III-3 LATERAL EARTH PRESSURES AND RETAINING WALLS

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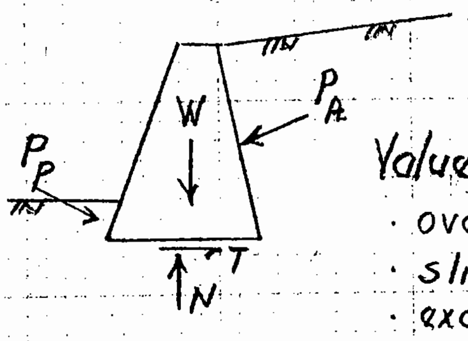
1. BACKGROUND ("Dry Soil," $\sigma = \sigma'$)

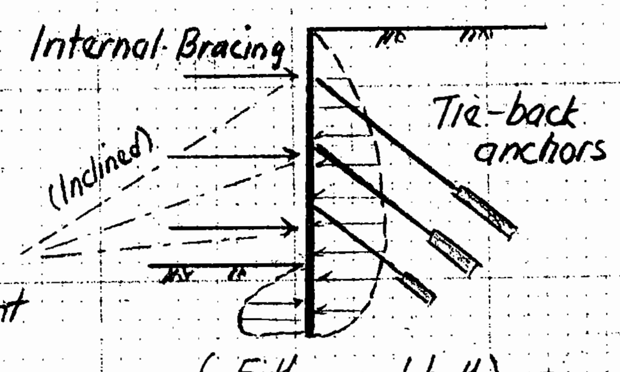
1.1 Typical Problems

Gravity Retaining Wall



Values of P_A & P_P →
- overturning
- sliding
- excessive settlement

Sheet Pile Excavation



1.2 Theoretical vs. Conventional Design Practice

1.361-1.366 focus is on (2)c

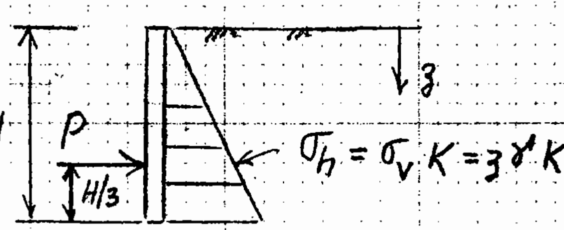
(1) Because problem is indeterminate since stresses = f(wall movements), need finite element analyses with advanced soil model (e.g., MIT-S1) to solve theoretically, which is expensive. Also details of actual construction are usually important, but difficult to predict and model.

(2) Hence conventional practice usually: a) predicts earth pressures on wall assuming that surrounding soil is in a state of failure (called the plastic zone); b) checks overall equilibrium (limiting equilibrium mechanics) } called "limit design"
c) adds a "factor of safety" to guard against collapse and uses empirical charts to estimate strut loads, wall & ground movements, etc.

2. RANKINE STATES OF STRESS: HORIZONTAL GROUND

2.1 Introduction

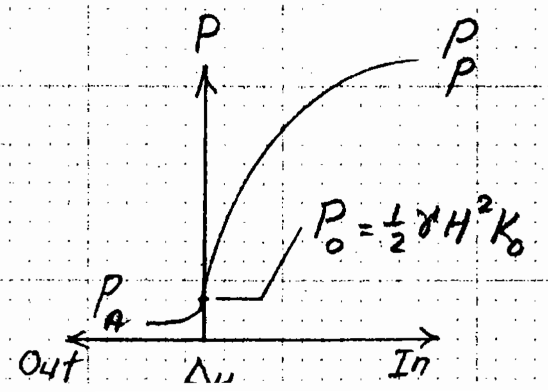
"Frictionless" Wall



(1) Rankine = failure condition with same stresses on all parallel planes (i.e., vertical or sloping)

