PROBLEM SET 5 - SOLUTIONS

Comments on Problem Set 5

PROBLEM 1:

- To solve part (a), you apply Bernoulli between the reservoir and the vena contracta location. Thus, you calculate the velocity at vena contracta, V_{VC}. The discharge is $Q=V_{VC}A_{VC}$, where $A_{VC}=C_V A_P$ is the area at vena contracta, $A_P=\pi D^2/4$, and $C_V=0.6$ (or 0.61) is the contraction coefficient. Some people applied Bernoulli between the reservoir and the center of the orifice, and the multiplied the resulting velocity by A_P to obtain Q. This is incorrect: You cannot apply Bernoulli between the reservoir and the orifice because flow at the orifice is not well behaved (streamlines are not straight and parallel). For this reason, pressure is not linear, and the pressure at the center of gravity of the orifice is not atmospheric.

- In part (c), pressure at vena contracta must be 0 (i.e., atmospheric). You could have predicted this without doing any calculation: Since you chose the length of the pipe to give the same discharge as in the free outflow, the conditions at vena contracta with and without pipe are identical, and therefore the pressure is the same, 0.

- Pipe velocities are on the order of 1 m/s. Values between 0.1 m/s and 10 m/s are thus reasonable. But if you obtain a pipe velocity of 0.001 m/s, you probably did something wrong (unless it's reasonable to expect almost no flow in the pipe). Same if you obtain a velocity of 100 m/s.

- Negative pressures in pipes are possible, since we are talking of gauge pressures (relative to atmospheric). If you get negative pressures, you should check for cavitation. However, absolute pressure is always positive (there is no traction in fluids!) and, for this reason, the minimum gauge pressure you can obtain is $-p_{atm}$ (abs) $\simeq$ -101300 Pa. Therefore, if you obtain p=-200kPa, you must have done something wrong, and you should check your calculations (or, if you are in a test and have no time, write down that the result is obviously wrong for the reasons mentioned above).

PROBLEM 2:

- To solve this problem, you apply Bernoulli between the section of the pipe right below the house (call it section 1) and the outflow (call it section 2). You cannot apply Bernoulli between the basement of the house (the point at $z = z_b = 3$ m) and the outflow, because these two points are not on a streamline. In fact, the water in the vertical tube from the house is stagnant (hydrostatic pressure). So, the tube it is working as a piezometer, indicating the value of $z+p/(\rho g)=z_b$. The total head at section 1 is therefore $z_b + V^2/(2g)$.

PROBLEM 3:

- In this problem you don't know the velocities in the pipes, so you cannot calculate the Reynolds numbers necessary to determine f_1 and f_2 and you have to iterate. The best way to iterate is the following: Assume rough turbulent flow (i.e., very large value of the Reynolds numbers). With the relative roughnesses of the pipes, enter the Moody diagram and obtain a first estimate of f_1

and f_2. Now you can calculate V_1 and V_2. With this, you calculate the Reynolds numbers and, using the Moody diagram, you get new values of $f_1$and f_2. And you keep doing this until your results converge.

- The minor loss in a sudden expansion from a smaller to a larger conduit, such as point B, is given by $(V_1-V_2)^2/(2g)$. There is a graph in the book that gives the value of K_L for this kind of expansion as a function of A_1/A_2. But who wants to use a graph when you have the exact analytical expression? (Particularly when the analytical expression is so simple as this one). Note that you can relate $V_1=4V_2$ by continuity, and the previous analytical expression gives you K_{L1} (to be multiplied by $V_1^2/(2g)$) or K_{L2} (to be multiplied by $V_2^2/(2g)$), as you prefer.

- Please take a careful look of my plot of the EGL and the HGL, and make sure you understand how to draw them, including all the detail features (minor losses, velocity head at vena contracta, etc.)

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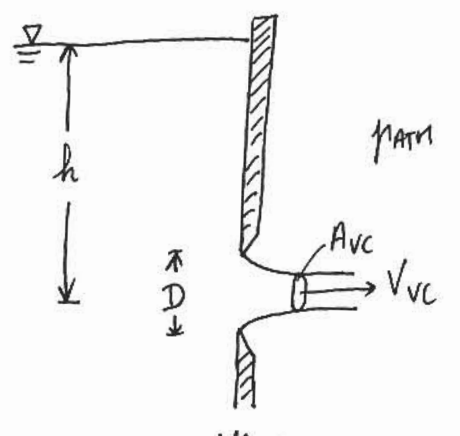
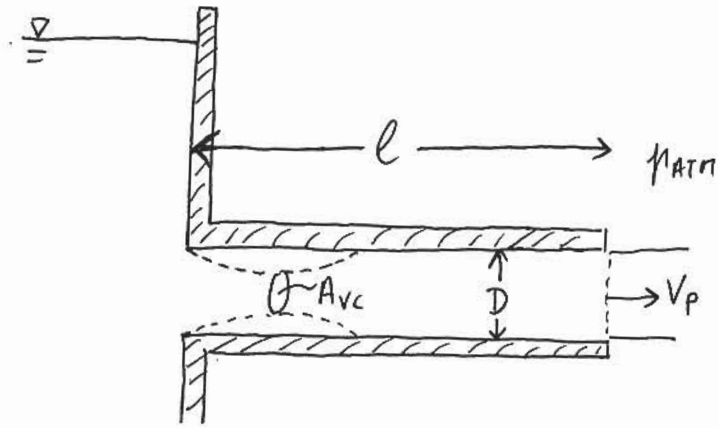
- PROBLEM N° 1:



