
Queueing Systems: Lecture 6

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Lecture Outline

- Congestion pricing in transportation: the fundamental ideas
- Congestion pricing and queueing theory
- Numerical examples
- A real example from LaGuardia Airport
- Practical complications

Reference: Handout on “Congestion Pricing and Queueing Theory” (on course website)

Congestion pricing: The basic observation

- The congestion costs due to any specific user have 2 components:
 - (1) Cost of delay to that user (internal cost)
 - (2) Cost of delay to all other users caused by that user (external cost)
- At congested facilities, this second component can be very large
- A congestion toll can be imposed to force users to experience this cost component (to “internalize the external costs”)

Economic principle

Optimal use of a transportation facility cannot be achieved unless each additional (marginal) user pays for all the additional costs that this user imposes on all other users and on the facility itself. A congestion toll not only contributes to maximizing social economic welfare, but is also necessary to reach such a result. (Vickrey, 1967, 1969; Carlin + Park, 1970)

Two hard technical problems

- In practice it is very hard to:
 - (1) Estimate external marginal delay costs (extensive data analysis and/or simulation have been typically needed – subtle issues);
 - (2) Determine equilibrium congestion tolls (trial-and-error approach that may take long time to converge)
- Queueing theory has much to offer (especially with regard to the first problem) under certain conditions.

Computing Internal and External Costs

Consider a queueing facility with a single type of users in steady-state. Let

c = delay cost per unit time per user

C = total cost of delay per unit time incurred in the system

Then: $C = cL_q = c\lambda W_q$

and the marginal delay cost, MC , imposed by an additional (“marginal”) user is given by:

$$MC = \frac{dC}{d\lambda} = c W_q + c\lambda \frac{dW_q}{d\lambda}$$

↑ ↑ ↑
Marginal Internal External
cost cost cost

Numerical Example

- Three types of aircraft; Poisson; FIFO service
 - _ Non-jets: $\lambda_1 = 40$ per hour; $c_1 = \$600$ per hour
 - _ Narrow-body jets: $\lambda_2 = 40$ per hour; $c_2 = \$1,800$ per hour
 - _ Wide-body jets: $\lambda_3 = 10$ per hour; $c_3 = \$4,200$ per hour
 - _ Total demand is: $\lambda = \lambda_1 + \lambda_2 + \lambda_3 = 90$ per hour
- pdf for service times is uniform
 - _ U[25 sec, 47 sec]
 - _ $E[S] = 36$ sec = 0.01 hour; $\mu = 100$ per hour
$$\sigma_S^2 = \frac{22^2}{12} = 40.33 \text{ sec}^2 = 3.11213 \times 10^{-6} \text{ hours}^2$$
- Note: We have a M/G/1 system

Numerical Example [2]

$$W_q = \frac{\lambda \cdot [E^2[S] + \sigma_S^2]}{2 \cdot (1 - \rho)} = \frac{90 \cdot [(0.01)^2 + 3.11213 \times 10^{-6}]}{2 \cdot (1 - 90/100)} \approx 0.0464 \text{ hours} \approx 167 \text{ sec}$$

Define: $c = c_1 \frac{\lambda_1}{\lambda} + c_2 \frac{\lambda_2}{\lambda} + c_3 \frac{\lambda_3}{\lambda}$

$$C = c \cdot L_q = c \cdot \lambda \cdot W_q = (c_1 \cdot \lambda_1 + c_2 \cdot \lambda_2 + c_3 \cdot \lambda_3) \cdot W_q = \bar{c} \cdot W_q$$

Or: $C = \bar{c} \cdot W_q = (\$138,000) \cdot (0.0464) = \$6,400$

$$\frac{dW_q}{d\lambda} = \frac{E^2[S] + \sigma_S^2}{2 \cdot (1 - \rho)} + \frac{\lambda \cdot [E^2[S] + \sigma_S^2]}{2 \cdot (1 - \rho)^2} \cdot \frac{1}{\mu} \approx 5.1556 \times 10^{-6} \text{ hours} \approx 18.6 \text{ sec}$$

Numerical Example [3]

$$\frac{dC}{d\lambda_1} = c_1 \cdot W_q + \bar{c} \cdot \frac{dW_q}{d\lambda} \approx \$28 + \$711 = \$739$$

↑
↑
internal
external cost=
cost
congestion toll

$$\frac{dC}{d\lambda_2} = c_2 \cdot W_q + \bar{c} \cdot \frac{dW_q}{d\lambda} \approx \$85 + \$711 = \$796$$

$$\frac{dC}{d\lambda_3} = c_3 \cdot W_q + \bar{c} \cdot \frac{dW_q}{d\lambda} \approx \$198 + \$711 = \$909$$

Generalizing to m types of users...