(2) Active
- "Small" outward Δ_H → plastic zone
- Mobilized soil τ decreases P

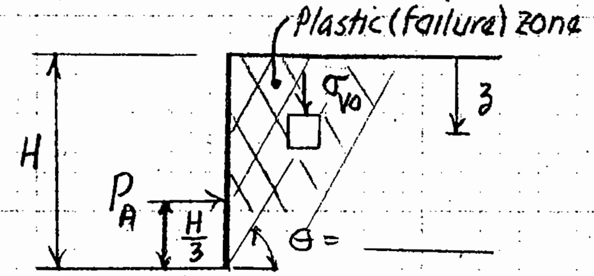
(2) Passive
- "Large" inward Δ_H → plastic zone
- Mobilized soil τ increases P



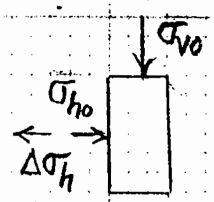
Part III–3 EARTH PRESSURES

2.2 Rankine Active (NC sand; γ', ϕ', $\sigma = \sigma'$)

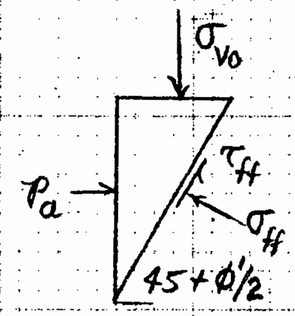
Field condition



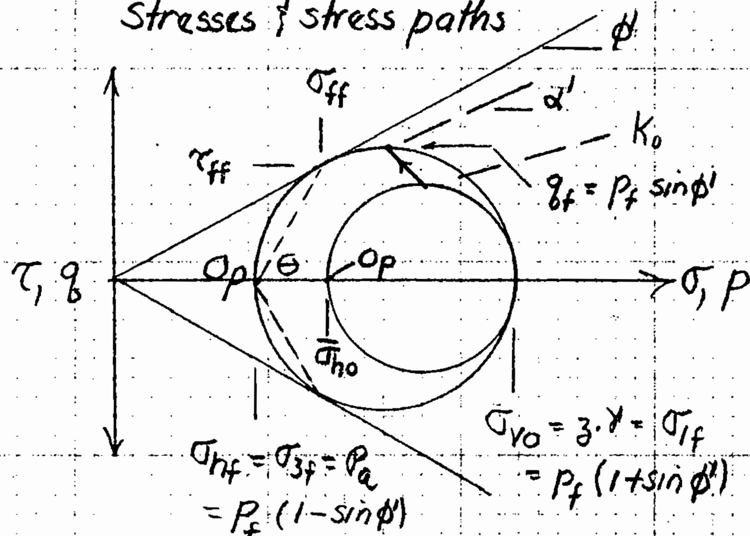
Lab test



TC (Unloading)
[Actually plane strain]