Free outflow
$l=0$



Piped outflow
$l=l$

a) For free outflow, Bernoulli from large container ($H_C = h = 3m$) to vena contracta ($H_{VC} = H_C$, short transition of converging flow) gives

$$H_{VC} = \frac{V_{VC}^2}{2g} + \frac{p_{VC}}{\rho g} + z_{VC} = \frac{V_{VC}^2}{2g} = H_C = 3 \Rightarrow V_{VC} = \sqrt{2gh} = 7'67 \ m/s$$

$$A_{VC} = C_V A_0 = C_V \frac{\pi}{4} D^2 = 0'6 \frac{\pi}{4} (0'1)^2 = 4'71 \cdot 10^{-3} \ m^2$$

$$\underline{\underline{Q_0}} = V_{VC} A_{VC} = 7'67 \cdot 4'71 \cdot 10^{-3} = \underline{\underline{3'61 \cdot 10^{-2} \ m^3/s}}$$

b) For piped outflow, Bernoulli from pipe outlet to atmosphere ($H_P = V_P^2/2g + 0 + 0$) to vena contracta in pipe ($H_{VC} = H_C =$ = head in large container $= h$) with losses (expansion from vena contracta and pipe friction losses) gives

$$H_{VC} = H_C = h = H_P + \Delta H = \frac{V_P^2}{2g}\left(1 + K_{exp} + f\frac{l}{D}\right)$$

$$V_p = \frac{Q_0}{A_p} = \frac{3'61 \cdot 10^{-2}}{\pi/4\ (0'1)^2} = 4'60\ m/s \ ; \ V_p^2/2g = 1'08\ m$$

$$K_{exp} = \left(\frac{1}{C_V} - 1\right)^2 = \left(\frac{1}{0'6} - 1\right)^2 = (1'667 - 1)^2 = 0'44$$

$$Re = \frac{V_p D}{\nu} = \frac{4'6 \cdot 0'1}{10^{-6}} = 4'6 \cdot 10^5 \ ; \ \varepsilon/D = \frac{0'2}{100} = 2 \cdot 10^{-3} \xRightarrow{\text{MOODY}} f = 0'024$$

$$h = 3 = 1'08\left(1 + 0'44 + 0'024\,\frac{l}{0'1}\right) \Rightarrow \underline{\underline{l = 5'57\ m}}$$

c) For condition considered in (b), the situation in vena contracta is the same as in the free outflow case (same V_{VC}), and therefore $\underline{\underline{p_{VC} = 0}}$ (gauge)

After vena contracta:

$$H_{after} = H_{VC} - K_{exp}\frac{V_p^2}{2g} = h - K_{exp}\frac{V_p^2}{2g} = 3 - 0'44 \cdot 1'08 = 2'52\ m$$

$$H_{after} = \frac{V_p^2}{2g} + \frac{p_{AFTER}}{\rho g} + 0 = 2'52 \Rightarrow \underline{\underline{p_{\substack{AFTER\\VC}} = 14100\ Pa}}$$

d) As seen in (b), Bernoulli between vena contracta and outlet to atmosphere gives

$$\frac{V_p^2}{2g}\left(1 + K_{exp} + f\,\frac{l}{D}\right) = h$$

Therefore, if l decreases, V_p must increase so that the headloss between these two points is constant. That is, if l is smaller than in (b), the discharge would be $\underline{\underline{\text{larger than } Q_0}}$

- PROBLEM N° 2 :

Bernoulli from H (house) to O (outlet) gives

$$H_H = \frac{V_H^2}{2g} + \frac{p_H}{\rho g} + z_H = \frac{V_0^2}{2g} + \frac{p_0}{\rho g} + z_0 + \Delta H f_{H\to 0}$$

$V_H = V_0$ (by continuity) ; $p_0 = p_{atm} = 0$ (gauge)

$z_H + \frac{p_H}{\rho g}$ = piezometric head at house = z_b
(when basement drain is about to back-up)

$$\Delta H f_{H\to 0} = f \frac{l}{D} \frac{V_0^2}{2g}$$

Therefore :

$$z_b = z_0 + f \frac{l}{D} \frac{V_0^2}{2g} \Rightarrow z_b - z_0 = f \frac{l}{D} \frac{V_0^2}{2g}$$

$$V_0 = \sqrt{\frac{2g(z_b - z_0)}{l/D}} \frac{1}{\sqrt{f}} = \sqrt{\frac{2 \cdot 9'8 \cdot (3-1)}{2000/0'6}} \frac{1}{\sqrt{f}} = \frac{0'108}{\sqrt{f}} \ (m/s)$$

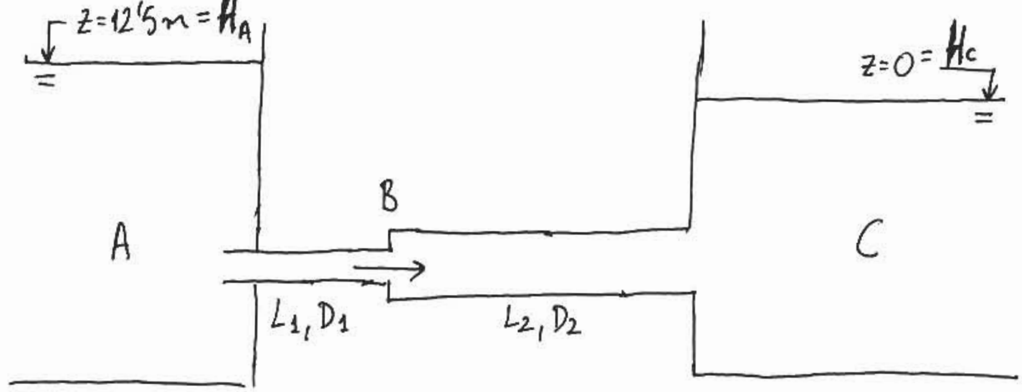
For sewer we have $\varepsilon/D = 0'06/60 = 0'001$. Moody for R.T. flow gives $f = 0'020 \Rightarrow V_0 = 0'108/\sqrt{0'020} = 0'76$ m/s.

