Page No

1 INTRODUCTION

Objectives of "Generalized" Model

1) Mathematical model (set of constitutive eqn) to represent stress-strain-strength behavior in reasonable fashion for various stress paths & stress systems (b & δ) for BOTH drained & undrained conditions

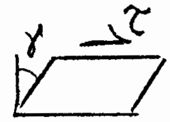
2) Reasonable number input parameters that have physical significance & can be measured!

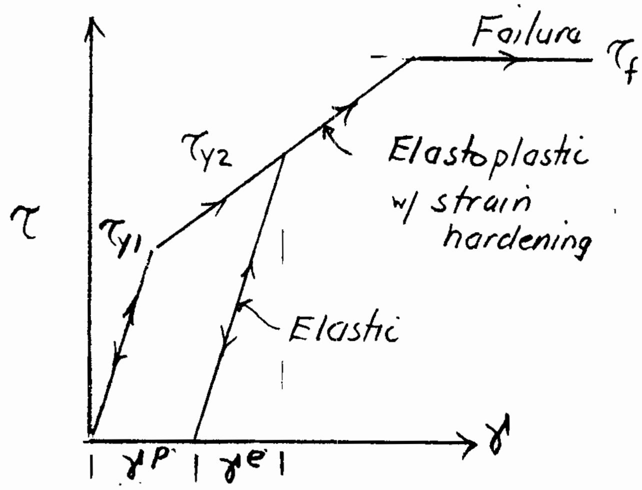
(Simplest "model" = linear, elastic, isotropic → 2 parameters)

2. PLASTICITY THEORY (Elasto-Plastic Model)

2.1 Yielding vs Failure

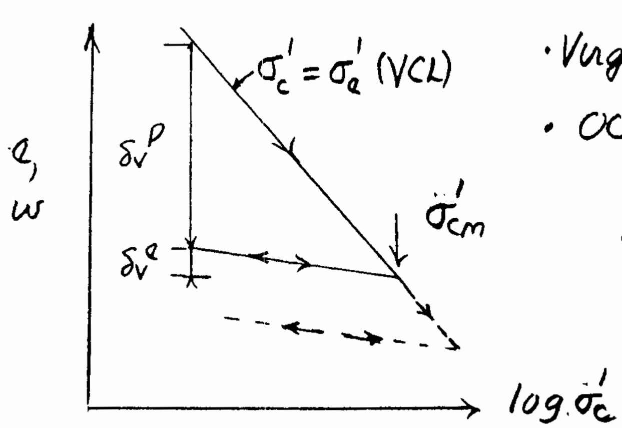
a) Simple shear





- Yielding = transition from elastic (all recoverable) to plastic (irrecoverable) strains
- Plastic strains → increase in τ_y via "hardening law"
- Failure - i.e. Mohr-Coulomb (continued deformation at constant stress)

b) Isotropic consolidation test



- Virgin → plastic (δ_v^p) + some elastic strains (δ_v^e)
- OC → only elastic, δ_v^e
- Simple "isotropic" hardening law with $\sigma_y = \bar{\sigma}'_{cm}$

NOTE: Continuous yielding on VCL = Virgin Compr. Line

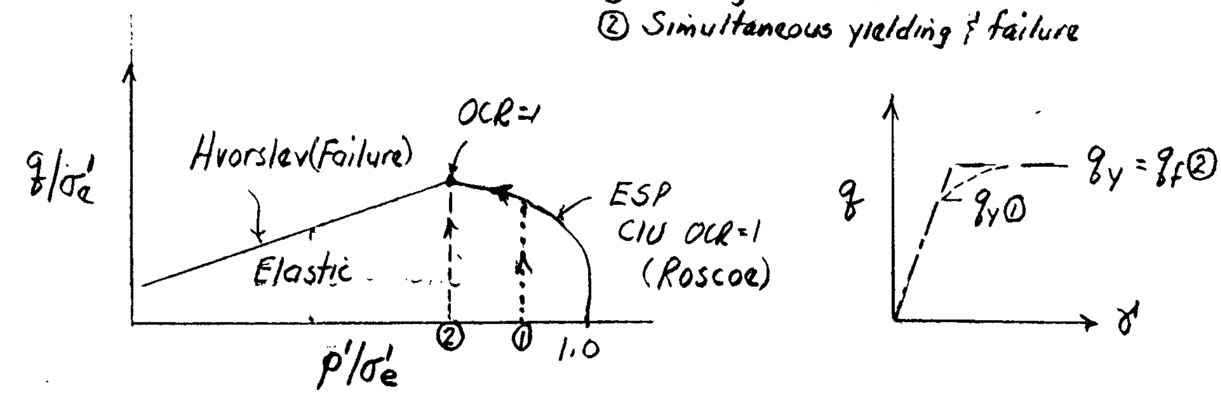
2.2 State Variables (SV)

Define "state" of soil . Simple Clay → $w - q - p'$ ($w = f(\sigma'_e)$)
MCC → $e - \bar{q} - \bar{p}$

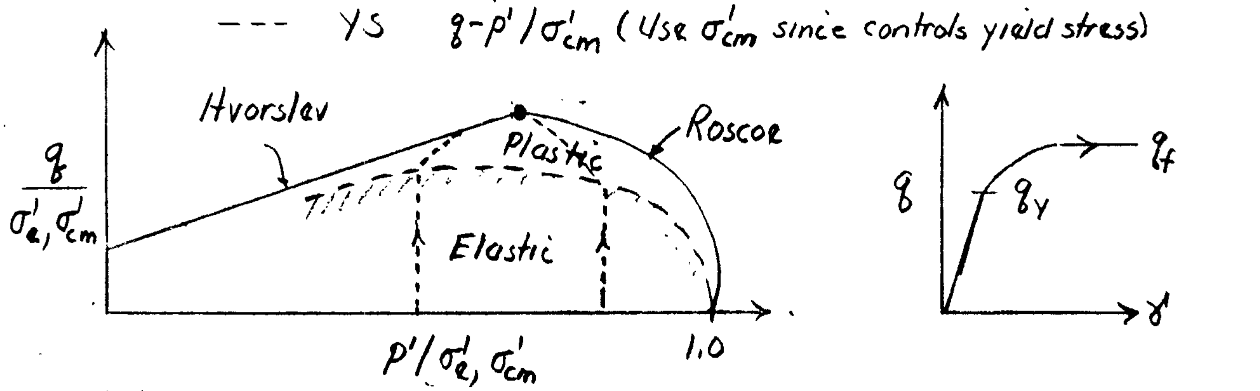
2.3 State Boundary Surface & Yield Surface (Locus-Envelope)

- Consider CIU PSC with elastic $A_e = 0.5$ ($b = 0.5$)

(a) Simplified Behavior SBS = YS → Elastic behavior inside SBS = YS
① Yielding before failure
② Simultaneous yielding & failure



(b) More Realistic Behavior : Yielding before hit SBS
—— SBS $q - p'/\sigma'_e$
- - - YS $q - p'/\sigma'_{cm}$ (Use σ'_{cm} since controls yield stress)



Plastic Potential = surface that is ⊥ to plastic strain increments (Sheet A)

2.4 Flow Rule (Set of eqn. = Plastic Potential) δ_v^p & $\delta\gamma^p$

- Gives direction & magnitude of plastic strain increments when SV hits YS, i.e. during yielding
- "Associated": same eqn. plastic potential & yield surface

2.5 Hardening Law

- Governs changes in shape & location of YS resulting from plastic deformations

3. MODIFIED CAM-CLAY (MCC)

3.1 Background

- First generalized model; developed at Cambridge Univ (Roscoe*, Schofield, Wroth, Poorooshasb, Burland*) in 1960's ±
- Intended to model results from CIUC & CIDC tests on remolded clay, especially at low OCR. But can predict Plane strain, CK_0UC/D, etc. ISOTROPIC

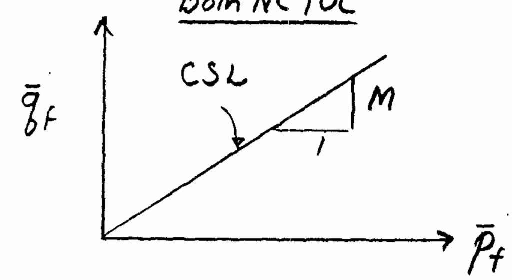
3.2 State Variables

For TX test**

$e \qquad \bar{q}' = (\sigma_1 - \sigma_3) \qquad \bar{p}' = \sigma'_{oct} = \frac{1}{3}(\sigma'_1 + \sigma'_2 + \sigma'_3)$