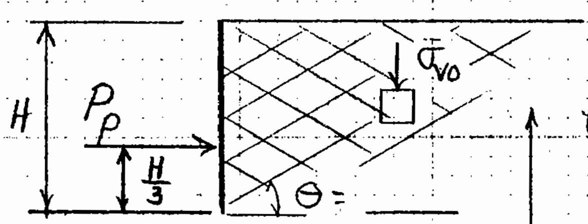
Stresses & stress paths

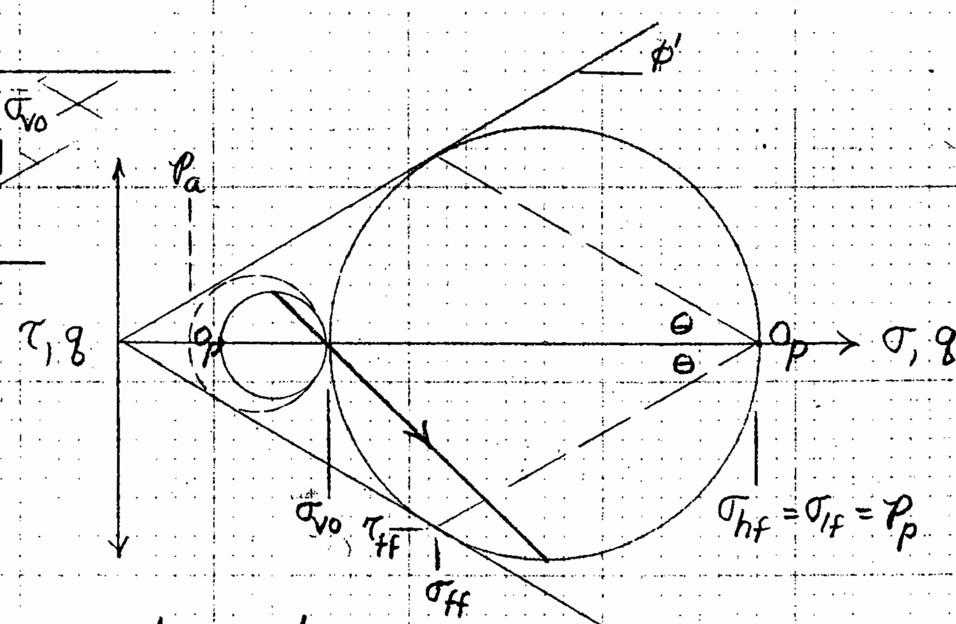


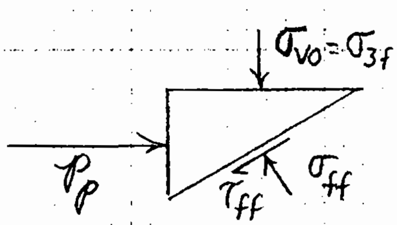
Remarks

- Limiting condition from development of plastic zone
- Coef. of active stress $= K_a$
  $= \sigma'_{hf}/\sigma'_{vf} = \sigma'_{3f}/\sigma'_{1f}$
  $= \tan^2(45 - \frac{\phi'}{2}) = \frac{(1 - \sin\phi')}{(1 + \sin\phi')}$
- $p_a = K_a \bar{\sigma}'_{vo}$; $P_A = \frac{1}{2}\gamma H^2 K_a$
- Θ = inclination of slip lines that intersect at $90 \pm \phi'°$

2.3 Rankine Passive (different scale; $\sigma = \sigma'$)





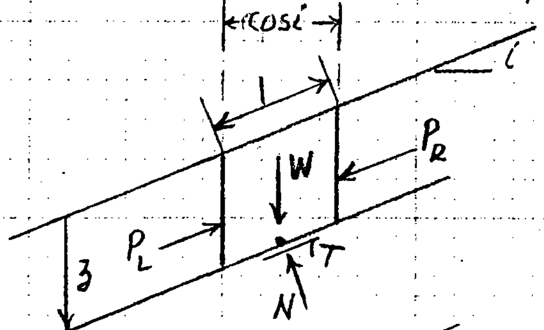


- Coef. of passive stress $= K_p = \frac{\bar{\sigma}'_{hf}}{\bar{\sigma}'_{vf}} = \frac{\bar{\sigma}'_{1f}}{\bar{\sigma}'_{3f}} = \tan^2(45 + \frac{\phi'}{2}) = \frac{(1 + \sin\phi')}{(1 - \sin\phi')} = 1/K_a$
- $p_p = z\gamma K_p \rightarrow P_p = \frac{1}{2}\gamma H^2 K_p$

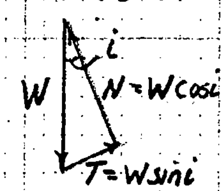
Part III-3 EARTH PRESSURES

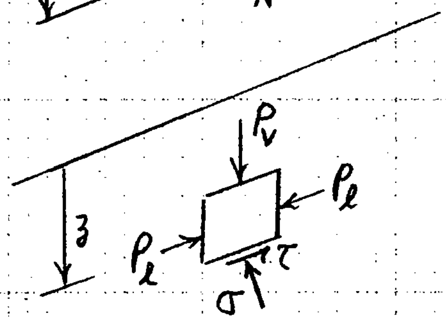
3. RANKINE STATES OF STRESS: SLOPING GROUND

3.1 Stresses in Infinite Slopes: General ($\sigma = \sigma'$)



- $W = z\gamma' \cos i$
- Reasoning → $P_L = P_R$ at inclination i
- ∴ $N = W/\cos i$ & $T = W \sin i$





