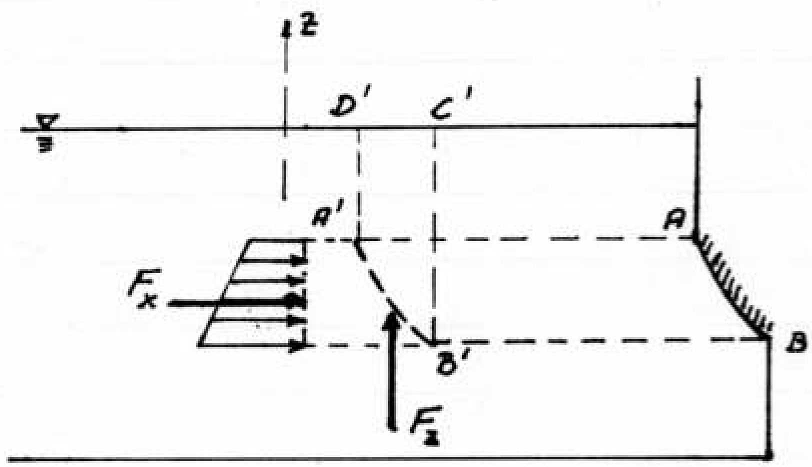
LECTURE #4

1.060 ENGINEERING MECHANICS II

HYDROSTATIC PRESSURE FORCES ON A CURVED SURFACE



We seek the pressure force on AB

A'B' represents the horizontal translation of AB to a location where A'B' is surrounded by fluid.

Since $p + \rho g z$ = constant (hydrostatics) and a horizontal translation preserves "z" along AB, the pressure forces on A'B' are identical to those on AB

From considerations of force equilibrium in the horizontal direction it follows that

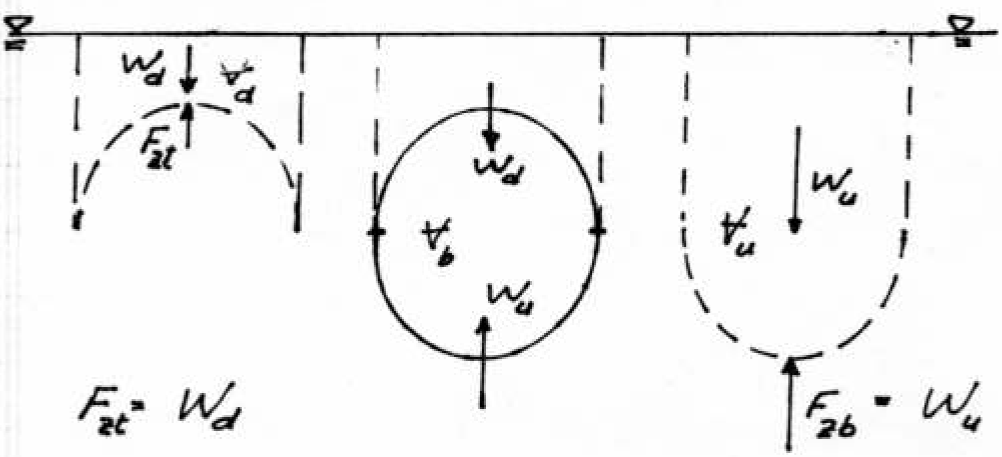
F_x = horizontal force on AB = pressure force on AB's projection onto a vertical plane.

F_z = vertical force on AB = vertical force on $A'B'$ = Weight of fluid above $A'B'$ (even if there was "air" above AB!) in the volume $A'B'C'D'$.

The line of action of F_x is obtained as the line of action of F_x on AB's projection on a vertical plane, i.e. using the rules for plane surfaces

The line of action of F_z passes through the center of gravity of the volume $A'B'C'D'$ above $A'B'$. - when translated back to the location of AB.