3.3 Failure Law (Surface = Envelope)

Both NC & OC



Extended von Mises

$\bar{q}_f = \bar{p}_f M$

at Critical State Line (CSL) → unique $e_f - \bar{q}_f - \bar{p}_f$

Sheet B

Not realistic

$$\begin{cases} b = 0\ (TC) & M = 6\sin\phi'/(3-\sin\phi') \quad \text{or} \quad \sin\phi' = 3M/(6+M) \\ b = 0.5 & M = \sqrt{3}\sin\phi' \\ b = 1.0\ (TE) & M = 6\sin\phi'/(3+\sin\phi') \quad \text{or} \quad \sin\phi' = 3M/(6-M) \end{cases}$$

→ For $M = 1.2$ $\phi' = 30°$ TC increasing to $\phi' = 48.6°$ TE
$M > 1.5$ > 36.9 → $\phi'_{TE} > 90°$

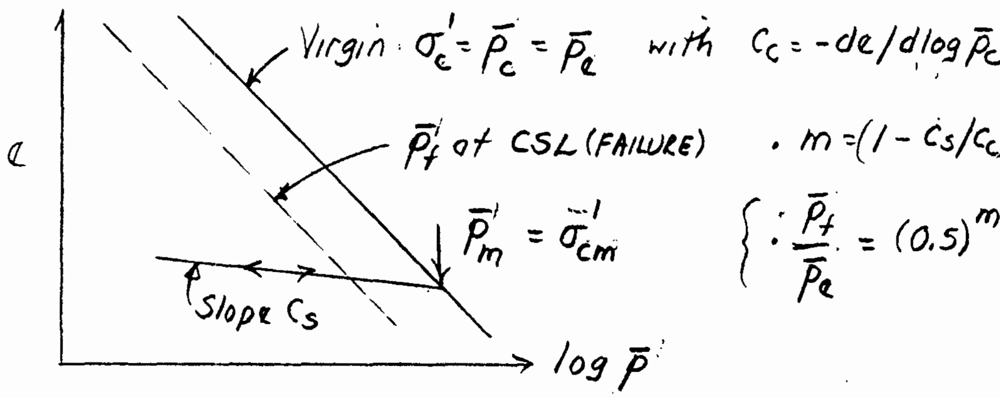
- Note: As applied in practice, usually use ϕ' triaxial compression to compute M

** For $0 < b < 1.0$ $\bar{q} \to q^* = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2 + (\sigma_1-\sigma_3)^2 + (\sigma_2-\sigma_3)^2}$
Extended von Mises: $q^*/\sigma'_{oct} = M$ (Bishop, 1971) } Sheet B

* Roscoe, K.H & Burland, J.B. (1968), "On the Generalized Stress-Strain Behaviour of 'Wet' Clay", in Engineering Plasticity, Cambridge Univ. Press, pp. 535-609. (MCC Input Parameters = ϕ'_{TC} + isotropic e vs. log $\bar{p}$ + ν')

3.4 Compressibility & Hardening Law - $K_c = 1.0$

{MCC actually uses e vs. $\ln \bar{p}$ → $\lambda = -de/d\ln \bar{p}_c = C_c/2.3$ & $\kappa = C_s/2.3$}



• $m = (1 - C_s/C_c)$

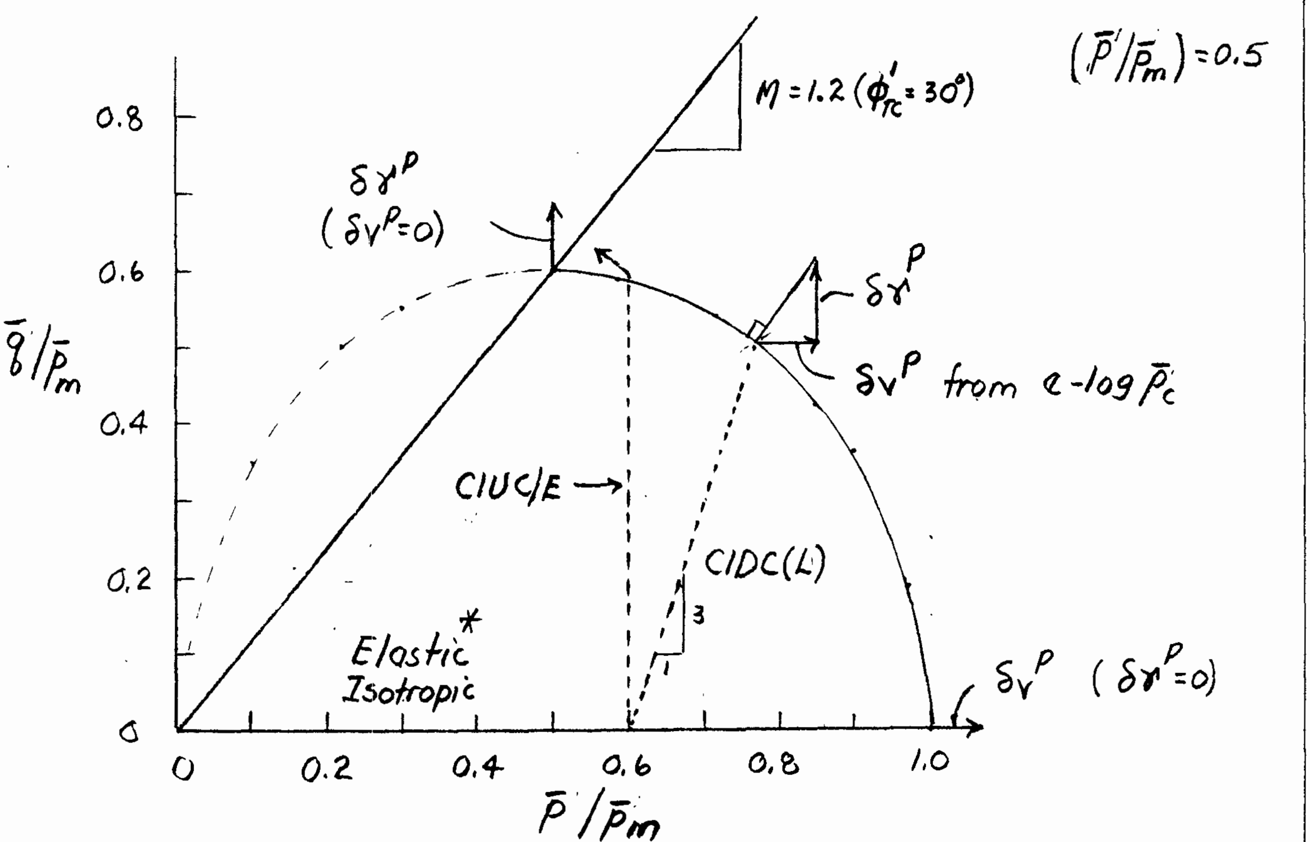
$\left\{ \therefore \dfrac{\bar{p}_f}{\bar{p}_e} = (0.5)^m \right\}$ later

• Isotropic $\underbrace{\text{hardening law}}_{\text{size of YS}}$ with end of YS at $\bar{p}'_m$

3.5 Yield Surface & Associated Flow Rule

• Eqn for Yield surface $\left\{ \dfrac{\bar{p}}{\bar{p}_m} = \left(\dfrac{M^2}{M^2 + R^2} \right) \right\}$ $R = \bar{q}/\bar{p}$
is ellipse & also
equals "plastic potential" →
associated flow rule (equals "normality")

