Homework Set #2

Problem 1

A machine to detect improper welds in a fabricating shop detects 80 percent of all improper welds, but it also incorrectly indicates an improper weld on 5 percent of all satisfactory welds. Past experience indicates that 10 percent of all welds are improper.

(a) What is the probability that a weld that the machine indicates to be defective is in fact satisfactory?
(b) What is the probability that a weld which the machine indicates to be satisfactory is in fact defective?
(c) Compare the probabilities in (a) and (b) and comment on their relative magnitudes.

Problem 2

The service stations along a highway are located according to a Poisson process in space, with an average of 1 service station in 10 miles. Because of a gas shortage, there is a probability of 0.3 that a service station would have no gasoline available. Assume that the availabilities of gasoline at different service stations are statistically independent.

(a) What is the probability that there is *at most* 1 service station in the next 15 miles of highway?
(b) What is the probability that none of the next 3 stations has gasoline for sale?
(c) A driver on this highway notices that the fuel gauge in his car reads empty; from experience he knows that he can go another 15 miles. What is the probability that he will be stranded on the highway without gasoline?[1]

Problem 3

During a 2-month hurricane season, severe hurricanes at a given location occur at Poisson times with a rate $\lambda = 1$ event/month. Last year, 4 hurricanes occurred and the local press has blamed "changes in the climate" for what they reported as an extremely severe hurricane season. From a statistical point of view, how unusual are seasons of this or higher severity? Would you agree or disagree with the press that this was an exceptionally severe season?

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[1] An important result for Poisson processes is that, if a Poisson process with rate λ is "thinned" randomly with probability p (meaning that each point of the process is eliminated with probability p independently of the other points), then the remaining points still form a Poisson process with a reduced mean rate of $(1-p)\lambda$. Apply this result to answer question 1(c).