<u>Buoyancy</u> (Archimedes' Law)

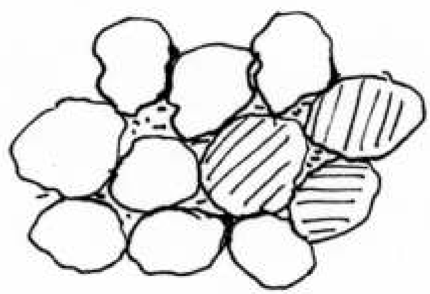


$F_{zt} = W_d$ $\qquad$ $F_{zb} = W_u$

<u>Net force on submerged body</u> $= W_u - W_d = \rho g (\forall_u - \forall_d) = \rho g \forall_b =$
<u>Weight of fluid displaced by the body</u>
<u>Line of action is vertical</u> (upwards) and passes <u>through the center of gravity of $\forall_b$</u>:

The Effective Stress in Soil Mechanics

A fully saturated soil deposit may be conceptualized as a solid matrix consisting of individual soil particles with the porespace between the soil grains filled with a fluid.



If the porespace occupied by fluid is simply connected, i.e. one can get from any point of the fluid to any other point of the fluid without ever being forced to leave the fluid, then the pressure in the fluid (if both fluid and soil matrix is at rest) is governed by hydrostatics, i.e.

$$p + \rho g z = \text{Constant in the pore fluid}$$

If we now consider a single soil particle and assume that it is completely surrounded by fluid, except for a few points where it touches neighboring soil particles, then each soil particle experiences an upward buoyancy force

$$f_b = \rho g \forall_s$$

where $\forall_s$ = volume of the soil particle.

If the density of the solid making up the soil particle is ρ_s, then the weight of the soil grain is

$$f_g = \rho_s g \forall_s$$

and acts vertically downward.

The effective weight of a single soil particle, i.e.

$$f_s = f_g - f_b = (\rho_s - \rho) g \forall_s$$

is the weight that must be carried through forces transmitted at the solid-solid contact points of the soil particle.

Thus, as far as the solid soil matrix is concerned, the soil behaves as if its density were $\rho_s - \rho$ = submerged density in a fluid of density ρ.

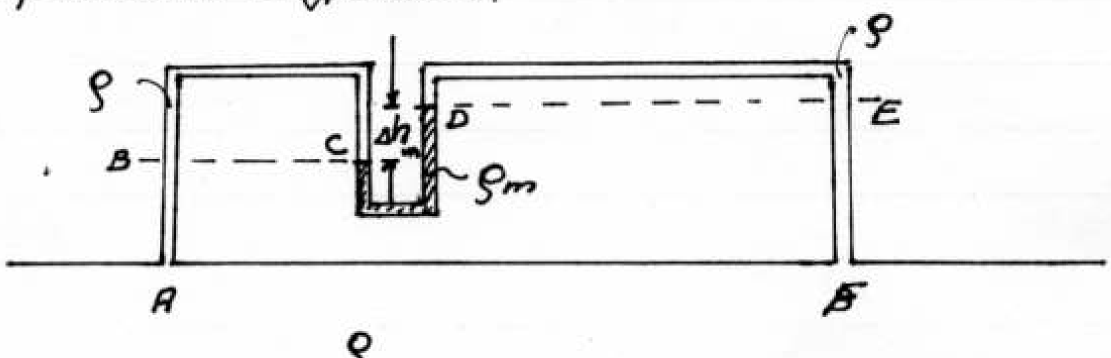
Vertical force equilibrium for a vertical column (of unit horizontal area) of a saturated soil now gives a vertical stress to be carried by the solid soil matrix, the effective stress in soil mechanics, is governed by:

$$\Delta \sigma_{ze} = (\rho_s - \rho) g (1 - n_p) h$$

where σ_{ze} = vertical effective normal stress, and n_p = porosity of the soil = soil volume /(soil + pore volumes), and h = vertical height of the column.

APPLICATION OF HYDROSTATICS

<u>Manometry</u> uses hydrostatics to obtain a non-intrusive measurement of a pressure or a pressure difference.



The manometer is connected to the fluid (of density ρ) through pressure taps at A and F. The manometer fluid has a density ρ_m ($> \rho$) and is located in a U-tube. The manometer reading is the difference between the manometer fluid elevations in the two legs of the U-tube, Δh_m.

To determine what Δh_m represents, we start at point A where

$$p = p_A$$

Going up into the tube leading from A to B we have (fluid at rest)

$$(p + \rho g z)_{in\ AB} = p_A + \rho g z_A =$$

$$p_B + \rho g z_B = p_C + \rho g z_C \Rightarrow p_C = p_A + \rho g (z_A - z_C) \quad (1)$$

where C is at the inface of ρ and ρ_m in the left leg of the manometer.

