1.050: Hydrostatics and Geostatics (HW#4)

Due: October 3, 2007

MIT – 1.050 (Engineering Mechanics I)
Fall 2007
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Team Building and Team Work: We strongly encourage you to form Homework teams of three students. Each team only submits one solution for correction. We expect true team work, i.e. one where everybody contributes equally to the result. This is testified by the team members signing at the end of the team copy a written declaration that "the undersigned have equally contributed to the homework". Ideally, each student will work first individually through the homework set. The team then meets and discusses questions, difficulties and solutions, and eventually, meets with TA or instructor. Important: Specify all resources you use for your solution.

The following set of exercises is designed to train you in the use of continuum model for the evaluation of the stress distribution in hydrostatics and geostatics engineering problems. For each exercise, show us how you came to your answer and result. We highly encourage you to make drawings where appropriate.

1. **Stress vectors and equilibrium**[*]:

Problem removed due to copyright restrictions.

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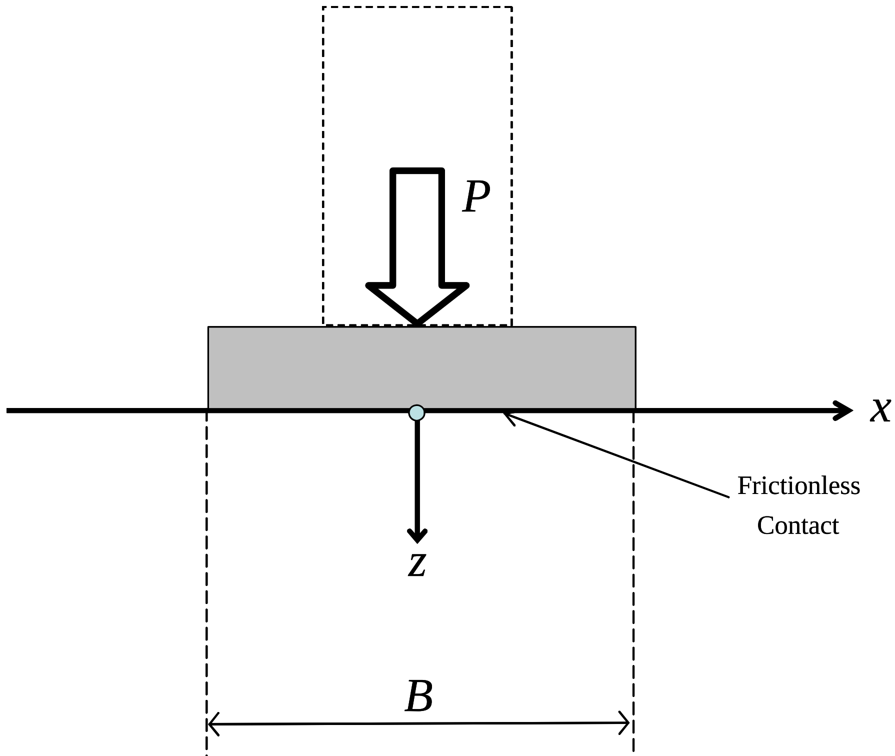
2. **Strip Foundation:** The sketch below shows a soil continuum below a strip foundation of width B in the x-z plane. The extension in the y-direction can be assumed to be infinite. In addition to its dead weight (specific weight ρg), the soil substrate is subjected to the line force P (of dimension $[P]=[F]L_y^{-1}$) of the strip foundation. The contact between the foundation and the soil substrate is assumed to be *frictionless*.



a. Specify precisely the conditions that a stress field $\underline{\underline{\sigma}}$ in the soil substrate needs to satisfy in order to be statically admissible.

b. To simplify the problem, we break the load (dead weight and foundation force) up in two separate load cases: the dead weight and the foundation load:

   i. *Load Case 1: Dead weight* - For the dead weight, consider a hydrostatic stress field of the form:

$$\underline{\underline{\sigma}}' = \sigma(z)\underline{\underline{1}}$$

   Determine the distribution o $\sigma(z)$ in response to the dead weight.

ii. *Load Case 2: Foundation Load* - In order to analyze the stress field in response to the foundation load, we break the soil domain up in three sub-domains, which have a common interface along $z > 0$ at $|x| = B/2$ (see sketch). In the two domains we consider the following *constant* diagonal stress fields:

$$|x| < B/2 : \underline{\underline{\sigma}}'' = a\underline{\underline{1}}$$

$$|x| > B/2 : \underline{\underline{\sigma}}'' = b\left(\vec{e}_x \otimes \vec{e}_x + \vec{e}_y \otimes \vec{e}_y\right) + c\vec{e}_z \otimes \vec{e}_z$$

Determine the three constant stress values *a*, *b*, *c* so that $\underline{\underline{\sigma}}''$ is statically admissible.

iii. *Superposition*: Why *must it be true* that the sum of the two previous stress fields $\underline{\underline{\sigma}}' + \underline{\underline{\sigma}}''$ is statically admissible?

c. It is common practice, in the engineering design of shallow foundation to neglect the dead weight of the soil. On the basis of your results, develop a rational criterion under which circumstances this assumption holds (approximately) true.