- Facility users of type i : arrival rate λ_i ;
 service time S_i with $\mu_i^{-1} = E[S_i]$;
 cost per unit of time in the system c_i

- For entire set of facility users, we have

$$\lambda = \sum_{i=1}^m \lambda_i \quad \frac{1}{\mu} = E[S] = \sum_{i=1}^m \left(\frac{\lambda_i}{\lambda} \times \frac{1}{\mu_i} \right)$$

$$\rho = \frac{\lambda}{\mu} = \sum_{i=1}^m \rho_i = \sum_{i=1}^m \frac{\lambda_i}{\mu_i} \quad c = \sum_{i=1}^m \left(\frac{\lambda_i}{\lambda} c_i \right)$$

Generalization (continued)

• As before: $C = cL_q = c\lambda W_q$

giving: $MC(i) = \frac{dC}{d\lambda_i} = c_i W_q + c\lambda \frac{dW_q}{d\lambda_i}$

• When we have explicit expressions for W_q , we can also compute explicitly the total marginal delay cost $MC(i)$, the internal (or private) cost and the external cost associated with each additional user of type i

Example

For an M/G/1 system:

$$MC(i) = \frac{dC}{d\lambda_i} = c_i \frac{\lambda \cdot E[S^2]}{2(1-\rho)} + c\lambda \frac{(1-\rho)E[S_i^2] + \frac{\lambda}{\mu_i} E[S^2]}{2(1-\rho)^2}$$

• Can extend further to cases with user priorities

Finding Equilibrium Conditions and Optimal Congestion Tolls!

We now generalize further: let x_i be the *total* cost perceived by a user of type i for access to a congested facility and let $\lambda_i(x_i)$ be the demand function for type i users.

$$x_i = IC_i + CT_i + K_i$$

IC_i = internal private cost; it is a function of the demand rates, $\lambda_i(x_i)$

CT_i = congestion toll imposed; equal to 0 in absence of congestion tolls; can be set arbitrarily or can be set as a function of the $\lambda_i(x_i)$ under congestion pricing schemes

K_i = any other charges that are independent of level of congestion

Finding Equilibrium Conditions and Optimal Congestion Tolls! [2]

- With m types of users, the equilibrium conditions for any set of demand functions, can be found by solving simultaneously the m equations:

$$x_i = c_i \cdot W_q[\hat{\lambda}(\hat{x})] + \left(\sum_{j=1}^m c_j \cdot \lambda_j(x_j) \right) \cdot \frac{dW_q[\hat{\lambda}(\hat{x})]}{d\lambda_i(x_i)} + K_i \quad \forall i$$

where $\hat{\lambda}(\hat{x}) = \{\lambda_1(x_1), \lambda_2(x_2), \dots, \lambda_m(x_m)\}$.

The missing piece: Demand functions can only be roughly estimated, at best!

