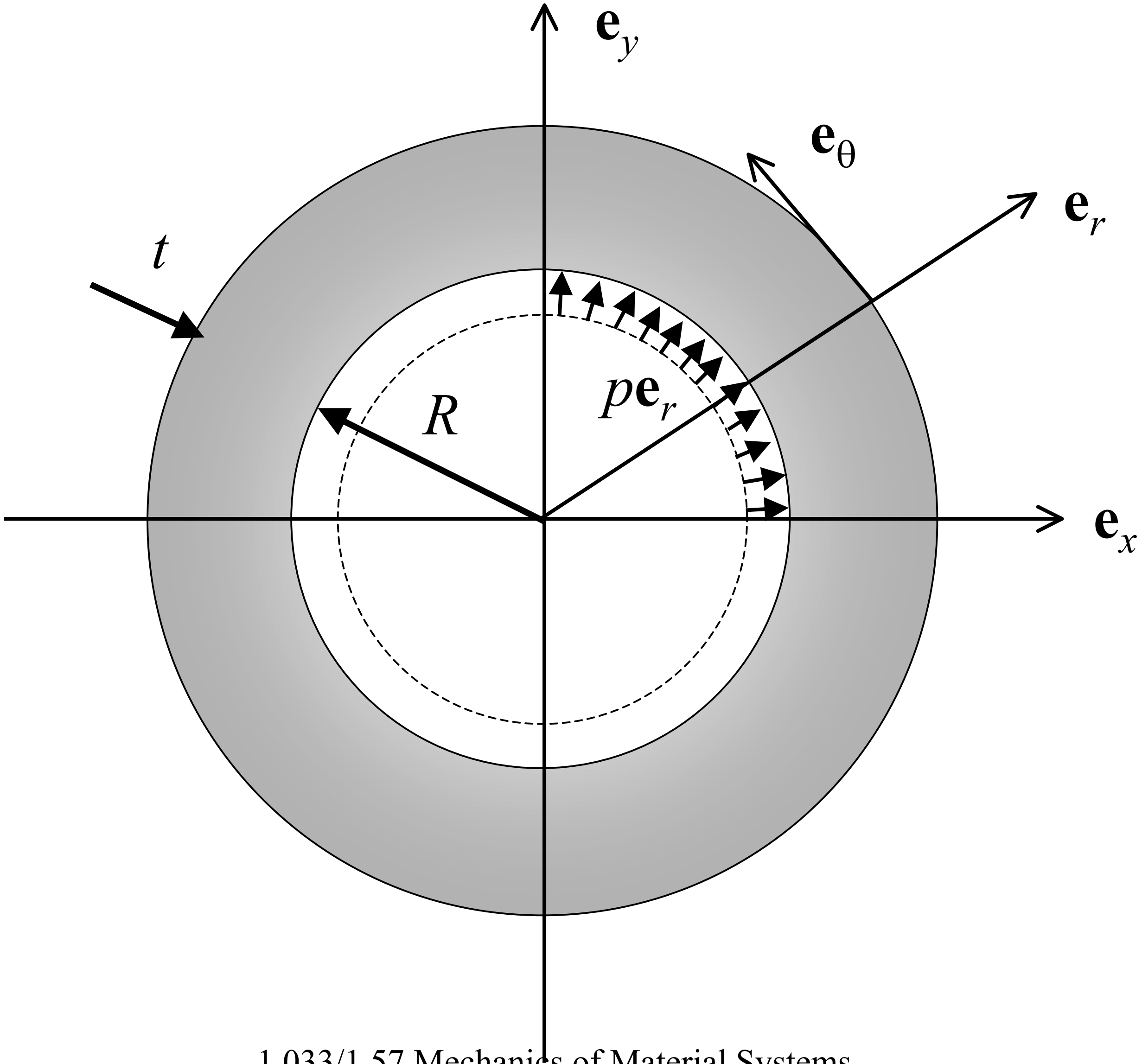
$$\boxed{\operatorname{div} \boldsymbol{\sigma} + \rho(\mathbf{f} - \boldsymbol{\gamma}) = 0}$$