3. **Borehole pipe**: The enclosed figure (in particular sketch (a)) shows a cylindrical borehole pipe used for CO_2 sequestration. The system has the following dimensions:

$$R = 0.5\,\text{m}$$
$$h = 40\,\text{m}$$

The pipe holds liquid CO_2, with a density of $\rho_{CO_2} = 1.10\,\text{g/cm}^3$, and it is subjected to a pressure $p_0 = 300\,\text{kPa}$. This pressure is used to pump some liquid CO_2 into the ground upon the opening of an outlet door at the bottom end of the pipe (see sketchs (a) and (b)). For this exercise, we will only focus at the (hydro)static case where the liquid CO_2 is contained in the pipe under the prescribed pressure p_0 and the outlet door is closed.

a. Formulate precisely how you can write the stress tensor $\underline{\underline{\sigma}}$ prevailing in the liquid CO_2 for this particular case.

b. Write down the conditions in the borehole in order to be (hydro)statically admissible (Note: for this problem, use cylindrical coordinates). Using these conditions, determine the pressure distribution and specify:

   i. What is the pressure in the liquid CO_2 immediately below the free surface in the top end of the pipe?

   ii. What is the pressure at the bottom of the borehole pipe?

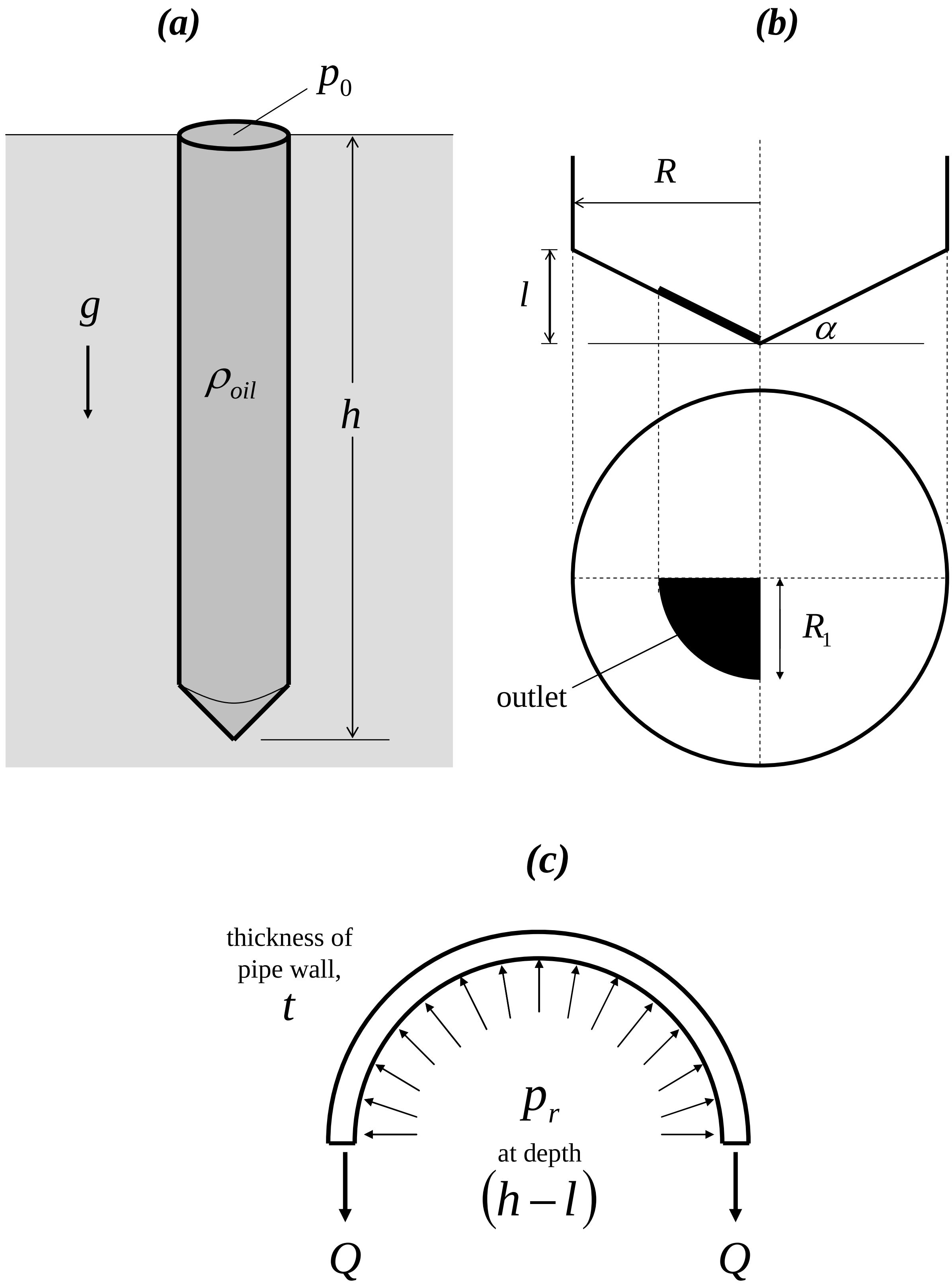
   iii. Sketch the stress distribution within the pipe (in radial and angular direction) as well as in the height direction.

   iv. Discuss how the stress distribution changes as a function of applied pressure on the top; for this purpose plot the ratio of pressure on the top over the pressure on the bottom as a function of the applied pressure on the top. Discuss the results.

c. Determine the resulting force that acts at the outlet door in the bottom of the pipe. The outlet door in the pipe is given in the sketch (b), where:

$$l = 1.0\,\text{m}$$
$$R_1 = 0.25\,\text{m}$$

d. *Strength of the pipe*: Using the hydrostatic state of the liquid CO_2 at depth $(h-l)$ in the radial direction, calculate the stress in the cylinder wall, $\hat{\sigma} = Q/t$ due to the constant internal pressure (see sketch (c)). The thickness of the pipe wall is $t = 2.5\,\text{cm}$.



Figure, Problem 3