Provided that surface tension can be neglected the pressure in ρ_m, just below C, is the same as in ρ, just above C, i.e. p_C. Thus, in the manometer fluid we have

$$p + \rho_m g z = p_C + \rho_m g z_C$$

and therefore

$$p_D + \rho_m g z_D = p_D + \rho_m g z_C + \rho_m g \Delta h_m =$$

$$p_C + \rho_m g z_C \Rightarrow p_D = p_C - \rho_m g \Delta h_m \quad (2)$$

Pressure being continuous across the inter= face between ρ_m and ρ in the right leg of the manometer gives

$$p + \rho g z = p_D + \rho g z_D$$

in the ρ-fluid leading from D to F. In particular we have

$$p_F + \rho g z_F = p_D + \rho g z_D \Rightarrow p_F = p_D + \rho g (z_D - z_F) \quad (3)$$

Combining (1), (2) and (3) we have

$$p_F = p_D + \rho g (z_D - z_F) = p_C - \rho_m g \Delta h_m + \rho g (z_D - z_F) =$$

$$p_A + \rho g (z_A - z_C) + \rho g (z_D - z_F) - \rho_m g \Delta h_m =$$

$$p_A + \rho g z_A - \rho g z_F + \rho g (\overset{\overset{\Delta h_m}{\shortparallel}}{z_D - z_C}) - \rho_m g \Delta h_m$$

or

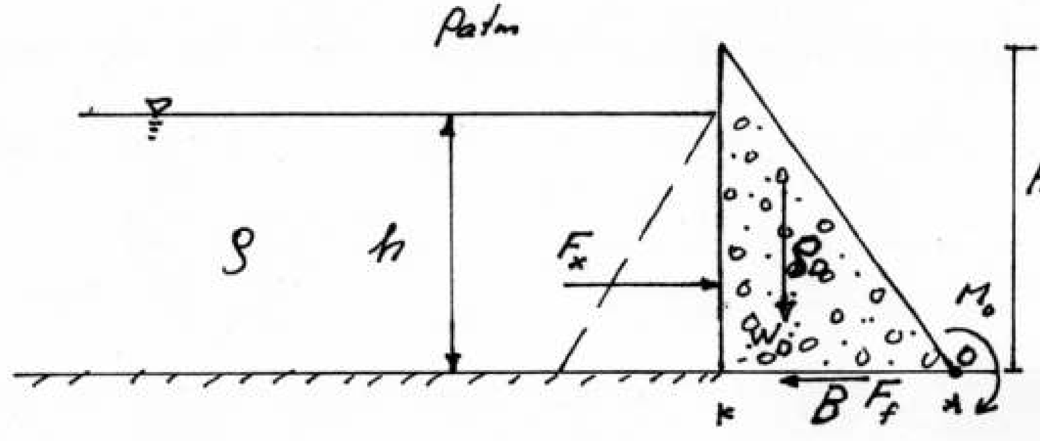
$$\underline{(\rho_m - \rho) g \Delta h_m = (p_A + \rho g z_A) - (p_F + \rho g z_F)}$$

Thus, the manometer reading Δh_m is a measure of the difference in $(p + \rho g z)$ between the two pressure taps at A & F.

- If $z_A = z_F$ or if z_A & z_F are known Δh_m gives the pressure difference between A & F

- If fluid below A & F is simply connected, i.e. one can get from A to F without ever leaving the ρ-fluid, then $p + \rho g z$ is constant if fluid is at rest and $\Delta h_m \equiv 0$. Thus, if $\Delta h_m \neq 0$: Fluid is moving!

Hydrostatic Forces on Dams



From 2-D hydrostatics we have (per unit length into paper):

$$F_x = \frac{1}{2}\rho g h^2 \Rightarrow \text{will try to make dam slide}$$

and

$$M_S = \frac{1}{3} h F_x = \frac{1}{6}\rho g h^3 \Rightarrow \text{will try to overturn the dam}$$

Dam will <u>fail by sliding</u> if

$$F_f = (\overbrace{\tfrac{1}{2}\rho_D g BH}^{W_D})\mu_f < F_x \Rightarrow \frac{F_f}{F_x} = \text{robustness against sliding}$$

where μ_f = coefficient of friction between dam and foundation surface.