Check if R.T. $\rightarrow Re = V_0 D/\nu = 0'76 \cdot 0'6 \cdot 10^6 = 4'6 \cdot 10^5 \rightarrow \sim$ Yes $\rightarrow$ Done.
($f = 0'020$ was right)

$$\underline{\underline{Q_0}} = V_0 A = 0'76 \frac{\pi}{4} (0'6)^2 = \underline{\underline{0'215 \ m^3/s}}$$

is the maximum allowable discharge.

- PROBLEM N°3:



a) Conservation of energy between A and C:

$$H_A = H_C + \Delta H_{f,1} + \Delta H_{f,2} + \sum \Delta H_m$$

$$H_A = H_C + f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} + f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g} + K_{LA}\frac{V_1^2}{2g} + \frac{(V_1-V_2)^2}{2g} + K_{LC}\frac{V_2^2}{2g}$$

Continuity: $V_1 = \frac{Q}{A_1}$, $V_2 = \frac{Q}{A_2}$

$$H_A - H_C = f_1 \frac{L_1}{D_1}\frac{Q^2}{2A_1^2 g} + f_2 \frac{L_2}{D_2}\frac{Q^2}{2A_2^2 g} + K_{LA}\frac{Q^2}{2A_1^2 g} + \frac{Q^2}{2g}\left(\frac{1}{A_1}-\frac{1}{A_2}\right)^2 + K_{LC}\frac{Q^2}{2A_2^2 g}$$

$$Q^2 = \frac{2g\,(H_A - H_C)}{f_1 \frac{L_1}{D_1 A_1^2} + f_2 \frac{L_2}{D_2 A_2^2} + \frac{K_{LA}}{A_1^2} + \left(\frac{1}{A_1}-\frac{1}{A_2}\right)^2 + \frac{K_{LC}}{A_2^2}}$$

$D_1 = 0'15\,m$, $D_2 = 0'30\,m$, $A_1 = 1'77 \cdot 10^{-2}\,m^2$, $A_2 = 7'07 \cdot 10^{-2}\,m^2$, $K_{LA} = 0'8$, $K_{LC} = 1$
$L_1 = 50\,m$, $L_2 = 100\,m$

FIG. 8.11 IN YOUNG ET AL.

$$Q^2 = \frac{245}{1'06 \cdot 10^6 f_1 + 6'67 \cdot 10^4 f_2 + 4547} \quad \left(m^6/s^2\right) \qquad [1]$$

To solve, we need to iterate:

- Initial values: Assume rough turbulent flow in both pipes.

PIPE 1 $\rightarrow \frac{\varepsilon}{D_1} = \frac{10^{-4}}{0'15} = 6'67 \cdot 10^{-4} \xrightarrow[R.T.]{MOODY} f_1 = 0'018$

PIPE 2 $\rightarrow \frac{\varepsilon}{D_2} = \frac{10^{-4}}{0'30} = 3'33 \cdot 10^{-4} \xrightarrow[R.T.]{MOODY} f_2 = 0'015$

From [1] $\rightarrow Q = 0'0997\ m^3/s$

- Iteration 1: $Q = 0'0997\ m^3/s$

PIPE 1 $\rightarrow V_1 = \frac{Q}{A_1} = 5'64\ m/s \Rightarrow \left.\begin{array}{l} Re = \frac{V_1 D_1}{\nu} = 8'5 \cdot 10^5 \\ \varepsilon/D_1 = 6'67 \cdot 10^{-4} \end{array}\right\} \xrightarrow{MOODY} f_1 = 0'018$

PIPE 2 $\rightarrow V_2 = \frac{Q}{A_2} = 1'41\ m/s \Rightarrow \left.\begin{array}{l} Re = \frac{V_2 D_2}{\nu} = 4'2 \cdot 10^5 \\ \varepsilon/D_2 = 3'33 \cdot 10^{-4} \end{array}\right\} \xrightarrow{MOODY} f_2 = 0'017$

From [1] $\rightarrow Q = 0'0995\ m^3/s$ (Convergence is good enough) $\rightarrow$

$\rightarrow \underline{\underline{Q \approx 0'10\ m^3/s}}$

b)

$$\Delta H_{f,1} = 0'018 \cdot \frac{50}{0'15} \cdot \frac{5'64^2}{2 \cdot 9'8} = 9'7\ m$$

$$\Delta H_{f,2} = 0'017 \cdot \frac{100}{0'30} \cdot \frac{1'41^2}{2 \cdot 9'8} = 0'5\ m$$

