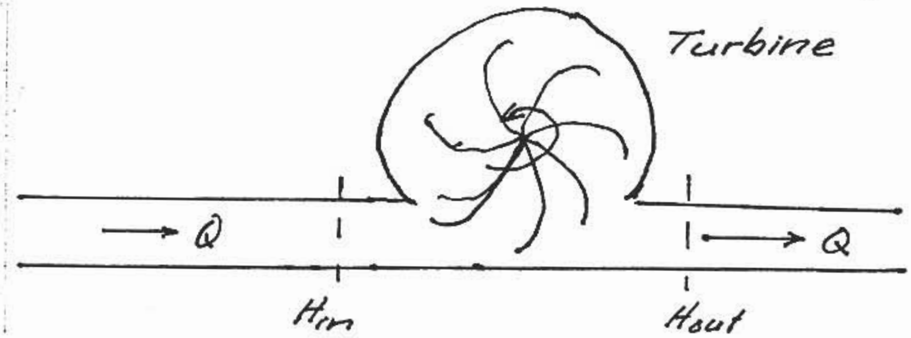
LECTURE # 19

1.060 ENGINEERING MECHANICS II

TURBINES

A turbine may be regarded as an "inverse pump" in that extracts energy (e.g. producing electicity) from the mechanical energy of a flow



H_{in} and H_{out} are obtained from pipe flow analyses identical to those performed for pumps.
Energy considerations the give:

$$\dot{E}_{in} - \dot{E}_{out} = \rho g Q \left(H_{in} - H_{out}\right) = \rho g Q H_T$$

where

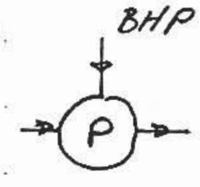
$$H_T = H_{in} - H_{out} = \text{Turbine Head}$$

Conversion of the flow power, $\rho g Q H_T$, to an alternative form of power is associated with a loss, so for turbines

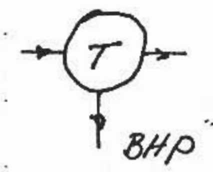
η = efficiency

$\underline{BHP}$ = Power Produced by Turbine = $\eta \rho g Q H_T$ $\quad \eta \leq 1$

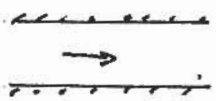
So what happens to the Energy Lost?



$$\eta \, BHP = \rho g Q H_p \qquad (1-\eta) BHP \text{ "lost"}$$



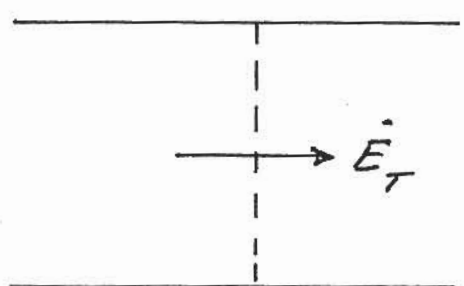
$$\eta \rho g Q H_T = BHP \qquad (1-\eta)\rho g Q H_T \text{ "lost"}$$



$$\rho g Q H_1 = \rho g Q H_2 + \rho g Q \Delta H \qquad \rho g Q \Delta H \text{ "lost"}$$

What actually happens to the "lost" energy?

ENERGY REVISITED



$$\dot{E}_{Total} = \dot{E}_{Mech} + \dot{E}_{internal} = \dot{E}_{org.} + \dot{E}_{disorg}$$

$$\dot{E}_{mech} = \dot{E}_{org} = \rho g Q H$$

$$\dot{E}_{internal} = \dot{E}_{disorg} = \rho Q \hat{u}$$

$\rho \hat{u}$ = internal (disorganized – thermal – heat) energy per unit volume of fluid.

$$\rho \hat{u} = \rho C_p T = \rho C_v T$$ [for incompressible fluid, water]

T = absolute temperature ($°K = 273.15 + °C$)

C_p = specific heat @ constant pressure } same for

C_v = " " " volume } an incompressible fluid.

$$[\rho C_p T] = \frac{kg}{m^3}[C_p]°K = \frac{\text{Energy}}{\text{volume}} = \frac{Nm}{m^3} = \frac{(kg\frac{m}{s^2})m}{m^3}$$

$$\underline{[C_p] = \frac{m^2}{s^2\ °K}}$$

For water: $C_p = C_v = 4{,}210 \frac{m^2}{s^2\ °K}$

ENERGY (TOTAL) CONSERVATION

$$\dot{E}_{Total} = \dot{E}_T = \rho Q[gH + \hat{u}]$$

$$\dot{E}_{T,in} - \dot{E}_{T,out} + \underbrace{\dot{H}_{add} - \dot{H}_{loss}}_{\text{Net inflow from boundaries}} = 0$$

$\vec{q} \cdot \vec{n} = 0$

If insulated pipe, then

$$\rho g Q(H_1 - H_2) = \rho Q(\hat{u}_2 - \hat{u}_1)$$

Rate of dissipation of mechanical energy = Rate of production of thermal (internal) energy

$$\underline{g(H_1 - H_2) = g\Delta H = \hat{u}_2 - \hat{u}_1 = C_p(T_2 - T_1)}$$

or

$$\Delta T = T_2 - T_1 = \frac{g\Delta H}{C_p}$$

Loss of head causes an increase in temperature! Material constants, e.g. ρ and ν, are functions of temperature; but we have treated them as constants. Is this justified?

<u>Pipe Flow Example</u>

$V = 1\,m/s$; $D = 0.025\,m$ (1"); $f = 0.02$; $l = 1{,}000\,m$.

$$\Delta H = \Delta H_f = f\,\frac{l}{D}\,\frac{V^2}{2g}$$

$$\Delta T = T_2 - T_1 = \frac{g\Delta H}{C_p} = \frac{f(l/D)V^2}{2C_p} = \frac{0.02 \cdot 1000 \cdot 40 \cdot 1}{2 \cdot 4000} = 0.1^\circ K$$

Negligible change in temperature. Indeed neglect of temperature variations is justified!