1.050 Solid Mechanics, Fall 2004
Problem Set#3 Solution

Problem 3.1

Figure 1.1 shows the sketch of a tree trunk with distributed load w_0 as the weight of the trunk + leaf. In a more realistic form, you can assume that the distributed load is not uniform with the largest load at the root of the trunk and reducing when advancing to the end of the trunk.
We get that

$$\Sigma M_A = w_0(L)\left(\frac{L}{2}\right) = w_0\left(\frac{L^2}{2}\right)$$

This is the moment acting at point A. This moment transfers to the root of the tree and depending on number of the limbs and offshoots, the moment acting at the root of the tree can be calculated. For a well grown tree (with evenly distributed numbers of limbs on every side), this resultant moment at the root is general low because of the roughly rotationally symmetric disposition of the limbs around and up the tree
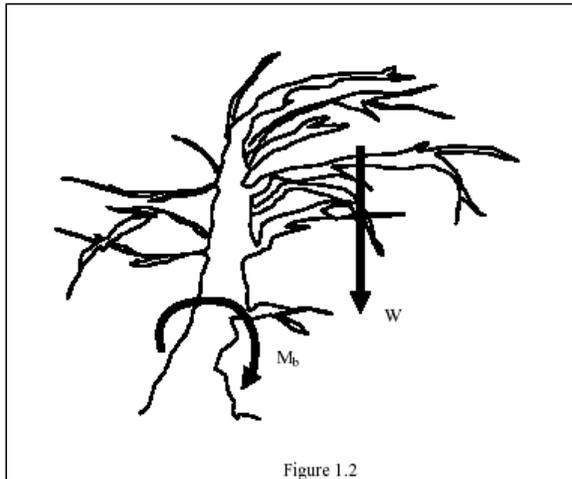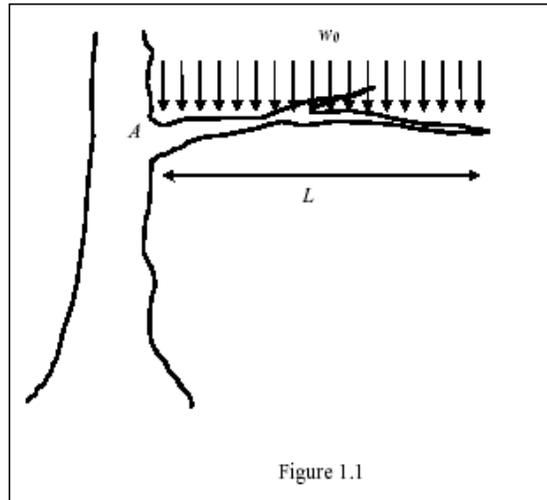
Figure 1.1

Figure 1.2

. However, this moment can be large for a tree that has uneven distribution of limbs, e.g., where there are many limbs on one side but none on the other, this moment can be very large (see figure 1.2 for illustration).

In a subsequent exercise, after we have determined how the local stress within a beam depends upon bending moment, we will model and estimate the bending moment at the root of the limbs as we go up the tree.

Problem 3.2

Figure 2.1 is a sketch of the FBD and forces that acting on the two gears at the top of the well. (There are other forces acting on the system, but we include only those that produce a moment about an axis perpendicular to the paper). Let the weight of the bucket full with water be W; the force that Ramelli exerts at the handle, $F_{Ramelli}$. We get from moment equilibrium of the large gear

Figure 2.1
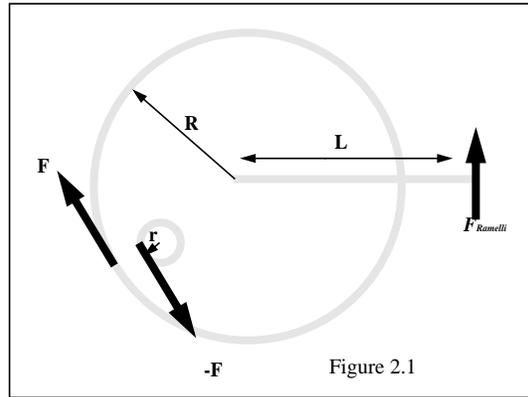
$$Moment_{from\ Ramelli} = F_{Ramelli}(L) = F(R)$$

$$F = \frac{F_{Ramelli}(L)}{R}$$

Now, this force F (its equal and opposite reaction) acts on the smaller gear with radius r and in turn this gear drives the pulley and retracts the rope to lift up the bucket of water. This force, F, times the radius, r, is equal to the he moment about the vertical shaft, which in the textbook we called M_T and was estimated to be $M_T = W(r_R)(r_s)/(r_N)$ (See textbook for definition of all radii appearing in this expression). So $F = M_T/r$ which means that

$$F_{Ramelli} = (M_T/r) \cdot \frac{R}{L}$$

Estimating L to be 6 ft, R to be 5 ft, and the radius of the pinion gear, r, at the base of the vertical shaft to be 3/4 ft, we claim that this 16th century source of power must push with a force of 22 pounds - which doesn't sound like an excessive demand - until one considers (a) how fast he went around, (b) how many hours a day he labored.