- P_v & P_ℓ are conjugate stresses (act ∥ to planes of other stresses)
- $P_v = z\gamma' \cos i$ = CONSTANT (independent of P_ℓ)

$\sigma = P_v \cos i = z\gamma' \cos^2 i$

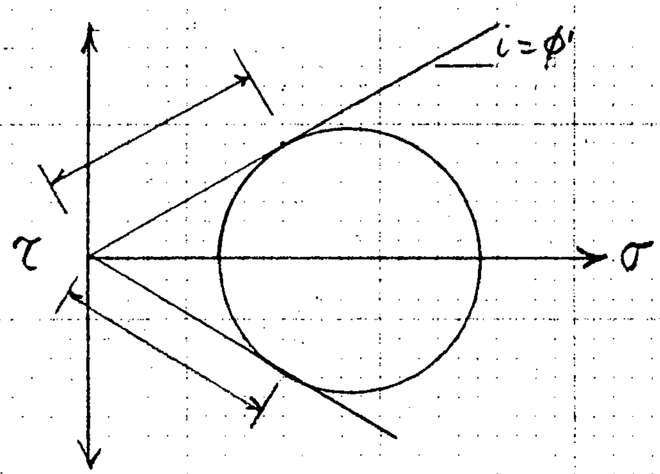
$\tau = P_v \sin i = z\gamma' \cos i \sin i$

$\rightarrow \tau/\sigma = \cos i \sin i / \cos^2 i = \tan i$

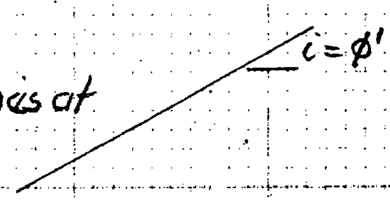
* P_ℓ indeterminate except at failure

3.2 Maximum Slope Angle (angle of repose for cohesionless soil)

$$\frac{\tau}{\sigma} = \tan i \rightarrow \frac{\tau_{ff}}{\sigma_{ff}} = \tan\phi'$$