At $R = M$ (failure)
$(\bar{p}'/\bar{p}_m) = 0.5$



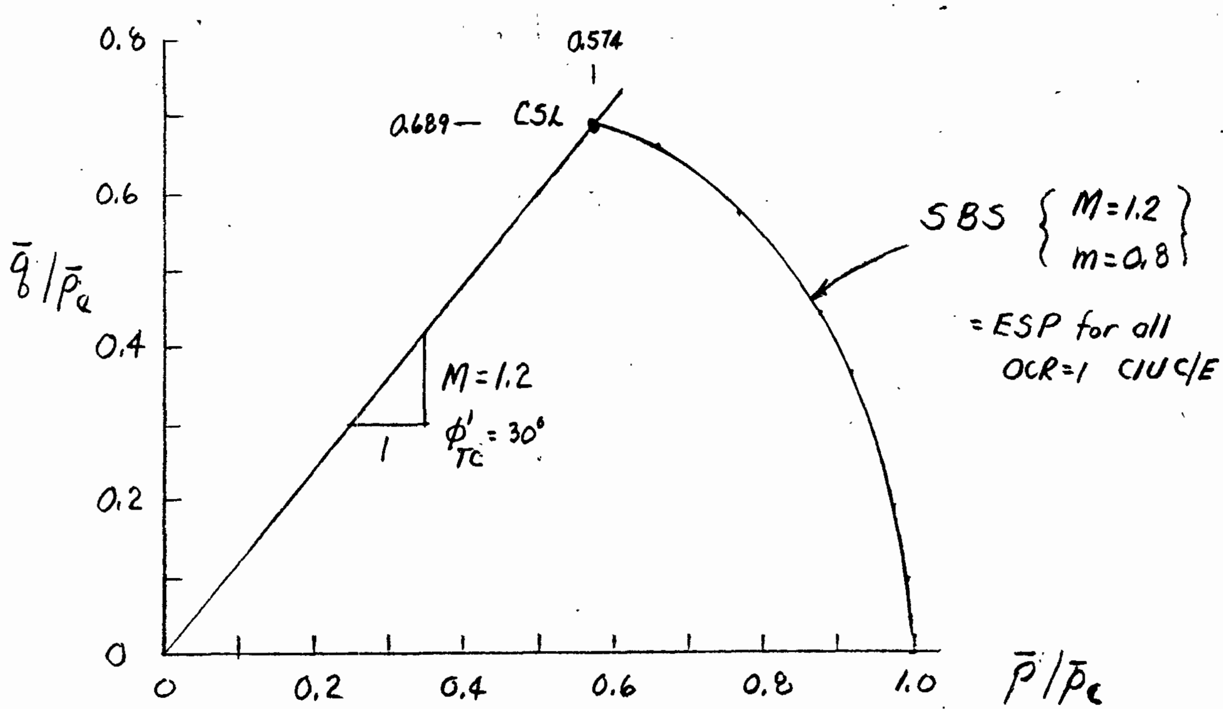
* Need input E' or ν' or G etc. (Already have $K' = \delta\bar{p}'/\delta v^e$ from C_s line)
↑ usually assumed

3.6 SBS = ESP for CIUC/E OCR=1 (Triaxial Tests)

(Follows from yield surface + flowrule + hardening law & failure law)

- Eqn for SBS $\dfrac{\bar{p}}{\bar{p}_e} = \left(\dfrac{M^2}{M^2+R^2}\right)^m$ $\quad m = 1 - C_s/C_c$, $\quad R = \bar{q}/\bar{p}$

- For $R = M$, i.e. at failure, $\bar{p}_f/\bar{p}_e = (0.5)^m \rightarrow$ CSL in 3.4



- SBS = ESP for OCR=1 CIUC/E + governs Δe for CIDC/E

3.7 Predicted CIU q_f/σ'_c for OCR=1

- $q_f = \frac{1}{2}\bar{q}_f$; $\bar{q}_f = \bar{p}_f M$; $\bar{p}_f = \bar{p}_e (0.5)^m = \sigma'_c (0.5)^m$

$\therefore q_f/\sigma'_c = \dfrac{M}{2}(0.5)^m$ for both CIUC & CIUE

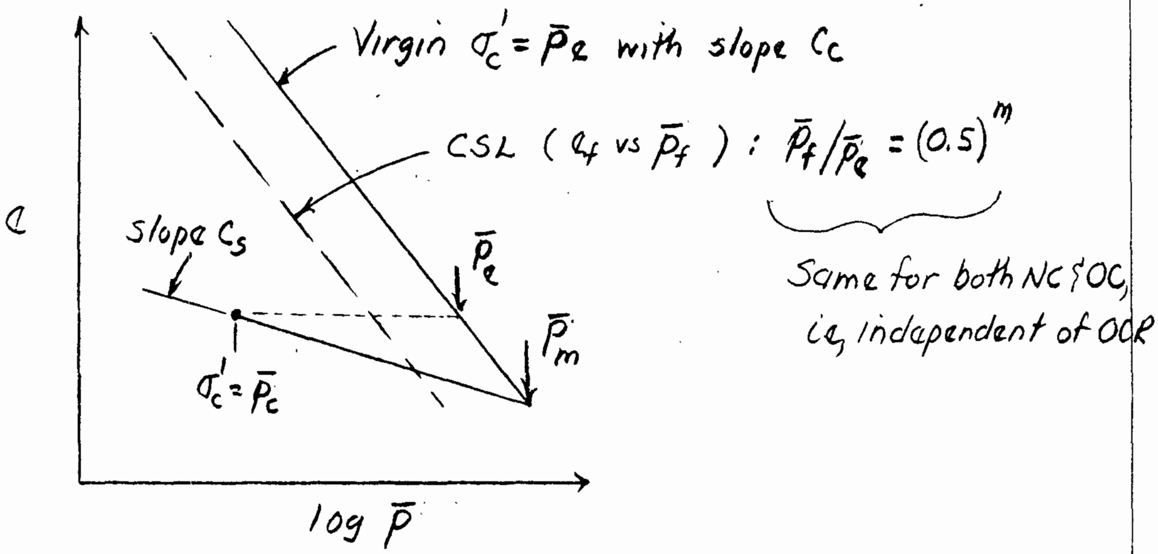
Examples for m=0.8

$\phi'(TC)$	M	q_f/σ'_c
25	0.984	0.283
30	1.200	0.345
35	1.418	0.407

See Sheet A for MIT ESP (q vs $p' \div \sigma'_c$)

3.8 Predicted CIU q_f/σ_c' vs OCR

(a) Graphical model



(b) Derivation of eqn.

(1) From 3.7 $\quad q_f/\bar{p}_e = \frac{M}{2}(0.5)^m$

(2) $\underset{\text{(swelling)}}{C_s \log \frac{\bar{p}_m}{\sigma_c'}} = \underset{\text{(virgin)}}{C_c \log \frac{\bar{p}_m}{\bar{p}_e}} \rightarrow \frac{C_s}{C_c} \log \underbrace{\frac{\bar{p}_m}{\sigma_c'}}_{= OCR} = \log \frac{\bar{p}_m}{\bar{p}_e}$

$$\log \frac{\bar{p}_m}{\bar{p}_e} = \log \frac{\bar{p}_m}{\sigma_c'} - \log \frac{\bar{p}_e}{\sigma_c'} \rightarrow \log OCR \underbrace{\left(1 - \frac{C_s}{C_c}\right)}_{= m} = \log \frac{\bar{p}_e}{\sigma_c'}$$

$$\therefore \frac{\bar{p}_e}{\sigma_c'} = (OCR)^m$$

(3) Multiply (1) by (2) $\rightarrow \frac{q_f}{\bar{p}_e} \times \frac{\bar{p}_e}{\sigma_c'} = \frac{M}{2}(0.5)^m (OCR)^m$

$$\rightarrow \boxed{q_f/\sigma_c' = \underbrace{\frac{M}{2}(0.5)^m}_{\text{Value at OCR=1 (= S in SHANSEP Eqn)}} (OCR)^m}$$

NOTES: (1) Similar form to 1.361, $\frac{s_u}{\sigma_c'} = S\,(OCR)^m$

(2) Tends to overpredict effect of OCR

(3) $s_u(TE) = s_u(TC)$ not realistic

3.9 Predicted ESP for CIU Tests at OCR ≥ 1

(Use Cambridge $\bar{q}$ vs $\bar{p}$ → TC = TE)

(1) At OCR = 2.0: Vertical ESP → simultaneous yielding and failure

(2) At OCR < 2.0

See Fig. 1 (Sheet C) for example with $\sigma_c' = 4$, $\sigma_{cm}' = 6$; OCR = 1.5

- Get vertical ESP until intersect YS for $\bar{p}_m = 6$
- Then follows ESP on end of SBS for $\bar{p}_e$ having same e_f $[\bar{p}_e = \bar{p}_c (OCR)^m]$ = NC CIU ESP with $\sigma_c' = \bar{p}_e$. Continued shearing → increasing $\bar{p}_m$

(3) At OCR > 2.0

See Fig. 2 (Sheet D) for ESP normalized to $\bar{p}_e$ at OCR = 1.0, 1.5, 2 ⋯ 10

(i.e., treat as results for tests with same e_f, but varying $\bar{p}_m$)