Problem 3.3

We first isolate the entire beam and determine the reactions:

$$\Sigma M_O = 0; \quad R_L(L) - Wa = 0$$

$$R_L = \frac{Wa}{L}$$

$$\Sigma F_y = 0; \quad R_O + R_L - W = 0$$

$$R_O = W - R_L = W - \frac{Wa}{L} = W\left(1 - \frac{a}{L}\right)$$

x<a
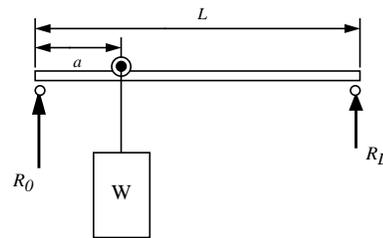
The figure at the left shows an isolation for x<a, to the left of the point of suspension. Equilibrium gives, taking moments about the cut point, x, ccw positive:
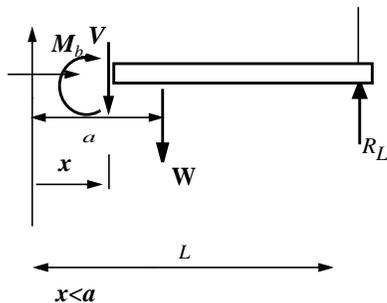
$$-V - W + R_L = 0$$

$$-M_b - Wa + (L - x) \cdot R_L = 0$$

NOTE: The figure shows the sign convention that we used for this class. It is important and recommended that you always state clearly your sign convention when you works on a problem set or an exam in this class to avoid any confusions.
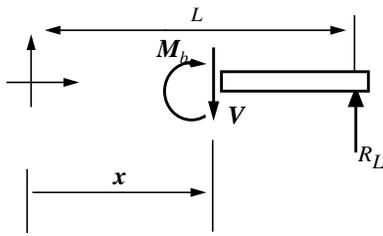
x>a

For x> a we have:

$$-V + R_L = 0$$

$$-M_b + (L - x) \cdot R_L = 0$$

Solving, we can sketch the Shear Force Diagram (SFD) and Bending Moment Diagram (BMD):

The dotted line in the BMD of the figure 3.2 shows the plot of the maximum moment as the trolley moves from one end to the other. You can find this plot by observing the equation of maximum M_b when the trolley is at any point on the beam.

$$\left|M_{b|_{max}}\right| = R_o \cdot a = W \cdot \left(1 - \frac{a}{L}\right) \cdot a$$

From this equation, you can see that the M_b is in the form of parabolic equation and the maximum moment always occurs where the trolley is located and it is increasing to the maximum value of $-R_O(L/2)$ when the trolley is located in the middle of the beam.

$$\left|M_{b|_{max}}\right|_{max} = W \cdot L^2 / 4$$
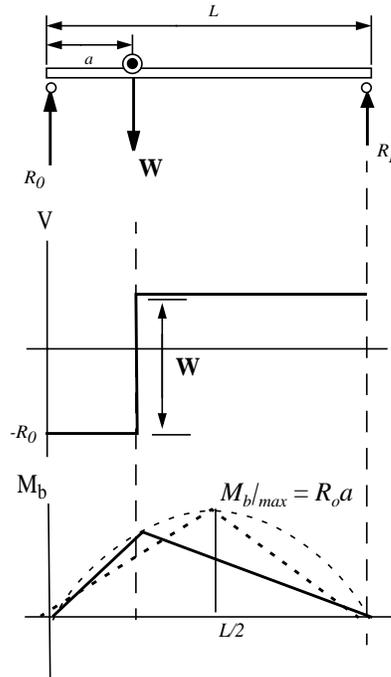
Problem 3.4

Figure 4.1 shows the FBD of the beam. We get that

$$\Sigma M_a = 0; \quad w_0(L - \beta L)\left(\beta L + \frac{L - \beta L}{2}\right) - R_b(\beta L) = 0$$

$$w_0(L - \beta L)\left(\frac{L}{2} + \frac{\beta L}{2}\right) = R_b(\beta L)$$

$$\frac{w_0 L^2}{2}(1 - \beta)(1 + \beta) = R_b(\beta L)$$

$$\frac{w_0 L^2}{2}(1 - \beta^2) = R_b(\beta L)$$

$$R_b = \frac{w_0 L}{2\beta}(1 - \beta^2)$$

$$\Sigma F_y = 0; \quad R_b - R_0 - w_0(L - \beta L) = 0$$

$$R_0 = R_b - w_0 L(1 - \beta)$$

$$R_0 = \frac{w_0 L}{2\beta}(1 - \beta^2) - w_0 L(1 - \beta)$$

$$R_0 = w_0 L(1 - \beta)\left[\frac{1 + \beta}{2\beta} - 1\right] = w_0 L(1 - \beta)\left[\frac{1 + \beta - 2\beta}{2\beta}\right]$$