An illustrative example from airports

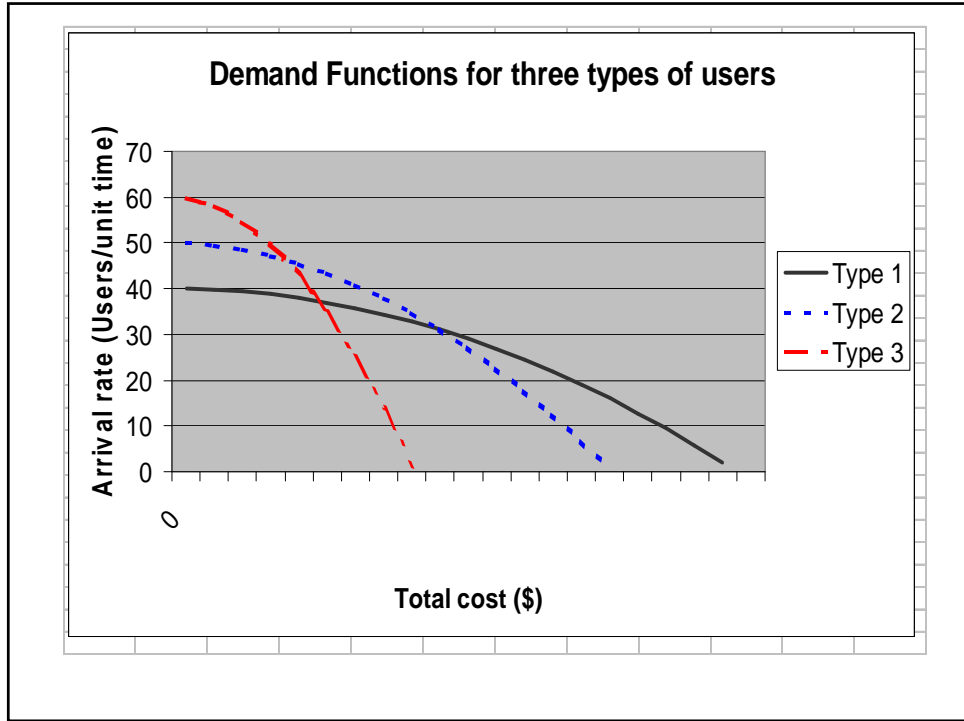
	Type 1 (Big)	Type 2 (Medium)	Type 3 (Small)
Service rate (movements per hour)	80	90	100
Standard deviation of service time (seconds)	10	10	10
Cost of delay time (\$ per hour)	\$2,500	\$1,000	\$400

Hypothetical Demand Functions

$$\lambda_1(x_1) = 40 - 0.001 \cdot x_1 - 0.00001 \cdot x_1^2$$

$$\lambda_2(x_2) = 50 - 0.003 \cdot x_2 - 0.00002 \cdot x_2^2$$

$$\lambda_3(x_3) = 60 - 0.01 \cdot x_3 - 0.00008 \cdot x_3^2$$

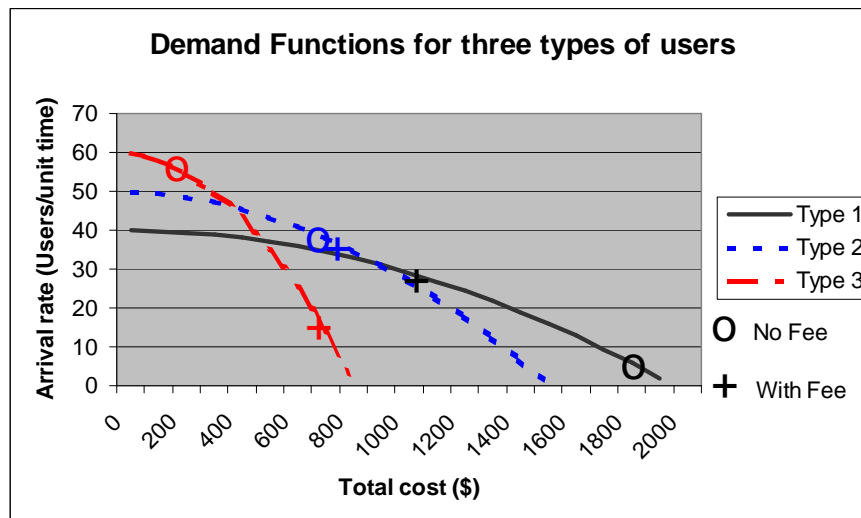


Case 1: No Congestion Fee

	Type 1	Type 2	Type 3
<i>No Congestion Fee</i>			
(1) Delay cost (IC) per aircraft	\$1,802	\$721	\$288
(2) Congestion fee	\$0	\$0	\$0
(3) Total cost of access [=(1)+(2)]	\$1802	\$721	\$288
(4) Demand (no. of movements per hour)	5.7	37.4	50.5
(5) Total demand (no. of movements per hour)	93.6		
(6) Expected delay per aircraft	43 minutes 15 seconds		
(7) Utilization of the airport (% of time busy)	99.2%		

Case 2: Optimal Congestion Fee

<i>Optimal Congestion Fee</i>			
(8) Delay cost (IC) per aircraft	\$135	\$54	\$22
(9) Congestion fee (CF)	\$853	\$750	\$670
(10) Total cost of access [=(1)+(2)]	\$988	\$804	\$692
(11) Demand (no. of movements per hour)	29.2	34.6	14.9
(12) Total demand (no. of movements per hour)	78.7		
(13) Expected delay per aircraft	3 minutes 15 seconds		
(14) Utilization of the airport (% of time busy)	89.9%		