Dynamic Resultant Theorem

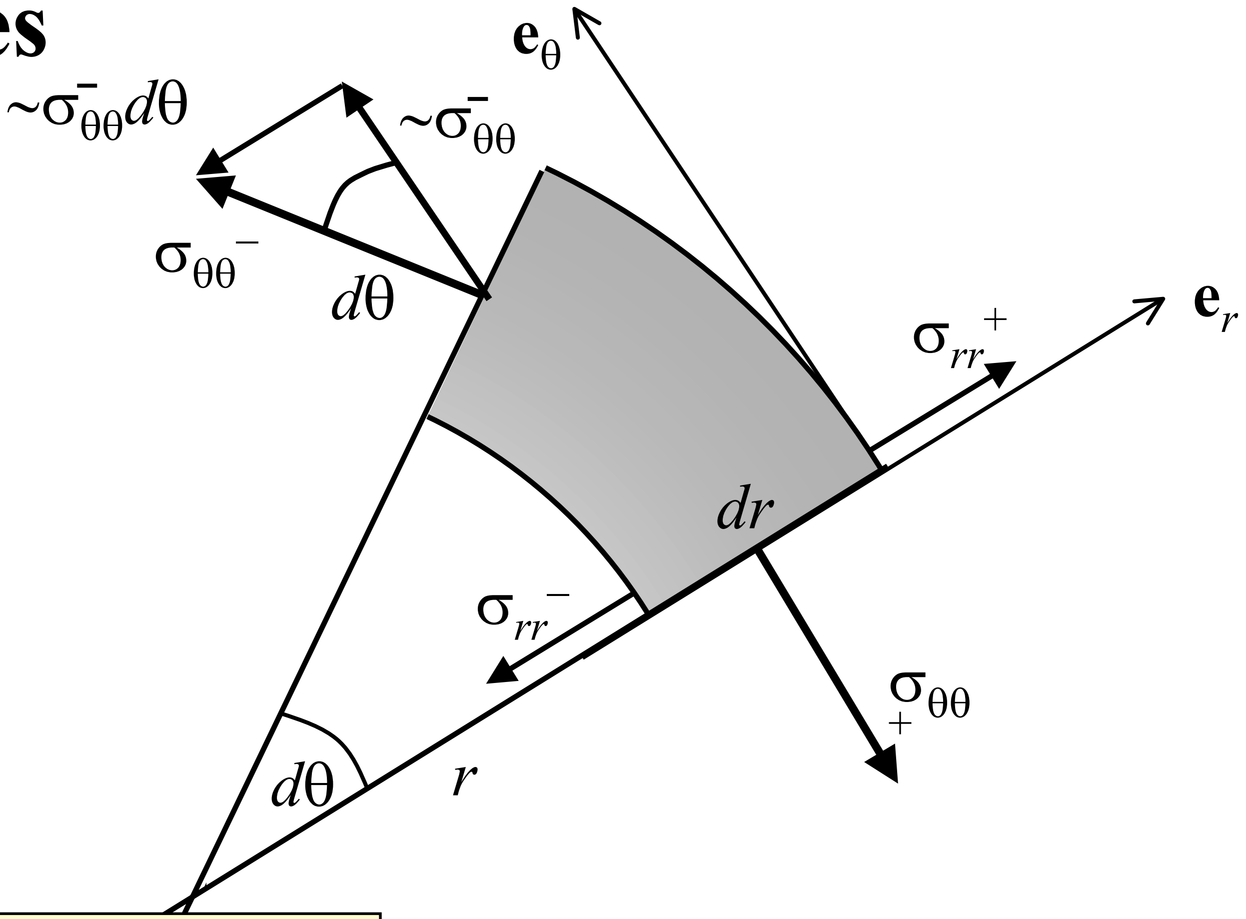
$$\boxed{\sigma_{ij} = \sigma_{ji}}$$

Dynamic Moment Theorem

Training Set: Pressure Vessel Formula



Momentum Balance in Cylinder Coordinates



Radial Direction:

$$\frac{\partial \sigma_{rr}(r)}{\partial r} + \frac{1}{r}[\sigma_{rr}(r)\text{-}\sigma_{\theta\theta}(r)] = 0$$

Vessel Formula *by* Reduction Elements