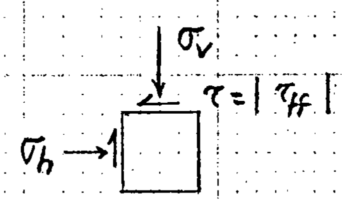


- Op =
- Slip lines at



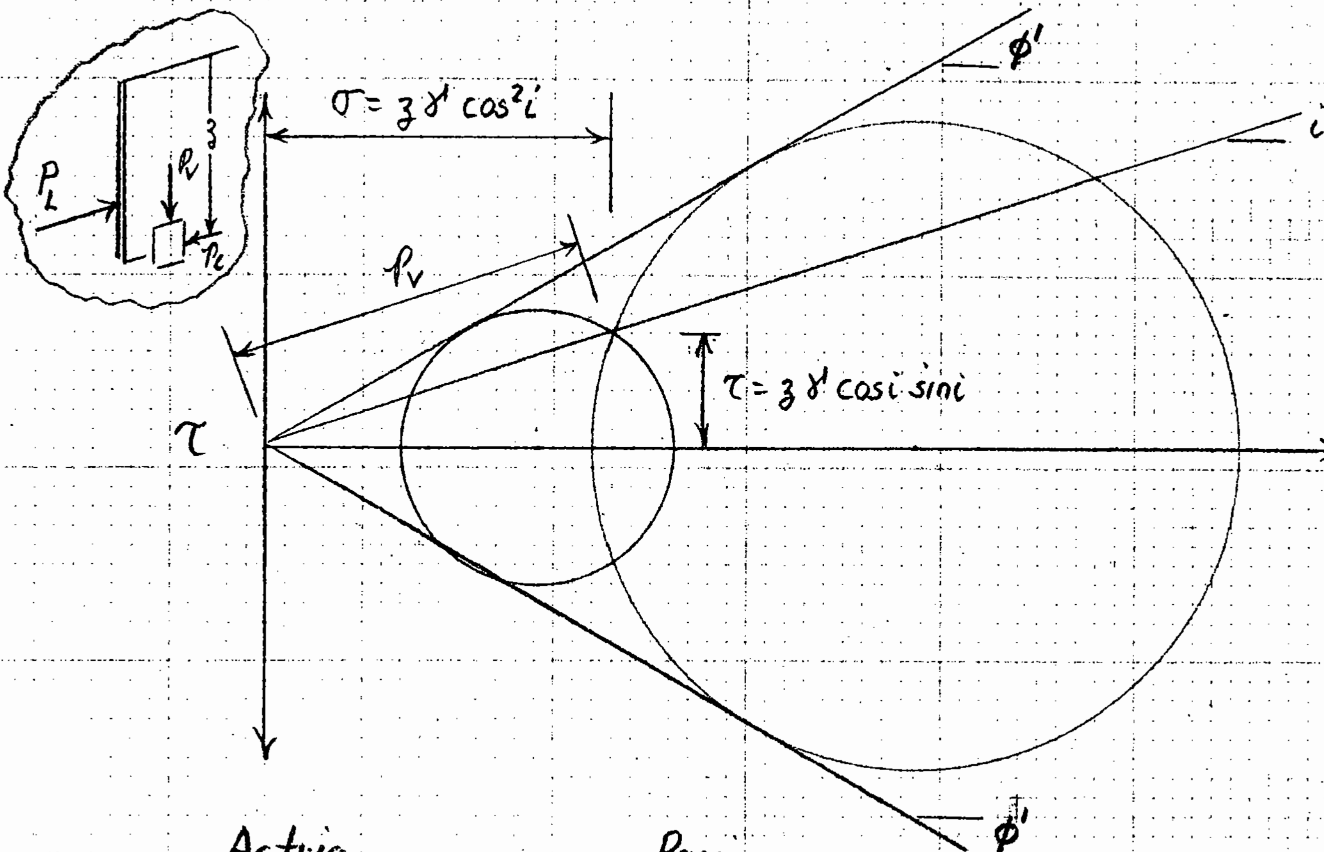
$$P_v = P_\ell = \sqrt{\sigma_{ff}^2 + \tau_{ff}^2} = z\gamma' \cos\phi'$$

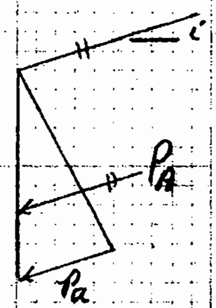
- Magnitudes of P_v & P_ℓ
- Magnitudes of σ_h & σ_v