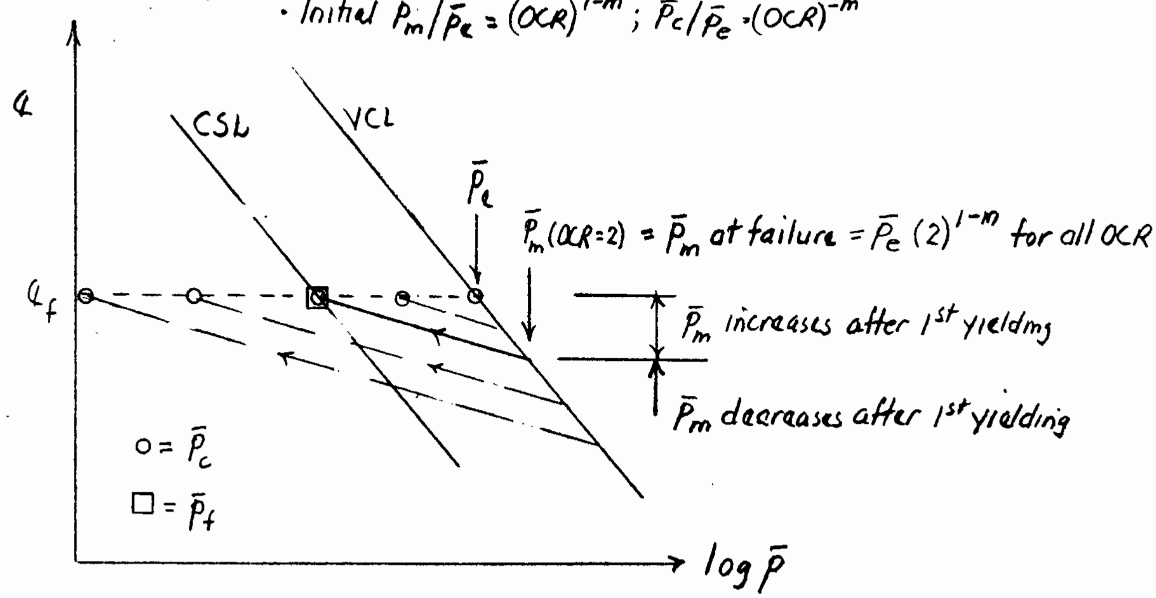
Eqn hold at all OCRs:

- At Consolidation: $\bar{p}_c/\bar{p}_e = 1/(OCR)^m = (OCR)^{-m}$
- At 1st yield: $\bar{p}_Y = \bar{p}_c$ since vertical ESP ; $\bar{q}_Y/\bar{p}_e = \dfrac{M\sqrt{OCR-1}}{(OCR)^m}$
- Thereafter follows SBS → $\dfrac{\bar{p}}{\bar{p}_e} = \left(\dfrac{M^2}{M^2+R^2}\right)^m$, where $R = \bar{q}/\bar{p}$
- At failure (CSL), $\bar{p}_f/\bar{p}_e = (0.5)^m \rightarrow \bar{p}_f/\underbrace{\bar{p}_{1st\,yield}}_{=\bar{p}_c} = (\tfrac{1}{2} OCR)^m$

continued shearing → decreasing $\bar{p}_m$

(4) Summary

- Initial $\bar{p}_m/\bar{p}_e = (OCR)^{1-m}$; $\bar{p}_c/\bar{p}_e = (OCR)^{-m}$



3.10 Summary of Input Parameters for MCC

(1) Compressibility ($K_c = 1$)

- NC e vs $\log \sigma'_c = \log \bar{p}_e \rightarrow C_c = -de/d\log\sigma'_c$ ($\lambda = -de/d\ln \bar{p}_e$)
- Swelling = recompression line $\rightarrow C_s$ ($\kappa = C_s/2.3$) & $m = (1 - C_s/C_c)$

(2) Failure envelope for NC clay, e.g. ϕ'_{TC}

- $M = \bar{q}_f/\bar{p}_f = f(\phi') = (6\sin\phi')/(3-\sin\phi')$ for $\phi' = \phi'_{TC}$

(3) One elastic parameter to combine with (1) to give elastic stress-strain behavior before yielding

$$\left\{ K' = \frac{\delta\bar{p}}{\delta v} = \frac{E'}{3(1-2\nu')} = \frac{2G(1+\nu')}{3(1-2\nu')} \right\}$$

usually selected

3.11 Summary of Relationships for CIUC/E Tests

(1) At Consolidation: $\bar{p}_c/\bar{p}_m = 1/OCR$; $\bar{p}_m/\bar{p}_e = (OCR)^{1-m}$; $\bar{p}_c/\bar{p}_e = (OCR)^{-m}$

(2) At 1st (initial) Yielding $[\bar{p}_y = \bar{p}_c = \sigma'_c = \bar{p}_m/OCR = \bar{p}_e(OCR)^{-m}]$

$$\therefore \frac{\bar{q}_y}{\bar{p}_c} = M\sqrt{OCR-1} \; ; \; \frac{\bar{q}_y}{\bar{p}_m} = \frac{M\sqrt{OCR-1}}{OCR} \; ; \; \frac{\bar{q}_y}{\bar{p}_e} = \frac{M\sqrt{OCR-1}}{(OCR)^m}$$

(3) At Failure $[\bar{q}_f = \bar{p}_f M \; ; \; \bar{p}_f/\bar{p}_e = (0.5)^m]$

- $$\bar{q}_f/\sigma'_c = \underbrace{\frac{M}{2}(0.5)^m}_{S}(OCR)^m$$
- $$\frac{\bar{p}_f}{\sigma'_c} = \left(\frac{OCR}{2}\right)^m$$ $\rightarrow$ OCR < 2 : decreasing $\bar{p}$ & increasing $\bar{p}_m$ of YS; $\rightarrow$ OCR > 2 : increasing $\bar{p}$ & decreasing $\bar{p}_m$ of YS
- $$\frac{\bar{p}_f}{\bar{p}_m} = (0.5)^m (OCR)^{m-1}$$

4. COMPARISON OF MCC WITH SIMPLE CLAY (CIUC)

4.1 s_u/σ_c' vs. OCR

(1) Selected parameters

- $\sin\phi'_{TC} = 0.3916 \rightarrow M = 0.9008 \simeq 0.90$
- $m = (1 - C_s/C_c) \rightarrow 0.65 \simeq$ ave. value at OCR = 8

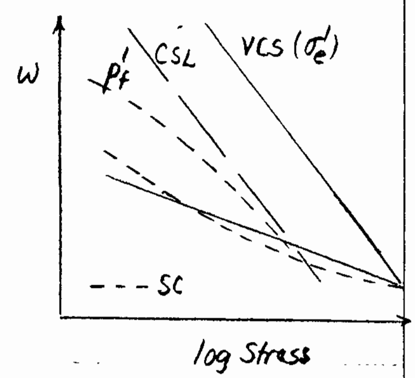
OCR	$(1 - C_s/C_c)$
2	0.73[5]
4	0.68
8	0.65
16	0.63

(2) Resultant ratios

OCR	q_f/σ_c' MCC	q_f/σ_c' SC	$\frac{MCC}{SC}$
1	0.287	0.290	0.99 } very close
4	0.706	0.696	1.01[5] } very close
12	1.442	1.166	1.24

Note: $m = 0.7 \rightarrow q_f/\sigma_c' = 1.58$ at OCR = 12 (+35%)

(3) Discussion → why MCC overpredicts s_u at high OCR



4.2 Stress-Strain and ESP at OCR ≥ 1

(1) See Fig 3 (Sheet E) for comparison. Stress-strain curves required computer program done by M. Kavvadas who developed MIT-E1 (for anisotropic NC clay)

(2) At OCR = 1; MCC → lower ESP, but stiffer response
" " = 2, MCC → linear ESP, same s_u and much stiffer, linear q vs ε_a

(3) At OCR = 8, MCC → ESP that goes far above Hvorslev envelope (with $A = A_e = 1/3$) before yielding; p' then increases significantly to reach "failure" at CSL

(4) Questions:
- Are CSL and Hvorslev concepts incompatible?
- Can difference be attributed to experimental errors?

5. DISCUSSION

5.1 Comments on MCC

(1) Represented very significant advance (in late 1960s) that wasn't appreciated by geotechnical engr. profession for > 10-15 yr.

(2) Highly innovative and elegant for its simplicity (but need computer program to obtain stress-strain curves). Formed the basis for Critical State Soil Mechanics (CSSM) that is still widely taught at some major universities (esp. in England) and assumed in practice (e.g., unique CSL).