$$\Delta H_{m_A} = 0'8 \cdot \frac{5'64^2}{2 \cdot 9'8} = 1'3\ m$$

$$\Delta H_{m_B} = \frac{(5'64 - 1'41)^2}{2 \cdot 9'8} \approx 0'9\ m$$

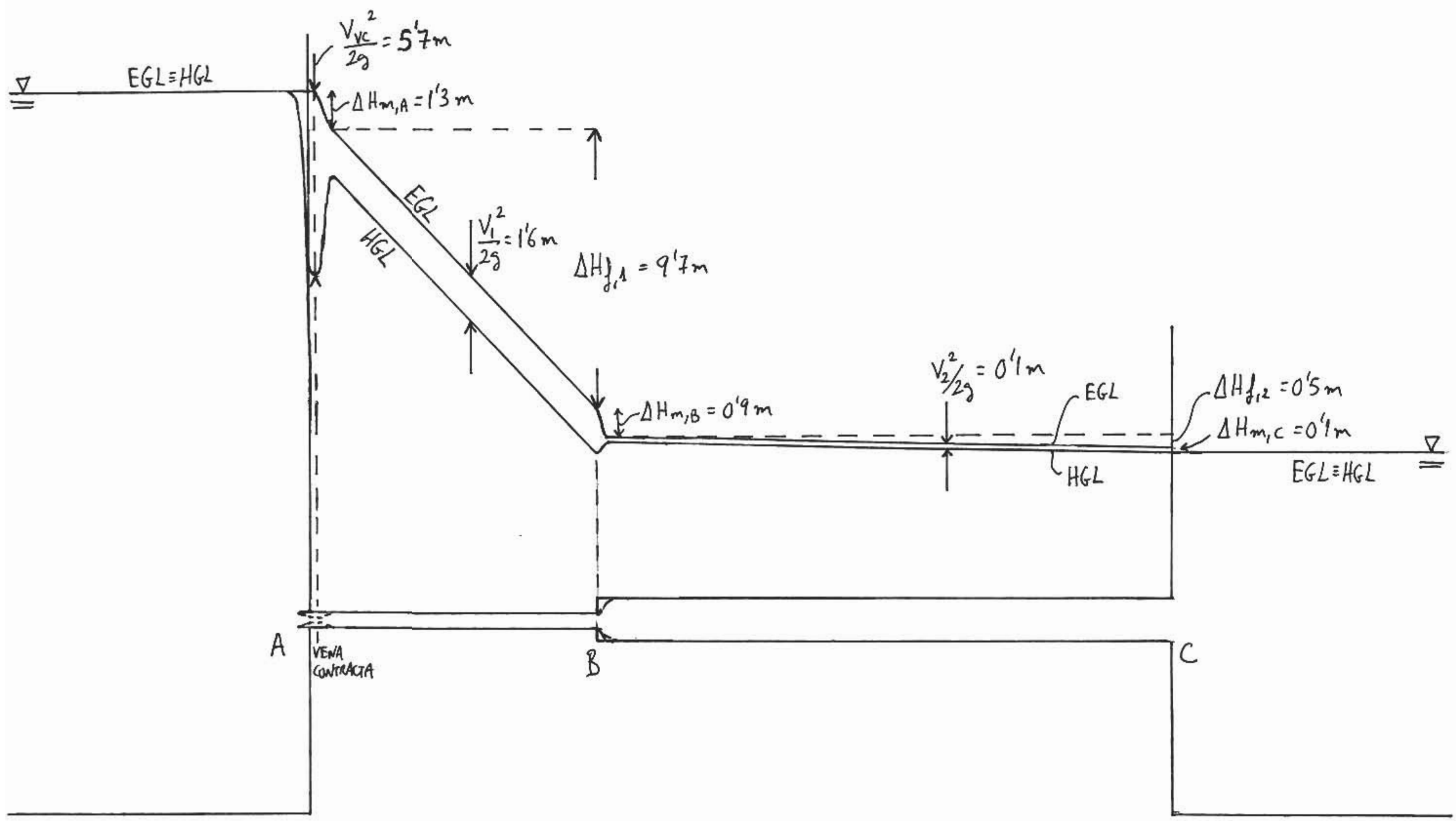
$$\Delta H_{m_C} = \frac{1'41^2}{2 \cdot 9'8} \approx 0'1\ m$$

$$\frac{V_1^2}{2g} = 1'6\ m$$

$$\frac{V_2^2}{2g} = 0'1\ m$$

$$K_{LA} = \left(\frac{1}{C_V} - 1\right)^2 \Rightarrow C_V = \frac{1}{1 + \sqrt{0'8}} = 0'53 \Rightarrow V_{VC} = \frac{V_1}{C_V} = 10'6\ m/s$$

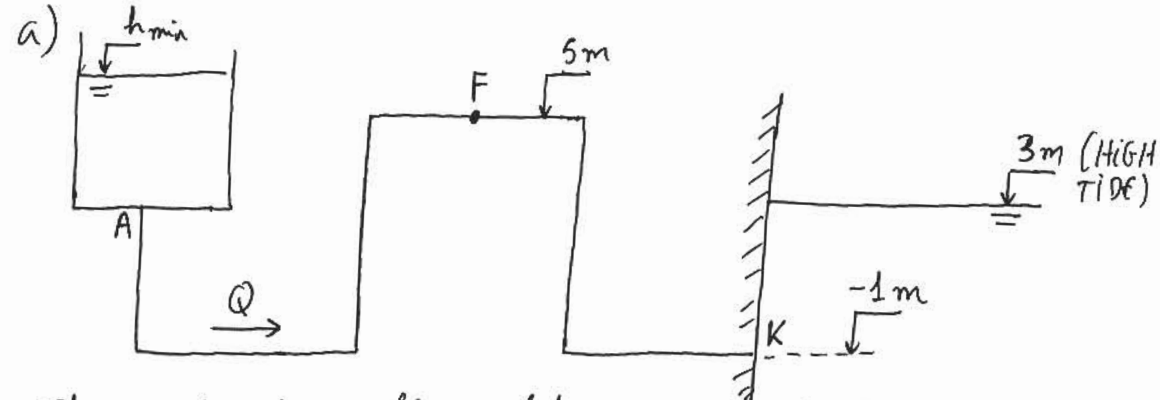
$$V_{VC}^2/2g = 5'7\ m$$



EGL≡HGL
$\frac{V_{vc}^2}{2g} = 5'7m$
$\Delta H_{m,A} = 1'3m$
EGL
HGL
$\frac{V_1^2}{2g} = 1'6m$
$\Delta H_{f,1} = 9'7m$
$\Delta H_{m,B} = 0'9m$
$V_2^2/2g = 0'1m$
EGL
HGL
$\Delta H_{f,2} = 0'5m$
$\Delta H_{m,C} = 0'1m$
EGL≡HGL
A
VENA CONTRACTA
B
C