Dam will <u>fail by overturning</u> (around "O") if

$$M_D = W_D \cdot \frac{2}{3}B = \frac{1}{6}\rho_D g B^2 H < M_S \Rightarrow \frac{M_D}{M_S} = \text{robustness against overturning}$$

For those who took 1.050 Engrng. Mech. I in Fall 2005, the damned dam whose stability we just analyzed looks awfully similar to the 3^{rd} problem in HW-3. In fact, the two problems are identical if $B = H = h$ and $\rho = \rho_0 = 2{,}200\ kg/m^3$.

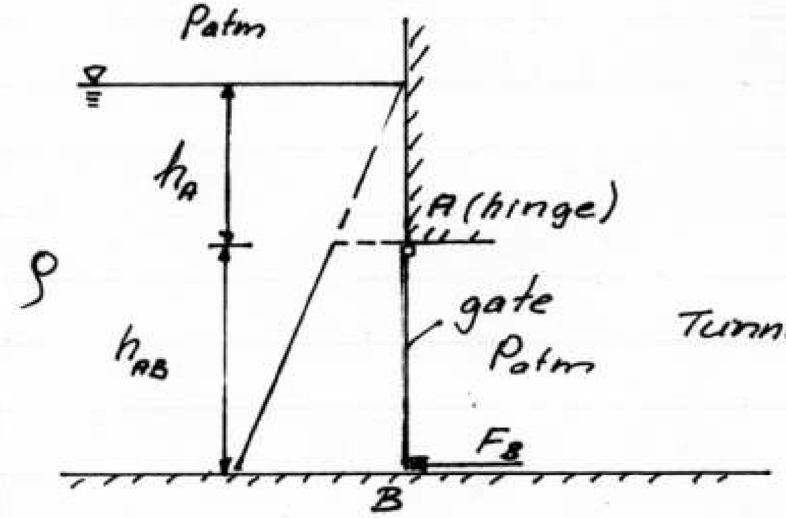
In HW-3, Problem No: 3, you obtained the stress field within the dam using the appropriate stress boundary conditions along the upstream face ($\sigma_{xx} = -p = \rho g z$, $\tau_{xz} = 0$) and the stress free inclined 45° "downstream" slope. In particular, you were asked about stresses along the dam-foundation contact line.

Here's something "fun" to do before the Red Sox take the field in Ft. Meyers.

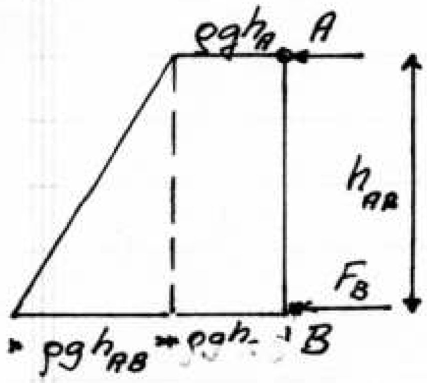
1) Show that your solution, obtained in 1.050, satisfied global equilibrium, i.e. $F_x = \int_0^B \tau_{zx}\, dx$ along rock-dam line and that $M_S - M_D = \int_0^B \sigma_{zz}\, dx = 0$ along rock-dam line.

2) Show that the stress field obtained in 1.050 could have been obtained by replacing the stress free boundary condition along the inclined downstream face of the dam with the global force and moment balance introduced in (1) above.

Hydrostatic Forces on Gates



Tunnel under dam



Moment around A (=0 since hinged):

$$\left(\frac{1}{2}\rho g h_{AB}^2\right)\left(\frac{2}{3}h_{AB}\right) + \left(\rho g h_A h_{AB}\right)\frac{1}{2}h_{AB} = h_{AB} F_B \Rightarrow \underline{\text{Gives } F_B}$$

Horizontal Force Equilibrium:

$$\frac{1}{2}\rho g h_{AB}^2 + \rho g h_A h_{AB} = F_A + F_B \Rightarrow \underline{\text{Gives } F_A}$$

As a check one may show that M_B, the moment of forces around B, is zero.

If the gate is very long in the plane into the paper the gate itself may be designed (structurally) using 1.050-knowledge, as if it were a simply supported beam spanning AB with load p ⊥ AB.