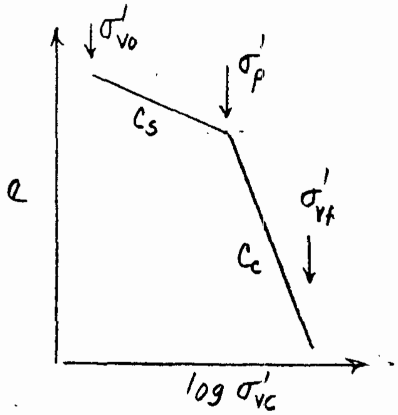
e.g., Wood, D.M. (1990). Soil Behavior and Critical State Soil Mechanics, Cambridge Univ. Press, 462 p

(3) Most popular & widely used clay model in finite element computer codes (e.g., ABAQUS,)
but using ϕ' = constant rather than M = constant

(4) However, has severe limitations when applied to natural OC clays
- Elastic behavior at OCR > 1
- Too high s_u at OCR > ≈ 2
- No s_u anisotropy

5.2 MCC 1-D Finite Element Consolidation Analyses

(1) Objective is to have correct e vs $\log \sigma'_{vc}$ for K_0 loading for each layer.



(2) How MCC operates: $\nu' = 0.3,\ M = 1.2,\ m = 0.8$

- $K_0(OC) = \dfrac{\nu'}{1-\nu'}$ $\rightarrow K_0 = 0.43$
- $K_0(NC) = f(\nu', M \,\&\, m)$ $\rightarrow K_0 = 0.65$

$\therefore$ OC clay will yield at $\sigma'_{vy} < \sigma'_p$

(3) Therefore need to select ν', M & m → same K_0 for OC & NC clay.
Will cover under Part C

5.3 MIT-E3 Model of Clay Behavior

(1) Formulation described by Whittle & Kavvadas (1994), ASCE JGE, 120(1), 173-198

(2) Three major components

1. Elasto-plastic model for NC clay that incorporates anisotropy and strain-softening
   - See Fig. 2, Sheet F1, for rotated yield surface = bounding surface
   - " Fig. 4, " , for NC CK_0UPSC/E

2. Eqn. to describe small strain nonlinearity and hysteretic response in unloading/reloading
   - See Fig 1a, Sheet F1, for hysteresis in e vs $\log \sigma_c'$
   - " Fig. 4, " , for nonlinear small strain behavior

3. Bounding surface plasticity for irrecoverable, anisotropic and path-dependent behavior of OC clays
   - See Fig 1b, Sheet F1, for plastic strain (Δ^P) for reloading to VCL
   - " Fig. 2, " " , bounding surface plasticity
   - " Fig. 4, " " , for anisotropy of OC clay

(3) Input parameters : Table 2, Sheet F2 → 15 parameters

a) 1-D consolidation data with measurement of K_0 → 7 parameters

b) CK_0UC at OCR = 1 & 2, CK_0UE at OCR = 1 → 6 parameters

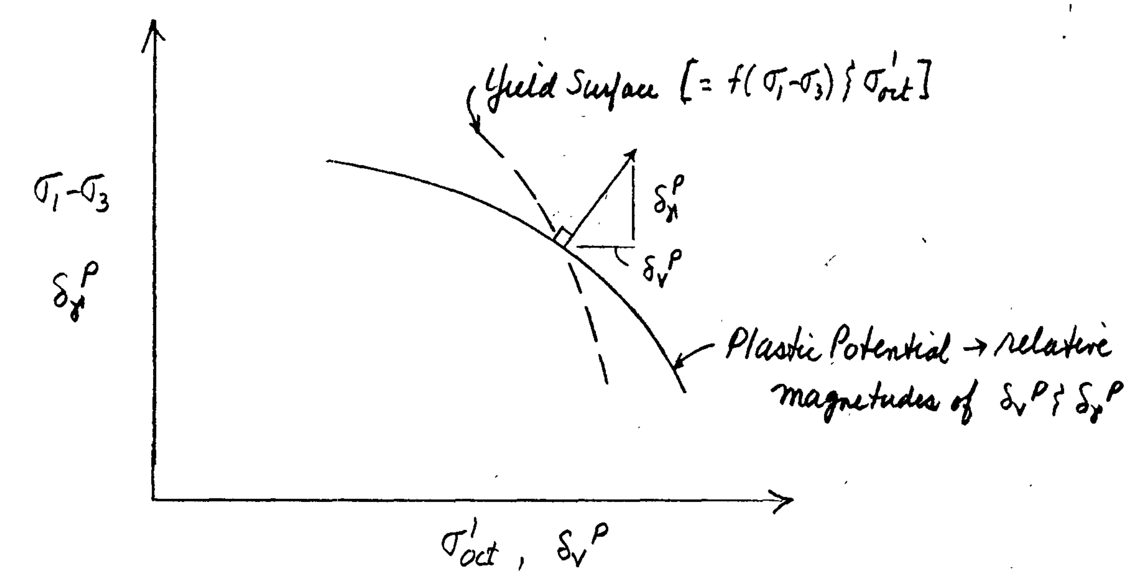
c) Resonant column or in situ shear wave velocity → 1 parameter (G_{max})

d) Special test to measure "evolving" anisotropy → 1 parameter (ψ_0) (rotation of bounding surface)

(4) See Tables 1 & 2 for values of parameters for three clays

Plastic Potential

Note: Associated flow rule = normality if Plastic Potential = Yield Surface

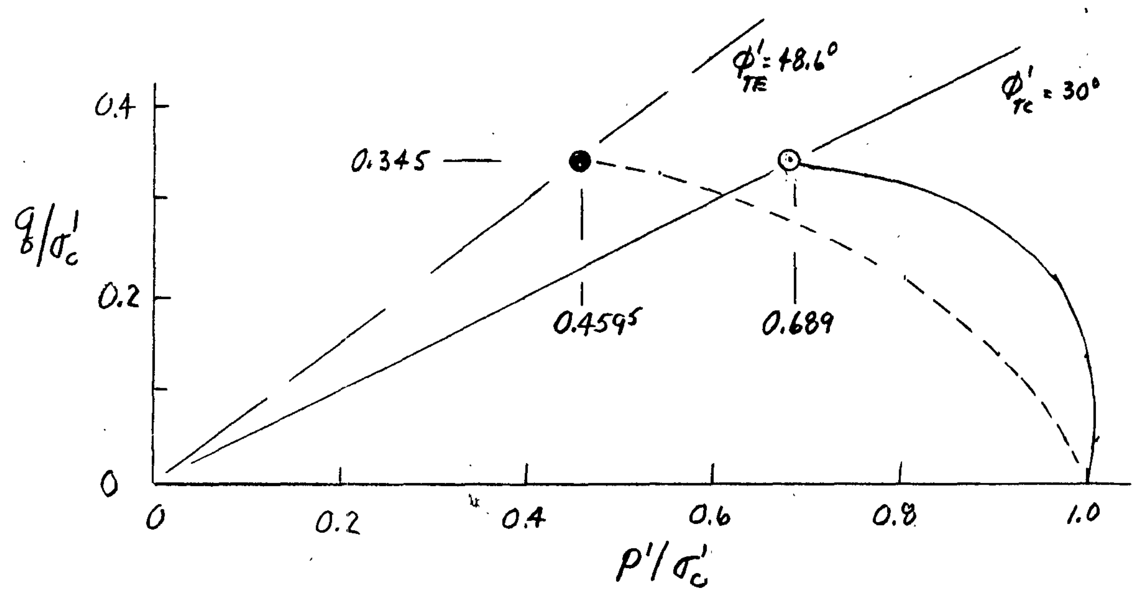


MCC CIUC/E OCR = 1 (M = 1.2 & m = 0.8)