- PROBLEM Nº 4:

a)



The most unfavorable conditions correspond to the high tide, since then the required value of $h = H_K + \text{LOSSES}$ is maximum. We calculate the friction factor:

$$\left.\begin{array}{l} Q = 0'5 \ m^3/s \\ D = 0'5 \ m \\ \varepsilon = 5\cdot 10^{-3} \ m \end{array}\right\} \quad V = \frac{Q}{\pi D^2/4} = \frac{0'5}{(\pi\cdot 0'5^2)/4} = 2'55 \ m/s \Rightarrow$$

$$\left.\begin{array}{l} \Rightarrow Re = \frac{VD}{\nu} = \frac{2'55\cdot 0'5}{10^{-6}} = 1'27\cdot 10^6 \\ \varepsilon/D = \frac{5\cdot 10^{-3}}{0'5} = 0'01 \end{array}\right\} \xrightarrow{\text{MOODY DIAGRAM}} f = 0'038$$

We apply energy conservation between A and K, accounting for friction and minor losses:

$$H_A = H_K + \Delta H_f + \sum \Delta H_m$$ (Take A before the inlet and K before the outlet).

FRICTION HEADLOSS: $\Delta H_f = f \frac{l_{AK}}{D}\frac{V^2}{2g} = f\cdot\frac{(l_{AB}+l_{BC}+\dots+l_{JK})}{D}\frac{V^2}{2g} =$

$$= 0'038\,\frac{(4+50+6+5+5+6+20)}{0'5}\cdot\frac{2'55^2}{2\cdot 9'8} = 2'42 \ m$$

MINOR LOSSES: $\sum\Delta H_m = (\underset{\substack{\uparrow \\ \text{inflow at A}}}{K_{LA}} + \underset{\substack{\uparrow \\ 90^\circ \text{ at B}}}{K_{LB}} + \dots + \underset{\substack{\uparrow \\ 90^\circ \text{ at J}}}{K_{LJ}})\frac{V^2}{2g} = (0'5+0'3+0'3+0'3+0'3+0'3)\frac{2'55^2}{2\cdot 9'8} =$

$$= 0'66 \ m$$

Therefore:

$$\underbrace{\underbrace{z_A + \frac{p_A}{\rho g}}_{= h_{min}} + \cancel{\frac{V_A^2}{2g}}^{\,0}}_{\text{Before the inlet}} = \underbrace{z_K + \frac{p_K}{\rho g} + \frac{V_K^2}{2g}}_{\text{Before the outlet}} + \Delta H_f + \sum\Delta H_m =$$

$$= -1 + \frac{1025\cdot 9'8\cdot 4}{1000\cdot 9'8} + \frac{2'55^2}{2\cdot 9'8} + 2'42 + 0'66 =$$

$$= \underline{\underline{6'51 \ m = h_{min}}}$$

b) $\underset{\text{HEAD AT F}}{H_F} = \underset{\text{HEAD AT A}}{H_A} - \overbrace{\left(\Delta H f_{A\to F} + \sum \Delta H_{m\,A,B,C,E}\right)}^{\text{HEADLOSS BETWEEN A AND F}} =$

$$= 6'51 - \Big(\underbrace{0'038\,\frac{(4+50+6+5)}{0'5}}_{\text{(friction)}} + \underbrace{(0'5+0'3+0'3+0'3)}_{\text{(minor losses)}}\Big)\frac{2'55^2}{2\cdot 9'8} = 4'41\,m$$

$$H_F = z_F + \frac{p_F}{\rho g} + \frac{V_F^2}{2g} = 4'41 \Rightarrow 5 + \frac{p_F}{9800} + \frac{2'55^2}{2\cdot 9'8} = 4'41 \Rightarrow$$

$\Rightarrow \underline{\underline{p_F = -9'03\ kPa}}$ (gauge) or $p_{F,ABS} = 101'3 - 9'03 = 92'3\ kPa$

We should check whether the pressure is low enough to cause <u>CAVITATION</u>. If cavitation occurs, air bubbles are formed. The collapse of these air bubbles generates very high pressures which may seriously damage the pipe.

Cavitation happens if $p_{ABS} < p_{VAPOR}$:

Young et al., Table B.2: $p_{VAPOR} = 2'3\ kPa$ at 20°C, $p_{VAPOR} = 7'4\ kPa$ at 40°C.

Since $p_{F,ABS} = 92'3\ kPa \gg \sim 7\ kPa \Rightarrow$ <u>NO CAVITATION</u>

c) Since, under the current conditions, $p_F < p_{ATM}$, when we open the well, air will enter the pipe and the syphon effect will be lost, leading to no discharge. Therefore, we need to increase the value of "h" to be able to discharge water. (Note: If we had $p_F > p_{ATM}$, the well would act as a piezometer and we would be able to discharge water in the same conditions as before).

We need to increase p_F so that $p_F \geq p_{ATM}$ and we get a discharge.

So, you can estimate h_{min} by assuming $p_F = 0$ and:

$$H_F = z_F + 0 + \frac{V^2}{2g} = 5 + \frac{2'55^2}{2 \cdot 9'8} = 5'33 \text{ m}$$

NOTE: $Q = 0'5$ m^3/s is constraint by the inflow to the collection chamber

$$\Delta H_{f_{A \to F}} + \sum \Delta H_{m_{A,B,C,E}} = 2'10 \text{ m}$$ (as calculated in (b)).

