Homework Set #7

Problem 1

Consider a sequence of random variables X_1, X_2, …, X_i, …, for example denoting the monthly profits of a supermarket chain. Suppose that $X_i \sim (m,\sigma^2)$ for all i and that the correlation coefficient between X_i and X_j, ρ_{ij}, depends only on the time lag |i-j| as

$$\rho_{ij} = 0.8^{|i-j|}$$

Using conditional SM analysis, calculate and plot, as a function of $k \geq 1$, the variances of $(X_{i+k}|X_i)$ and $(X_{i+k}|X_i,X_{i-1})$. Comment on the results.

Problem 2

X is an unknown quantity, say the compressive strength of a concrete column, with mean value m and variance σ^2. Several indirect measurements of X, in the form $Z_i = X + \varepsilon_i$ for i = 1, …, n, are made through a nondestructive technique.
Under the assumption that the ε_i are iid measurement errors with zero mean and common variance σ_ε^2, use conditional SM analysis to find the variance of $(X|Z_1,\ldots,Z_n)$. Plot this conditional variance against n for $\sigma^2 = 1$ and σ_ε^2 either 1 or 0.2.

Useful result on the inverse of covariance matrices with a special "equicorrelated" structure. The inverse of an n × n matrix $\underline{A}$ of the type:

$$\underline{A} = \sigma^2 \begin{bmatrix} 1 & & \rho \\ & \ddots & \\ \rho & & 1 \end{bmatrix}$$

is:

$$\underline{A}^{-1} = \frac{1}{\sigma^2(1-\rho)[1+(n-1)\rho]} \begin{bmatrix} [1+(n-2)\rho] & & -\rho \\ & \ddots & \\ -\rho & & [1+(n-2)\rho] \end{bmatrix}$$