Note: Using MIT q vs p' → $q_f/\sigma'_c = 0.344^6 \approx 0.345$

—○— TC : $(\Delta u - \Delta\sigma_3)/\sigma'_c = 0.655$ → $A_f = 0.95$

- - ● - - TE : " $= 0.885$ → " $= 1.28^5$



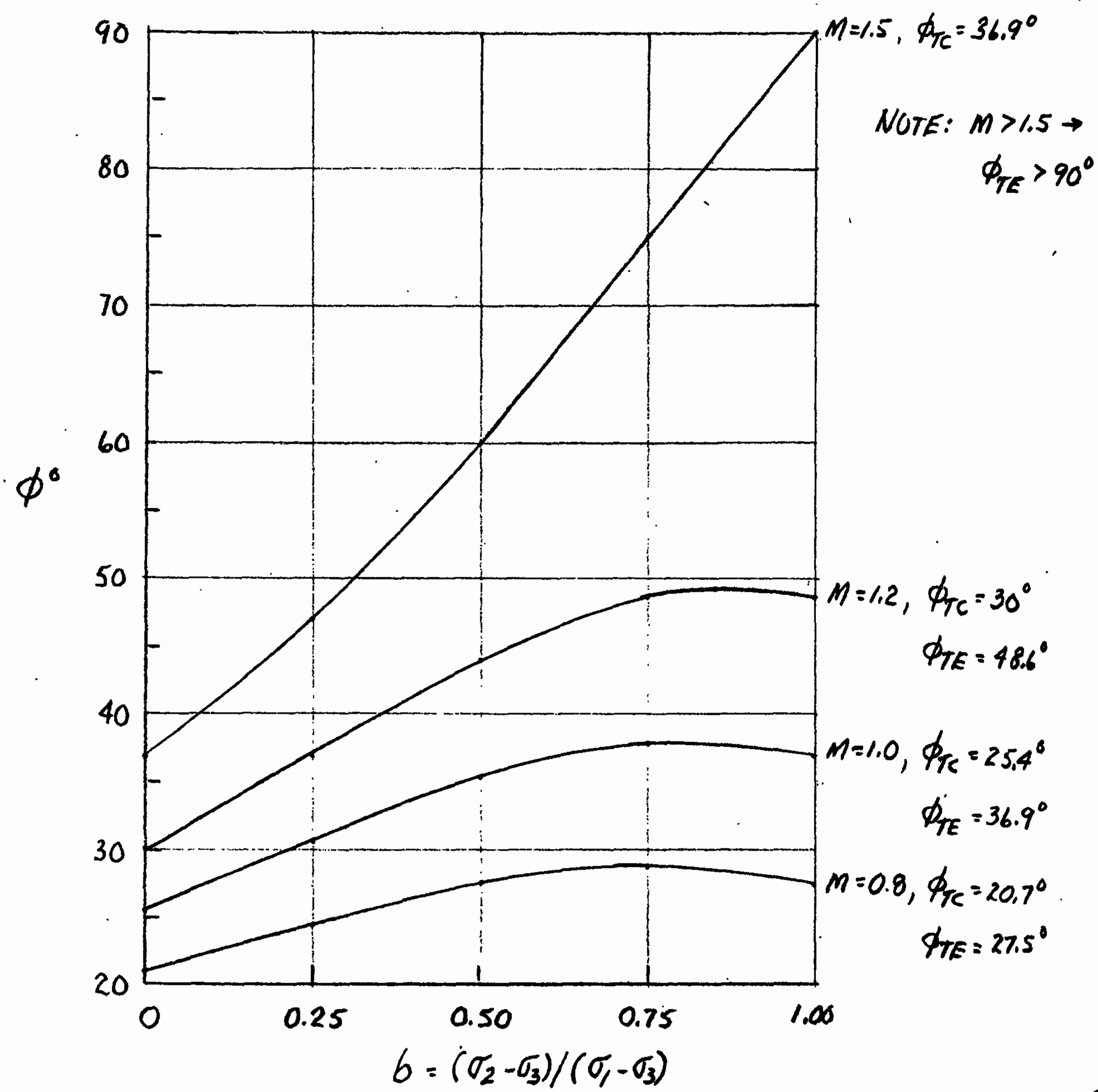
A

Extended von Mises Failure Criterion

From Bishop (1971) Roscoe Memorial Volume:

- $\dfrac{q^*}{\sigma_{oct}} = M$ with $q^* = \dfrac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_1-\sigma_3)^2+(\sigma_2-\sigma_3)^2}$

- Defining $b = \dfrac{\sigma_2-\sigma_3}{\sigma_1-\sigma_3} \rightarrow \sin\phi = \dfrac{3M}{M(1-2b)+6\sqrt{1-b+b^2}}$



B

CCL 2/22/87 2/90
1.322 2/96

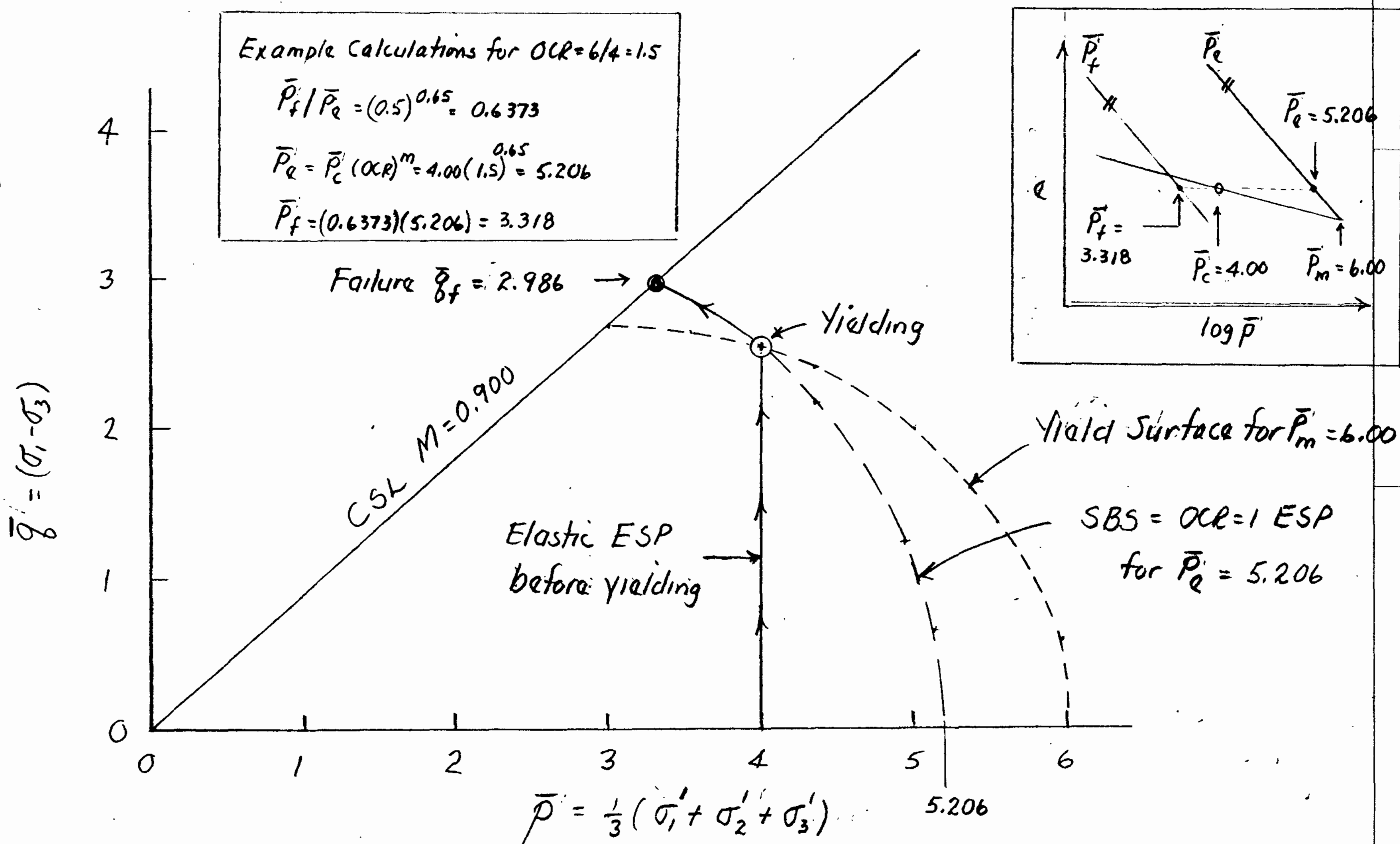
Fig. 1 ESP for CIUC/E OCR = 1.5 ($\bar{\sigma}_c' = 4.00$ & $\sigma_{cm}' = 6.00$)
Predicted via MCC for $M = 0.900$ and $m = 0.650$