Therefore

$$H_A = H_F + \Delta H_{f_{A \to F}} + \sum \Delta H_{m_{A,B,C,E}} = 5'33 + 2'10 = 7'43 \text{ m} \Rightarrow$$

$$\Rightarrow \underline{\underline{h \simeq 7'43 \text{ m}}}$$

This is an <u>approximation</u>, because the system works now in a funny way. Conservation of energy between F and K:

$$H_F = z_F + \frac{V^2}{2g} = 5'33 \text{ m}$$

$$H_K = z_K + \frac{p_K}{\rho g} + \frac{V^2}{2g} = 3'43 \text{ m}$$

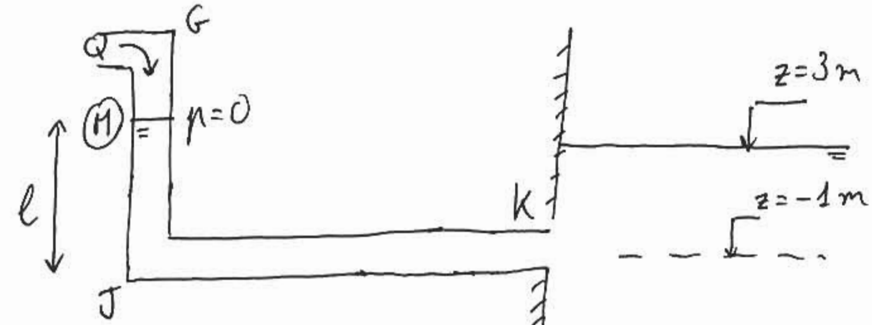
$$\Delta H_{f_{F \to K}} = f \frac{l_{FK}}{D} \frac{V^2}{2g} = 0'78 \text{ m}$$

$$\sum \Delta H_{m_{G,J}} = (K_{LG} + K_{LJ}) \frac{V^2}{2g} = 0'20 \text{ m}$$

$$H_F = 5'33 > H_K + \Delta H_{f_{F \to K}} + \sum \Delta H_{m_{G,J}} = 4'41 \text{ !!}$$

Full pipe flow along all the pipe is not possible. unless we install a valve. We need an <u>additional headloss</u> to satisfy conservation of energy. To get this additional loss, flow is going to happen in part of the pipe from F to G, and then water will go in free fall from G. until a point M, where full pipe flow is recovered.

Where is this point M? Consider the right part of the pipe, and determine at which point we need $p = 0 = p_{ATM}$ (gauge) to have a discharge $\varphi = 0'5\ m^3/s$:



Conservation of energy between M and K ($p_M = 0$; K is before the outlet):

$$H_M = z_M + \frac{V^2}{2g} = -1 + l + \frac{V^2}{2g} \qquad V = 2'55\ m/s$$

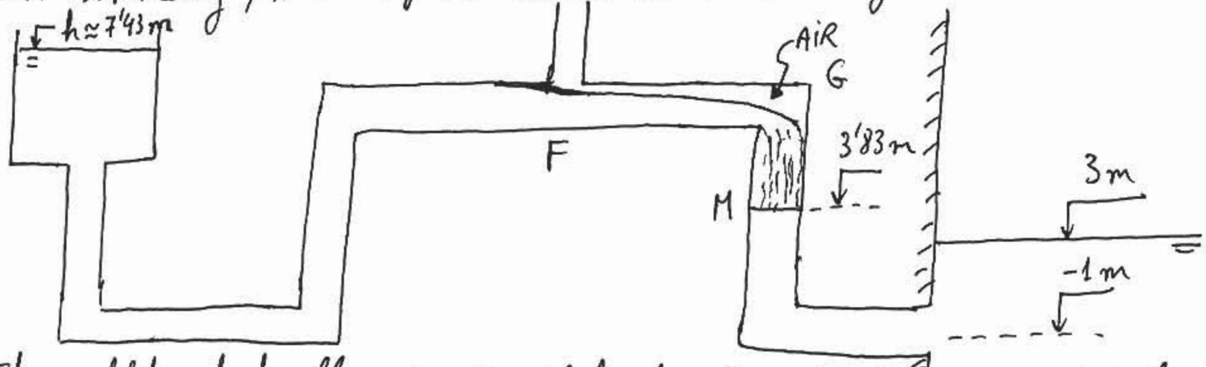
$$H_K = z_K + \frac{p_K}{\rho g} + \frac{V^2}{2g} = -1 + 4'1 + \frac{V^2}{2g}$$

$$\Delta H f_{M\to K} = f\,\frac{(l + L_{JK})}{D}\,\frac{V^2}{2g} = (0'076\,l + 1'52)\frac{V^2}{2g}$$

$$\Sigma \Delta H_{m\,M\to K} = \Delta H_{m\,J} = 0'3\,\frac{V^2}{2g}$$

$$H_M = H_K + \Delta H f_{M\to K} + \Sigma \Delta H_{m\,M\to K} \Rightarrow l = 4'83\ m$$

In summary, the system works in this way:



The additional headloss is provided by the turbulence generated by the free fall from G to M.