Part III-3 EARTH PRESSURES

3.3 Values of p_a and p_p for Vertical Walls ($i < \phi'$)

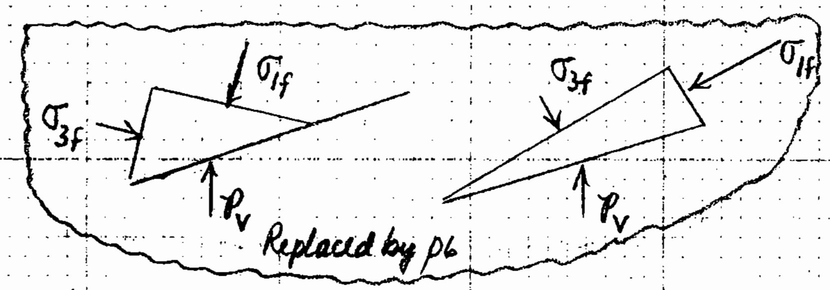


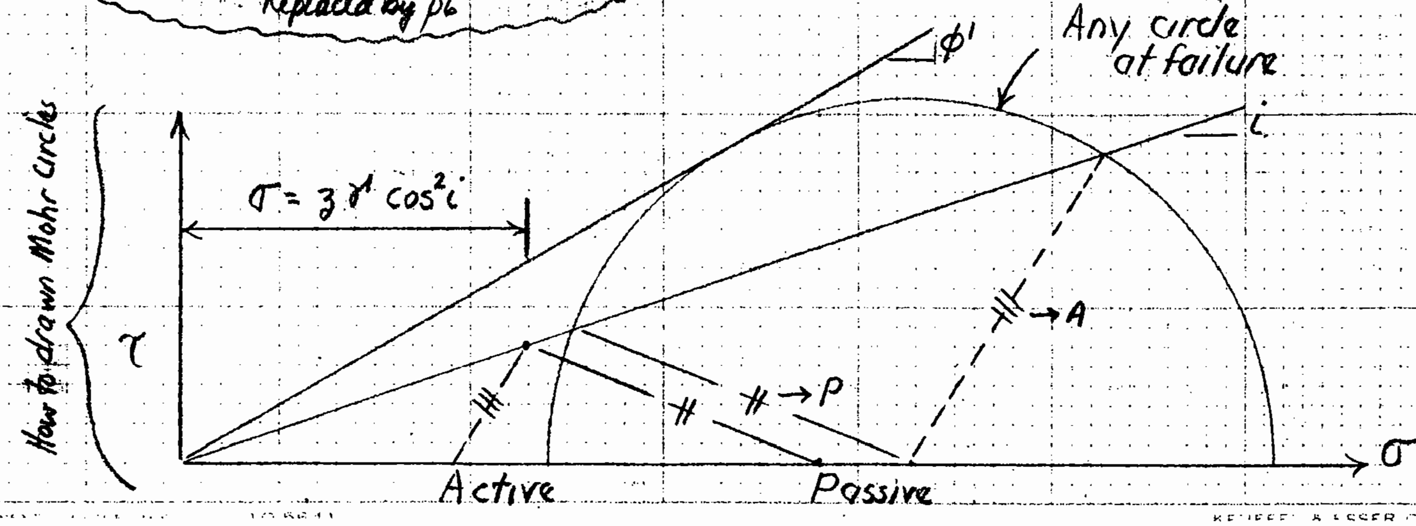


Active	Passive
· Op =	· Op =
· p_a =	· p_p =
· Slip lines	· Slip lines

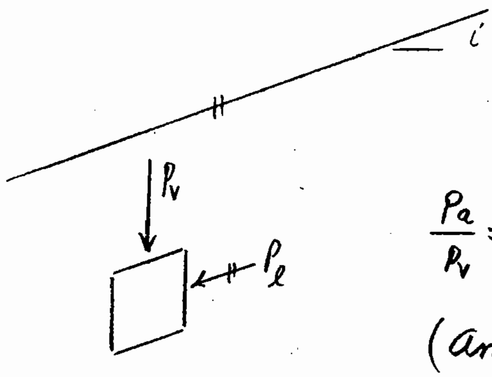
$P_A = \frac{1}{2}\gamma' H^2 K_a$ @ 1/3 point
$P \qquad\qquad P$