Example Calculations for OCR = 6/4 = 1.5

$\bar{p}_f' / \bar{p}_e' = (0.5)^{0.65} = 0.6373$

$\bar{p}_e' = \bar{p}_c' (OCR)^m = 4.00(1.5)^{0.65} = 5.206$

$\bar{p}_f' = (0.6373)(5.206) = 3.318$



C

CCL 2/25/93 1.322
2/24/96

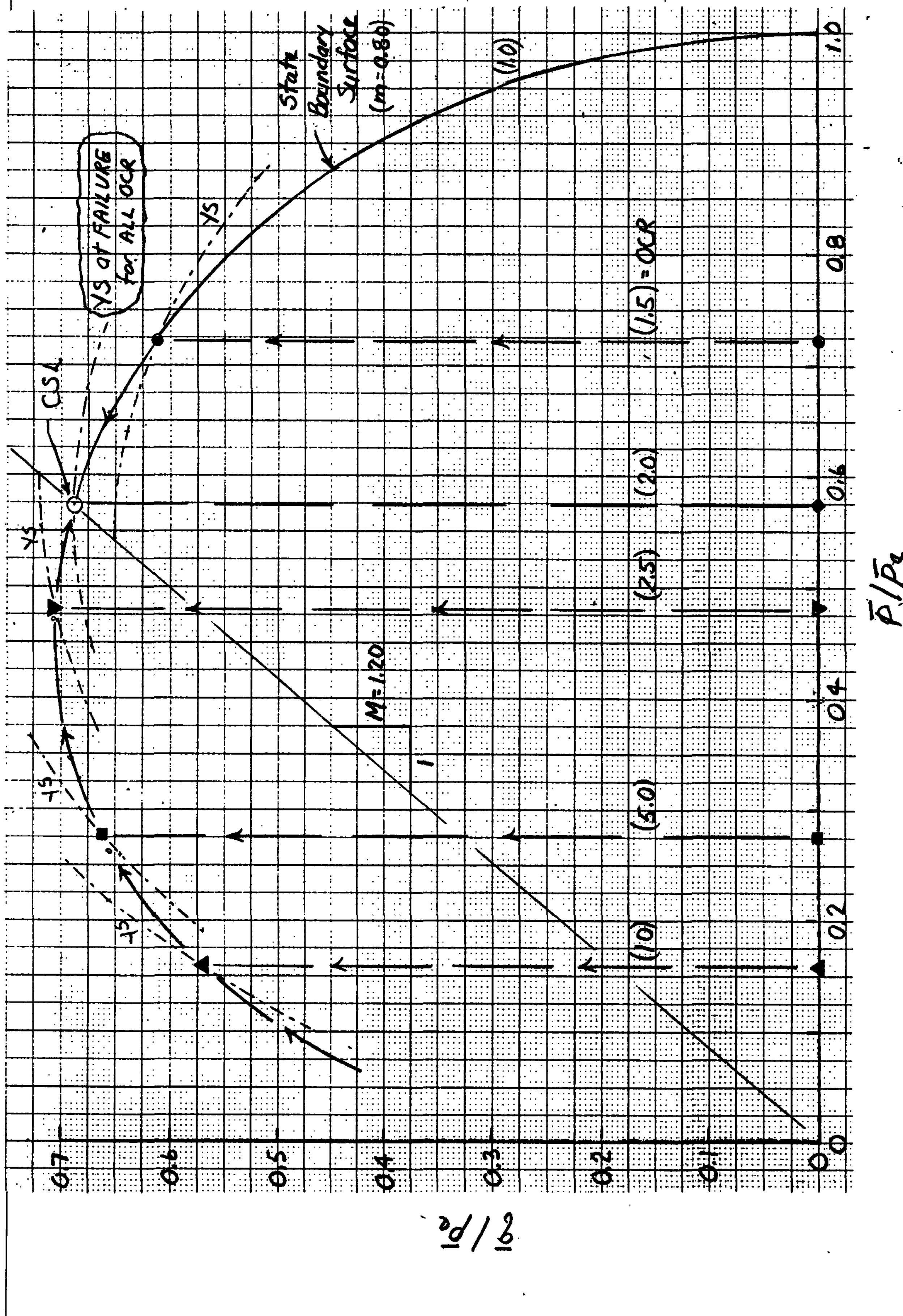


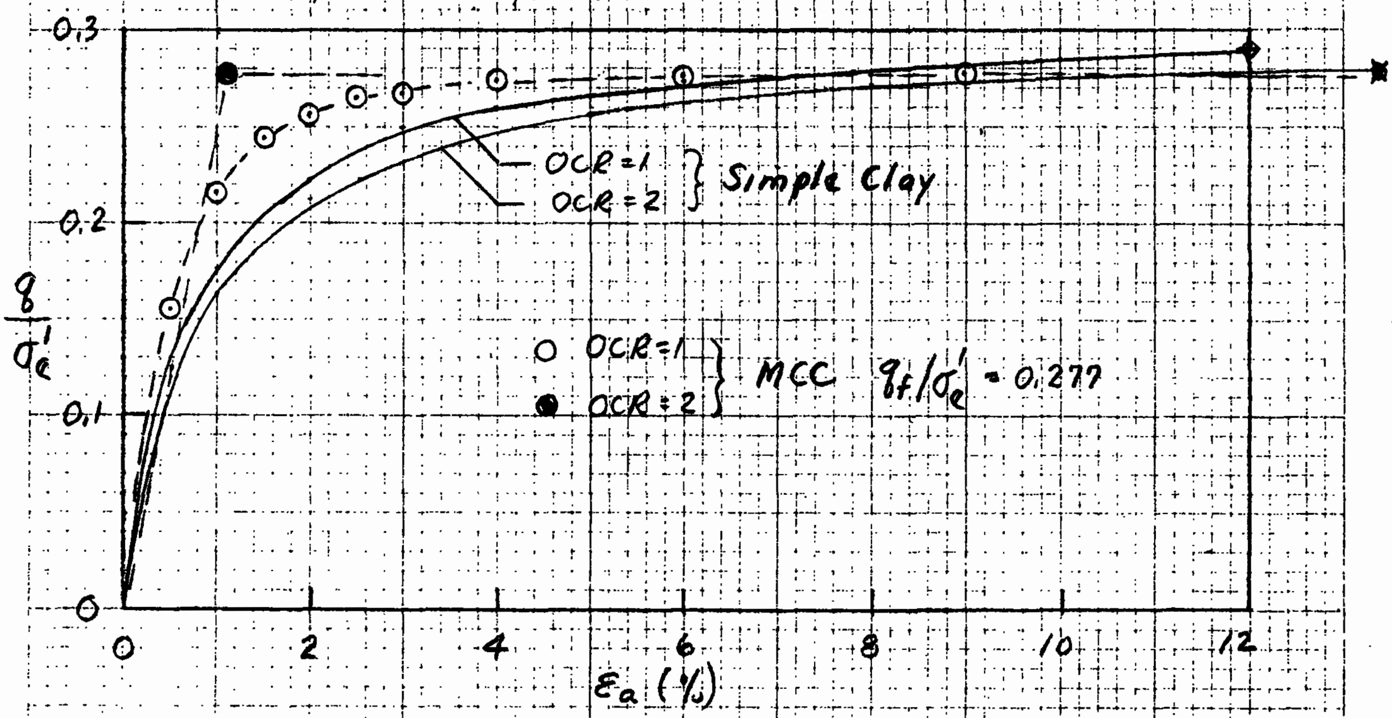
Fig. 2 ESP for CIUC/E: After Initial Yielding $\frac{\bar{p}}{\bar{p}_e} = \left(\frac{M^2}{M^2+R^2}\right)^m$