- PROBLEM N° 5:

a)

The flowrate Q_1 in the pipe of diameter D_1 must be equal to the average flowrate demand:

$$Q_1 = \text{average flowrate demand} = \frac{0'05 \cdot 12 + 0'1 \cdot 12}{24} = 0'075 \ m^3/s$$

$$V_1 = \frac{Q_1}{\pi D_1^2/4} = \frac{0'075}{\pi 0'25^2/4} = 1'53 \ m/s$$

$$\left.\begin{aligned} Re &= \frac{V_1 D_1}{\nu} = \frac{1'53 \cdot 0'25}{10^{-6}} = 3'82 \cdot 10^5 \\ \varepsilon/D &= 5 \cdot 10^{-4}/0'25 = 2 \cdot 10^{-3} \end{aligned}\right\} \xrightarrow{\text{MOODY}} f_1 = 0'024$$

Apply energy conservation between the upper reservoir and the outflow into the intermediate reservoir:

$$H_{in} = H_{out} + \Delta H_f + \Sigma \Delta H_m$$

$$50 = h_{max} + \left(0'024 \cdot \frac{1750}{0'25} + \underbrace{0'5}_{\uparrow \text{ assume sharp corners}} + 1\right) \cdot \frac{1'53^2}{2 \cdot 9'8} \Rightarrow \underline{\underline{h_{max} = 29'76 \ m}}$$

b)

At 8 PM, the intermediate reservoir is empty ($h = h_{min}$). From 8 PM, $Q_{in} = 0'075 \ m^3/s > Q_{out} = 0'05 \ m^3/s$ and the reservoir level increases, reaching the maximum ($h = h_{max}$) at 8 AM. From 8 AM to 8 PM, $Q_{in} = 0'075 \ m^3/s < Q_{out} = 0'1 \ m^3/s$ and the level decreases, reaching $h = h_{min}$ at 8 PM. Therefore:

$$\text{Volume} = (\underset{\uparrow Q_{in}}{0'075} - \underset{\uparrow Q_{out} \text{ at 8 PM-8 AM}}{0'05}) \cdot (\underbrace{12 \cdot 3600}_{\nwarrow \text{ time between 8 PM-8 AM}}) = 1080 \ m^3$$

$$\Delta h = \frac{\text{Volume}}{\text{area}} = \frac{1080}{15^2} = 4'8 \ m$$

$$\underline{\underline{h_{min}}} = h_{max} - \Delta h = 29'76 - 4'8 = \underline{\underline{24'96 \ m}}$$

c)

The most critical situation happens right before 8 PM ($t = 20\ h - 1$ second). At that instant, $h = h_{min}$ (minimum initial head) and $Q = 0'1\ m^3/s$ (maximum headloss).

We apply energy conservation between the inflow and the outflow of the pipe of diameter D_2:

$$\Delta H = \left(f\frac{L}{D} + 0'5 + 1\right)\frac{V^2}{2g} = h_{min} - 10 = 14'96\ m$$

Now we calculate D_2 by trial and error:

1ST TRY: $D_2 = 35\ cm$

$$\left.\begin{array}{r} Q = 0'1\ m^3/s \Rightarrow V = 1'04\ m/s \Rightarrow Re = 3'6\cdot 10^5 \\ \varepsilon/D = 0'0014 \end{array}\right\} \Rightarrow f = 0'022 \Rightarrow$$

$$\Rightarrow \Delta H_f = \left(f\frac{L}{D} + 1'5\right)\frac{V^2}{2g} = 1'64\ m < 14'96\ m$$

We can afford a larger headloss, so we can use a smaller (cheaper) D_2.

2ND TRY: $D_2 = 25\ cm$

$$\left.\begin{array}{r} Q = 0'1\ m^3/s \Rightarrow V = 2'04\ m/s \Rightarrow Re = 5'1\cdot 10^5 \\ \varepsilon/D = 0'002 \end{array}\right\} \Rightarrow f = 0'024 \Rightarrow$$

$\Rightarrow \Delta H_f = 9'49\ m < 14'96\ m \Rightarrow$ We can use a smaller D_2.

3RD TRY: $D_2 = 20\ cm$

$$\left.\begin{array}{r} Q = 0'1\ m^3/s \Rightarrow V = 3'18\ m/s \Rightarrow Re = 6'4\cdot 10^5 \\ \varepsilon/D = 0'0025 \end{array}\right\} \Rightarrow f = 0'025 \Rightarrow$$

$\Rightarrow \Delta H_f = 29'8\ m > 14'96\ m \Rightarrow$ This diameter would give $Q < 0'1\ m^3/s$.

Therefore, take the smallest diameter that works: $\underline{\underline{D_2 = 0'25\ m}}$.

d)

Apply energy conservation between the intermediate reservoir and the outflow to the channel:

$$H_{in} = H_{out} + H_f + \Sigma \Delta H_m \qquad (\text{Recall}: D_2 = 0'25 \text{ m})$$

- For $Q = 0'1 \text{ m}^3/\text{s}$, we have $V = 2'04 \text{ m/s}$, $f = 0'024$ (as calculated in (c)).

$$h = 10 + \left(0'024 \frac{450}{0'25} + 1'5\right) \frac{2'04^2}{2 \cdot 9'8} + \Delta H_{valve} \quad (\text{S.I.})$$

$$\Delta H_{valve} = h - 19'49 \ (\text{m}) \qquad [1]$$

- For $Q = 0'05 \text{ m}^3/\text{s}$ $\Rightarrow V = 1'02 \text{ m/s}$, $Re = 2'6 \cdot 10^5$, $\frac{\varepsilon}{D} = 0'002$, $f = 0'0245$