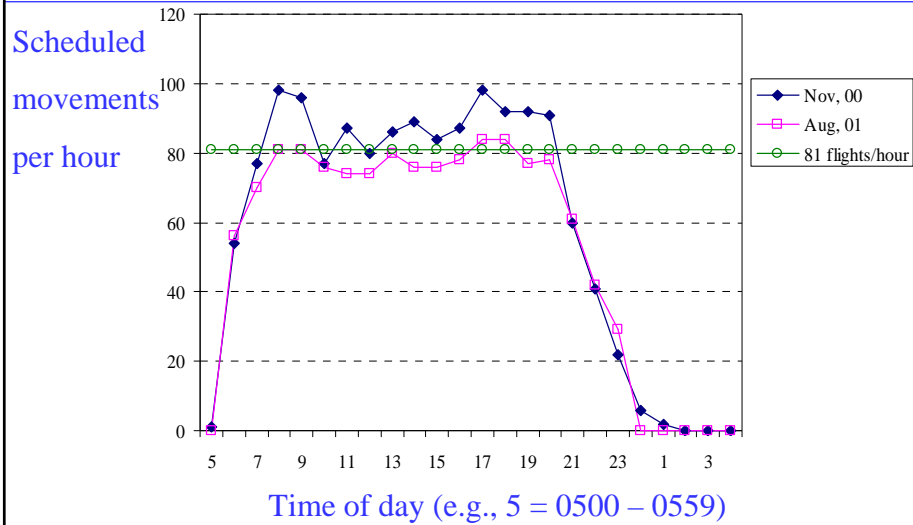
Important to note...

- The external costs computed in the absence of congestion pricing give only an upper bound on the magnitude of the congestion-based fees that might be charged
- These are *not* necessarily “equilibrium prices”
- Equilibrium prices may turn out to be considerably lower than these upper bounds
- Equilibrium prices are hard to estimate, absent knowledge of demand functions

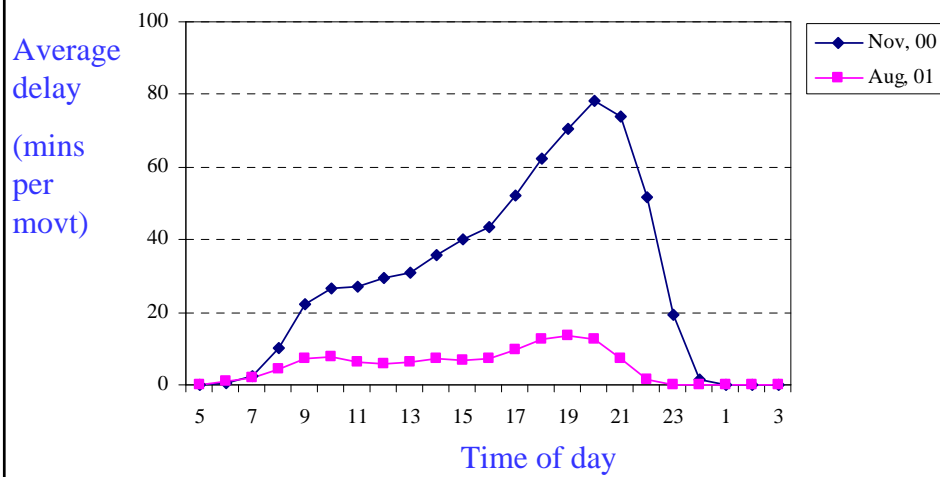
Case of LaGuardia (LGA)

- **Since 1969: Slot-based High Density Rule (HDR)**
 - _ DCA, JFK, LGA, ORD; “buy-and-sell” since 1985
- **Early 2000: About 1050 operations per weekday at LGA**
- **April 2000: Air-21 (Wendell-Ford Aviation Act for 21st Century)**
 - _ Immediate exemption from HDR for aircraft seating 70 or fewer pax on service between small communities and LGA
- **By November 2000 airlines had added over 300 movements per day; more planned: virtual gridlock at LGA**
- **December 2000: FAA and PANYNJ implemented slot lottery and announced intent to develop longer-term policy for access to LGA**
- **Lottery reduced LGA movements by about 10%; dramatic reduction in LGA delays**
- **June 2001: Notice for Public Comment posted with regards to longer-term policy that would use “market-based” mechanisms**
- **Process stopped after September 11, 2001; re-opened in 2004**