NOTES: P_A & P_P act ∥ to slope
Have $\tau \neq 0$ vertical wall



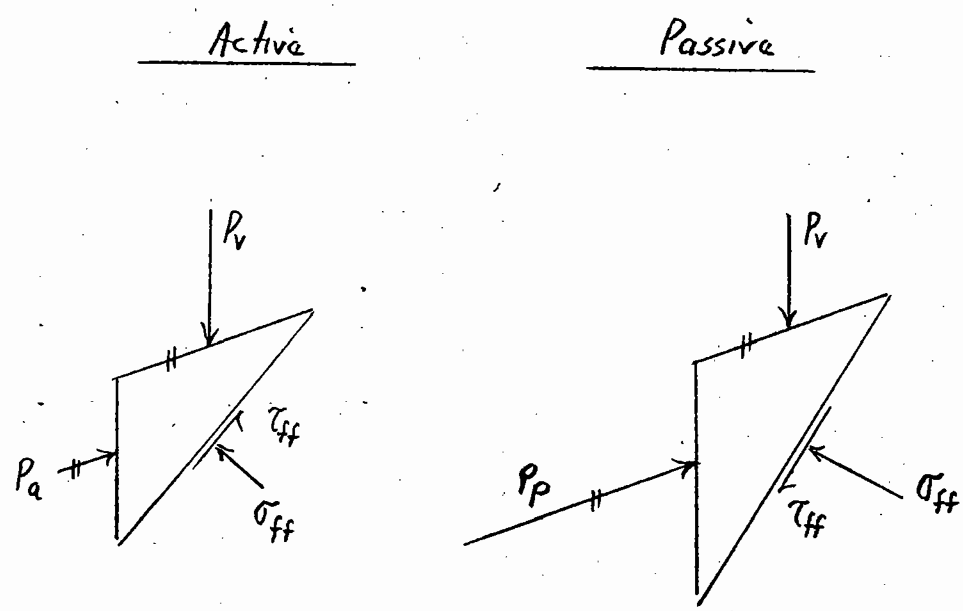


Supplement to 3.3 (Rankine Active & Passive for Vertical Wall)



$$\frac{P_a}{P_v} = \frac{P_v}{P_p} = \frac{\cos i - \sqrt{\cos^2 i - \cos^2\phi}}{\cos i + \sqrt{\cos^2 i - \cos^2\phi}}$$