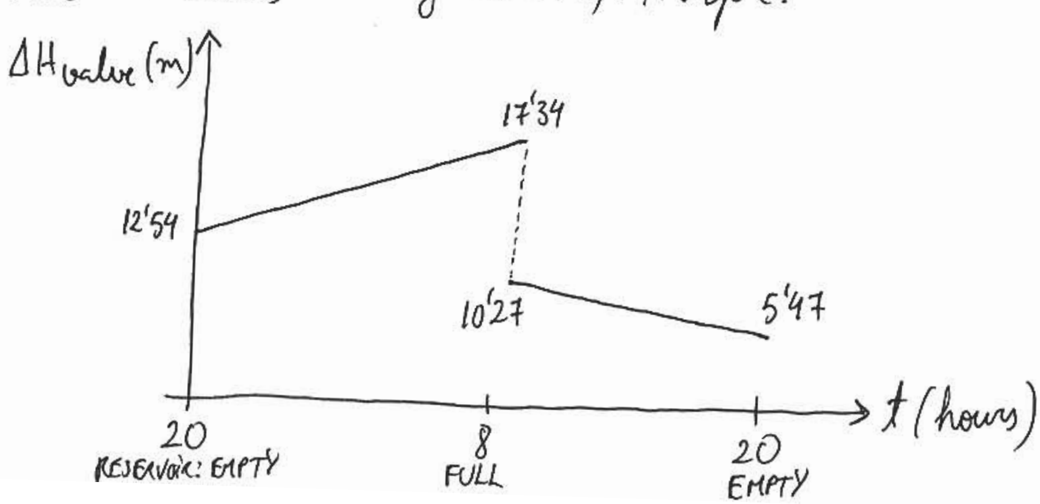
$$h = 10 + \left(0'0245 \frac{450}{0'25} + 1'5\right) \frac{1'02^2}{2 \cdot 9'8} + \Delta H_{valve} \quad (\text{S.I.})$$

$$\Delta H_{valve} = h - 12'42 \ (\text{m}) \qquad [2]$$

Using equations [1] and [2] we get:

t (hours)	20'01	7'99	8'01	19'99
h (m)	24'96	29'76	29'76	24'96
Q (m^3/s)	0'05	0'05	0'10	0'10
ΔH_{valve} (m)	12'54	17'34	10'27	5'47

In between these values, ΔH_{valve} varies linearly in time (since h varies linearly in time). Therefore:



- PROBLEM N° 6:

a) Applying conservation of energy between B and C, we calculate the head at B (note that, since we neglect minor losses at B, the head at B will be the same for the three pipes):

$$H_B = H_C + \Delta Hf_{B \to C} + \Delta H_{m_C}$$

$$H_C = 0$$

$$\left.\begin{matrix} Q_2 = 0'4 \ m^3/s \\ D_2 = 0'5 \ m \end{matrix}\right\} \Rightarrow V_2 = 2'04 \ m/s \Rightarrow \left.\begin{matrix} Re = 1'02 \cdot 10^6 \\ \varepsilon/D = 4 \cdot 10^{-3} \end{matrix}\right\} \Rightarrow f_2 = 0'028$$

$$\Delta Hf_{B \to C} = f_2 \frac{L_2}{D_2} \frac{V_2^2}{2g} = 0'028 \cdot \frac{3000}{0'5} \cdot \frac{2'04^2}{2 \cdot 9'8} = 35'67 \ m$$

$$\Delta H_{m_C} = \frac{V_2^2}{2g} = 0'21 \ m$$

$$H_B = 0 + 35'67 + 0'21 = 35'88 \ m$$

Now we apply energy conservation between B and E, and obtain D_3 by trial and error:

$$\Delta H_{BE} = \left(f_3 \frac{L_3}{D_3} + 1 \right) \frac{V_3^2}{2g} = H_B - H_E = 35'88 - 30 = 5'88 \ m$$

•) Try $D_3 = 0'2 \ m$: $V_3 = 1'59 \ m/s$ (for $Q_3 = Q_{3,min} = 0'05 \ m^3/s$) $\Rightarrow$

$$\Rightarrow \left.\begin{matrix} Re = 3'2 \cdot 10^5 \\ \varepsilon/D = 0'01 \end{matrix}\right\} \Rightarrow f_3 = 0'038$$

$$\Delta H_{BE} = \left(0'038 \cdot \frac{200}{0'2} + 1 \right) \frac{1'59^2}{2 \cdot 9'8} = 5'03 m < 5'88 m \Rightarrow$$

$\Rightarrow$ This diameter will provide $Q_3 > 0'05 \ m^3/s$. Let's try a smaller (cheaper) D_3.

• Try $D_3 = 0'175$ m : $V_3 = 2'08$ m/s, $Re = 3'6 \cdot 10^5$, $\varepsilon/D = 0'0114$, $f_3 = 0'038$.

$$\Delta H_{BE} = \left(0'038 \cdot \frac{200}{0'175} + 1\right) \cdot \frac{2'08^2}{2 \cdot 9'8} = 9'8 \text{ m} > 5'88 \text{ m}$$

We don't have enough ΔH_{BE} to spare. This diameter is too small and will provide $Q_3 < 0'05$ m³/s.
So take the smallest D_3 that works, i.e., $\underline{\underline{D_3 = 0'2 \text{ m}}}$

b) We chose $D_3 = 0'2$ m, so we have $Q_3 > 0'05$ m³/s.
Calculate Q_3: (Note that f_3 is insensitive to Re, since rough turbulent flow).

$$\Delta H_{BE} = 5'88 = \left(0'038 \cdot \frac{200}{0'2} + 1\right) \frac{V_3^2}{2g} \Rightarrow V_3 = 1'7 \text{ m/s} \Rightarrow$$

$$\Rightarrow Q_3 = 0'054 \text{ m}^3/\text{s}$$

Continuity at B dictates

$$Q_1 = Q_2 + Q_3 = 0'4 + 0'054 = 0'454 \text{ m}^3/\text{s}$$

Since $D_1 = 0'8$ m $\Rightarrow V_1 = 0'90$ m/s, $Re = 7'2 \cdot 10^5$, $\frac{\varepsilon}{D} = 0'0025$, $f_1 = 0'025$. Conservation of energy between A and B yields

$$H_{min} = H_B + \Delta H_f$$ (minor headlosses between A and B neglected).

$$\underline{\underline{H_{min}}} = 35'88 + 0'025 \cdot \frac{10000}{0'8} \cdot \frac{0'90^2}{2 \cdot 9'8} = \underline{\underline{48'8 \text{ m}}}$$