$$R_0 = \frac{w_0 L(1 - \beta)}{2\beta}(1 - \beta) = \frac{w_0 L}{2\beta}(1 - \beta)^2$$
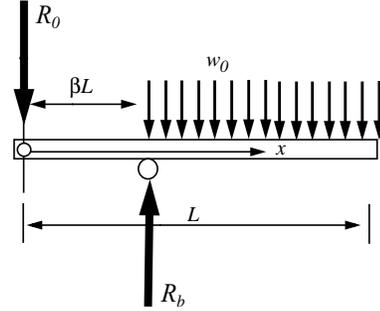
FBD 1: Cut at the left of the support B, $x < \beta L$;

$$\Sigma F_y = 0; \quad R_b - V - w_0(L - \beta L) = 0$$

$$V = R_b - w_0 L(1 - \beta)$$

$$V = \frac{w_0 L}{2\beta}(1 - \beta^2) - w_0 L(1 - \beta)$$

$$V = R_0$$

$$\Sigma M_b = 0; \quad M_b + w_0 L(1 - \beta)\frac{(L - \beta L)}{2} - V(\beta L - x) = 0$$

$$M_b = R_0(\beta L - x) - \frac{w_0 L^2}{2}(1 - \beta)^2$$

$$M_b = R_0(\beta L - x) - R_0(\beta L) = R_0(\beta L - x - \beta L)$$

$$M_b = -R_0(x)$$
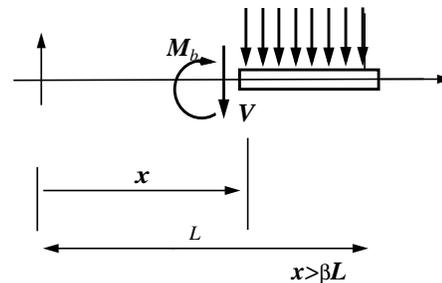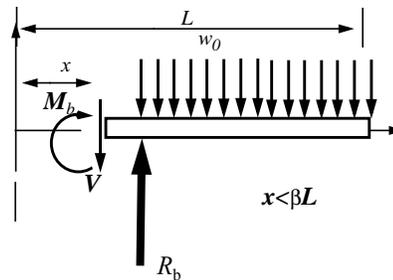
FBD 2: Cut at the right of the support B, x > βL

$$\Sigma F_y = 0; \quad V + w_0(L-x) = 0$$

$$V = -w_0(L-x)$$

$$\Sigma M_x = 0; \quad M_b + w_0(L-x)\frac{(L-x)}{2} = 0$$

$$M_b = -\frac{w_0}{2}(L-x)^2$$
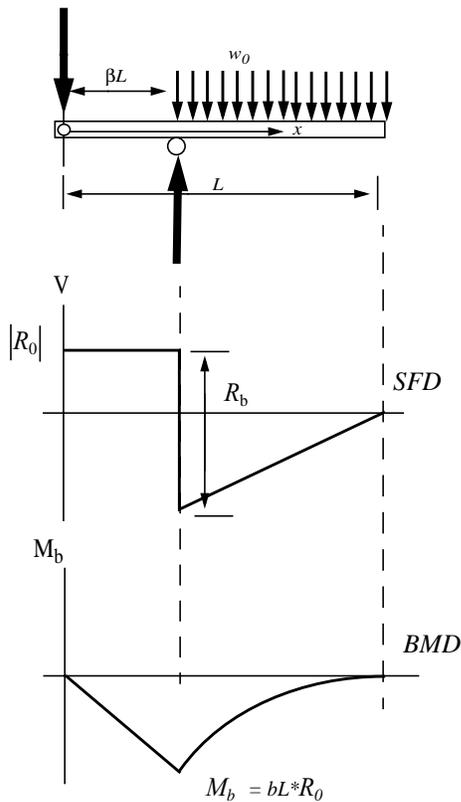
We plot of SFD and BMD.

Also, we can check that our equation is correct by checking whether $dM_b/dx = -V$;

For x < βL:

$$\frac{dM_b}{dx} = \frac{d(-R_0 x)}{dx} = -R_0 = -V$$

For x > βL:

$$\frac{dM_b}{dx} = \frac{d\left(-\dfrac{w_0(L-x)^2}{2}\right)}{dx} = w_0(L-x) = -V$$

$M_b = bL*R_0$