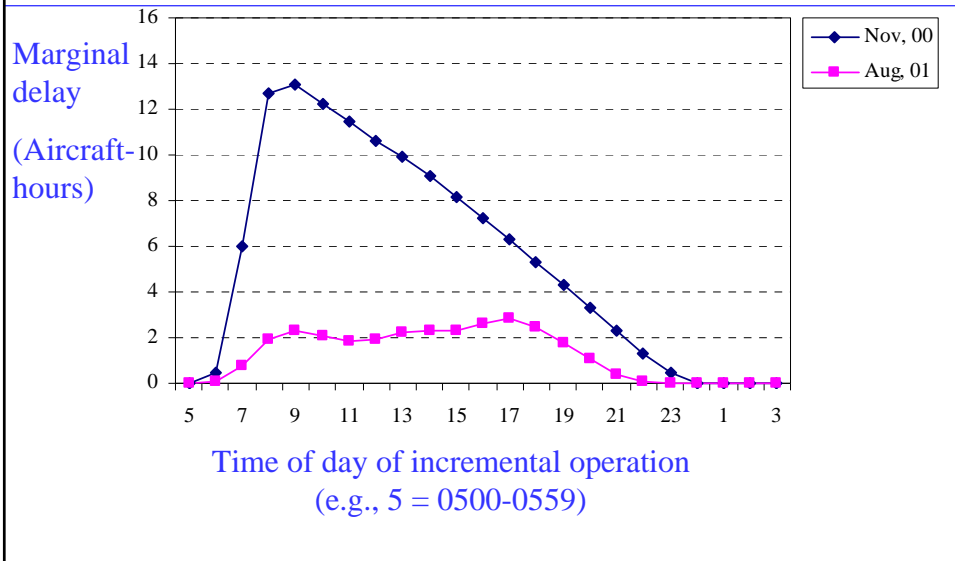
Scheduled aircraft movements at LGA before and after slot lottery



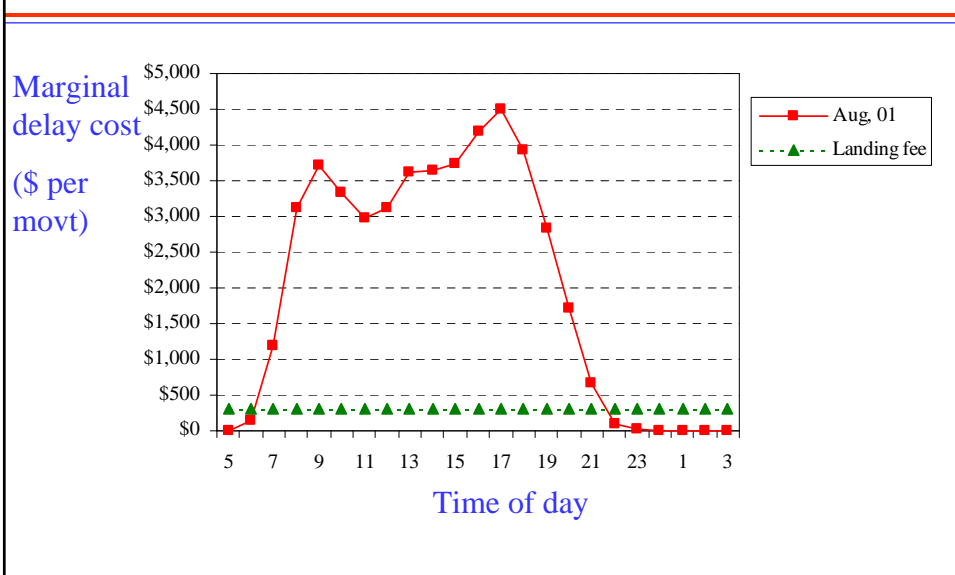
Estimated average delay at LGA before and after slot lottery in 2001



LGA: Marginal delay caused by an additional operation by time of day



LGA: Marginal external delay cost per additional movement vs. average landing fee per movement



Issues that arise in practice

- Toll may vary in time and by location**
- Facility users may be driven by “network” considerations**
- “Social benefit” considerations**
- Political issues**
- What to do with the money?**