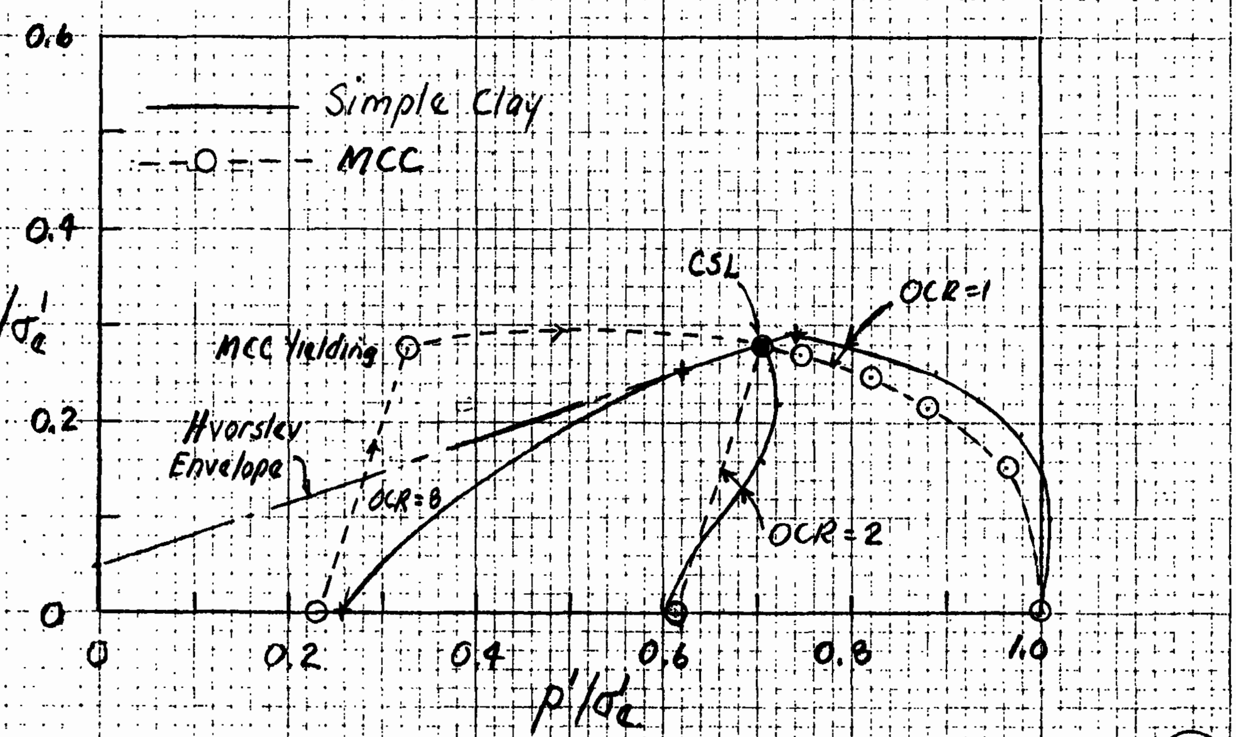
D

Fig. 3 Comparison of MCC vs Simple Clay CIUC

MCC { $M = 0.90$ ($\sin\phi' = 0.3913$), $e_o = 0.575$ at $\sigma'_c = 4.0$
$C_s/C_c = 0.30$ $C_s = 0.063$ $m = 0.70$ $\nu' = 0.30$

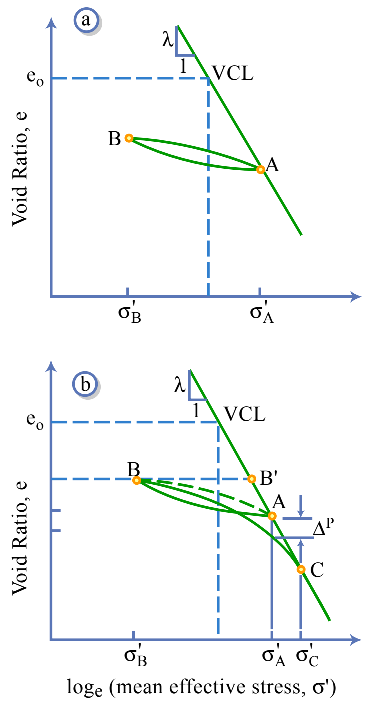
(By Kavvadas 1981)



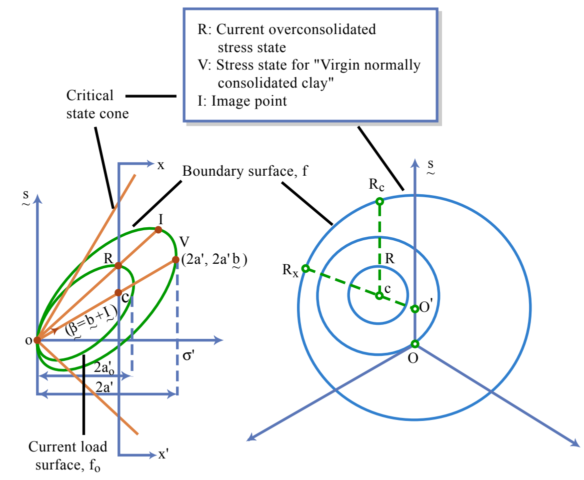


E

Adapted from Whittle & Kavvadas (1994)

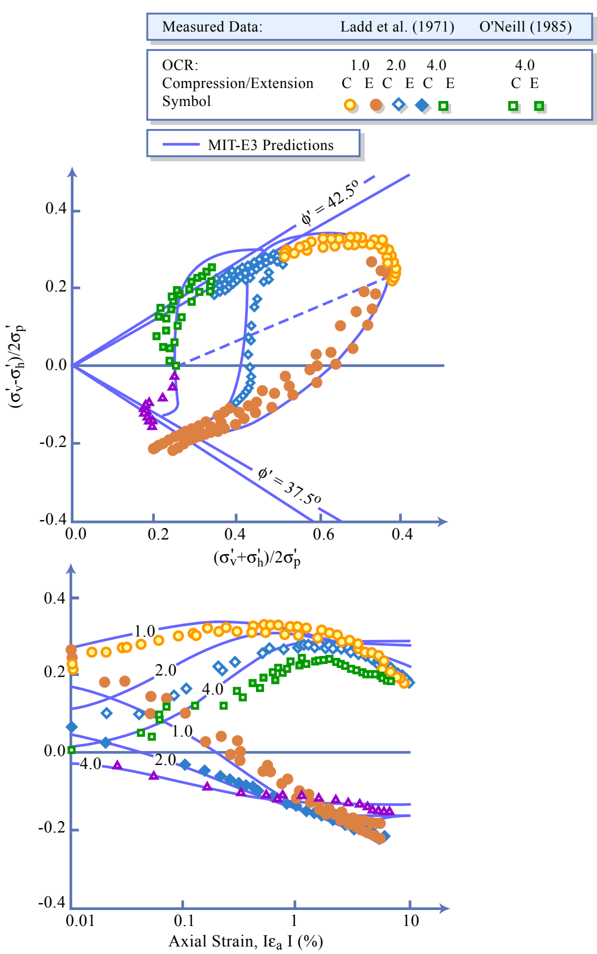


Conceptual Model of Unload-Reload Used by MIT-E3 for Hydrostatic Compression:
(a) Perfect Hysteresis; and (b) Hysteresis and Bounding Surface Plasticity.



Yield, Failure, and Load Surfaces used in MIT-E3 Model

Adapted from Whittle et al. (1994) ASCE JGE 120(1)



Evaluation of MIT-E3 Predictions of Undrained Plane Strain Shear Tests on K_o-Consolidated BBC

Figures by MIT OCW.

Whittle (1993) Geotechnique 43(2), 289-313.

Table 1. Average index properties of three clays

Property	Boston blue clay	Empire clay	London clay
w_L: %	42	76	75
w_P: %	21	26	28
I_P: %	21	50	47
I_L: %	95	36	5

Table 2. Input parameters for the MIT-E3 model

Test type	Parameter/ symbol	Physical contribution/ meaning	Boston blue clay	Empire clay	London clay
One-dimensional consolidation (oedometer CRS, etc.)	e_0	Void ratio at reference stress on virgin consolidation line	1·12	1·26	1·21
	λ	Compressibility of virgin normally consolidated clay	0·184	0·274	0·172
	C	Non-linear volumetric swelling behaviour	22·0	24·0	65·0
	n		1·60	1·75	1·50
	h	Irrecoverable plastic strain	0·2	0·2	0·1
K_0-oedometer or K_0-triaxial	K_{0NC}	K_0 for virgin normally consolidated clay	0·48	0·62	0·62
	$2G/K$	Ratio of elastic shear to bulk modulus (Poisson's ratio for initial unload)	1·05	0·86	0·99
Undrained triaxial shear tests: degrees OCR = 1; CK_0UC OCR = 1; CK_0UE OCR = 2; CK_0UC	ϕ'_{TC}	Critical state friction angles in triaxial compression and extension (large strain failure criterion)	33·4°	23·6°	22·5°
	ϕ'_{TE}		45·9°†	21·6°	22·5°
	c	Undrained shear strength (geometry of bounding surface	0·86	0·75	0·80
	s_t	Amount of post-peak strain softening in undrained triaxial compression	4·5	3·0	3·9
	ω	Non-linearity at small strains in undrained shear	0·07	0·20	0·20
	γ	Shear induced pore pressure for OC clay	0·5	0·5	0·5
Resonant column*	κ_0	Small strain compressibility at load reversal	0·001	0·0035	0·001
Drained triaxial	Ψ_0	Rate of evolution of anisotropy (rotation of bounding surface)	100·0	100·0	100·0

* Alternatively use field data from cross-hole shear wave velocity type tests.
† Recent data (Germaine, 1989) suggest $\phi'_{TE} \approx 40°$.