(Analytical solution to check graphical solution)*

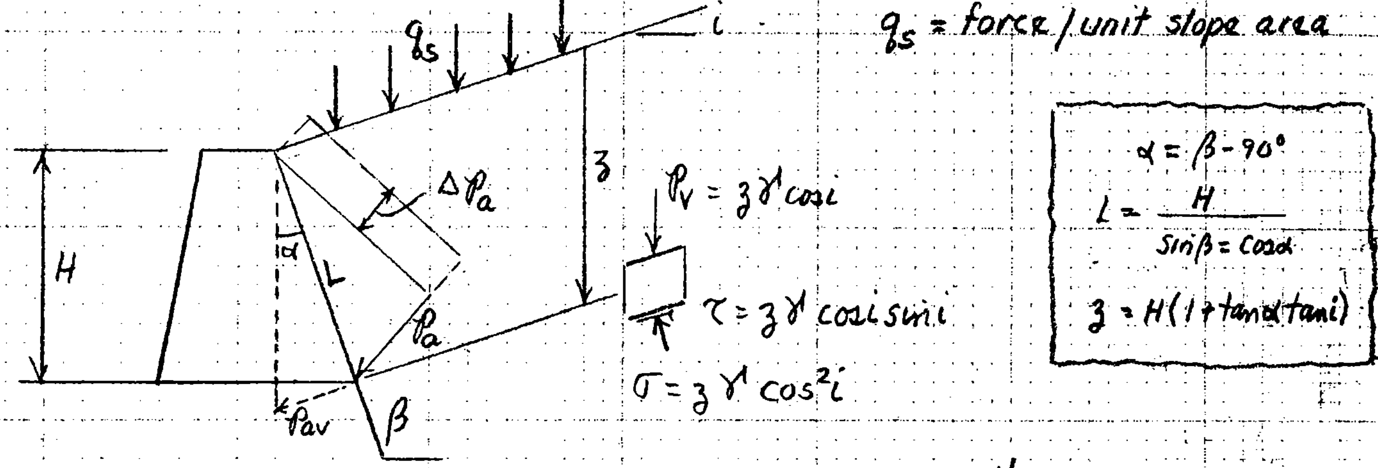


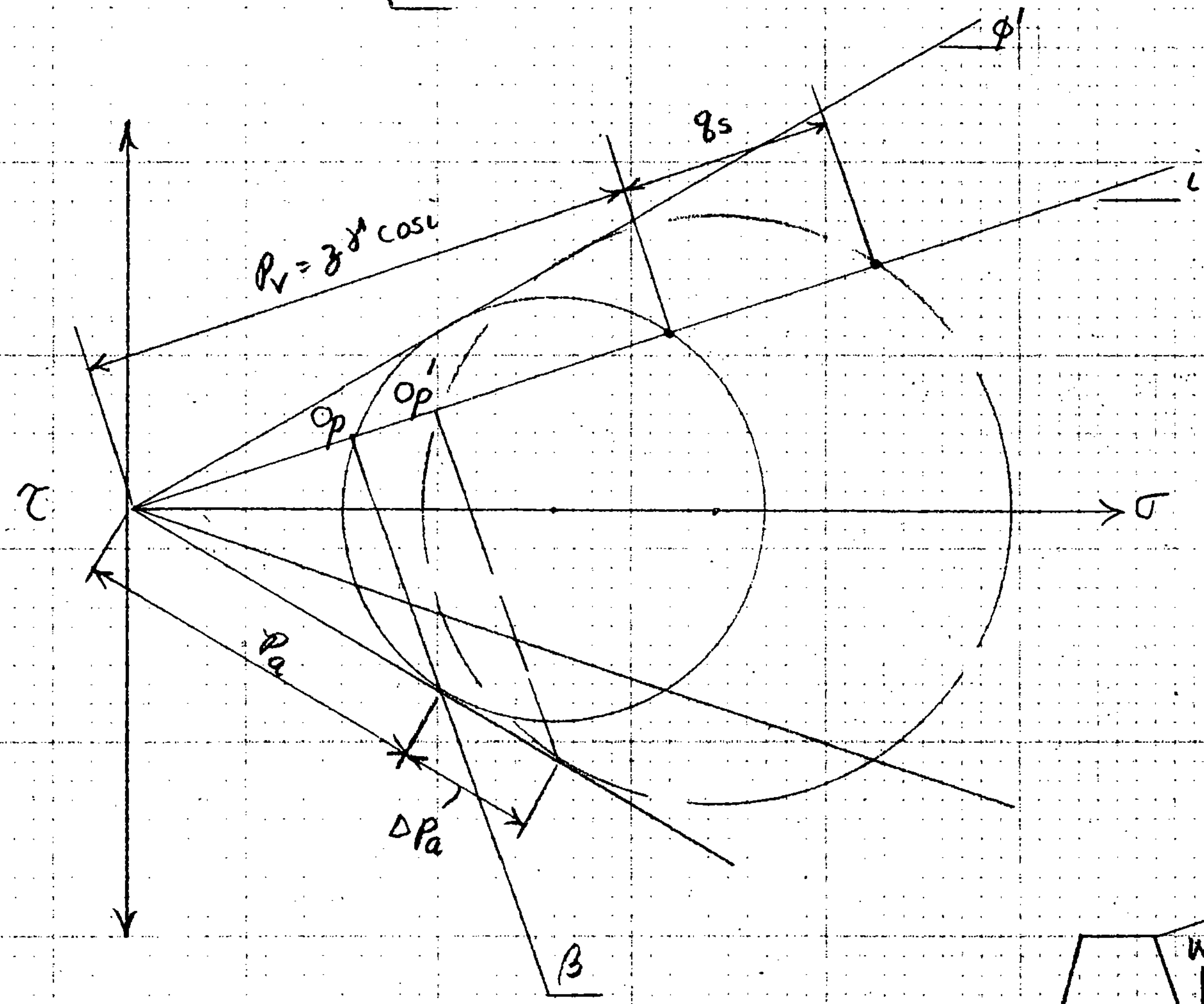
Required wall friction (ϕ_w)

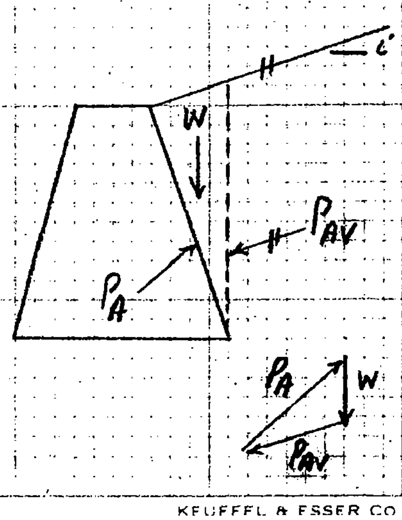
$\tan \phi_w =$

* From 1.361 student S. Kazzi (1986)

3.4 Values of p_a With Sloping Wall & Surcharge (Rankine)







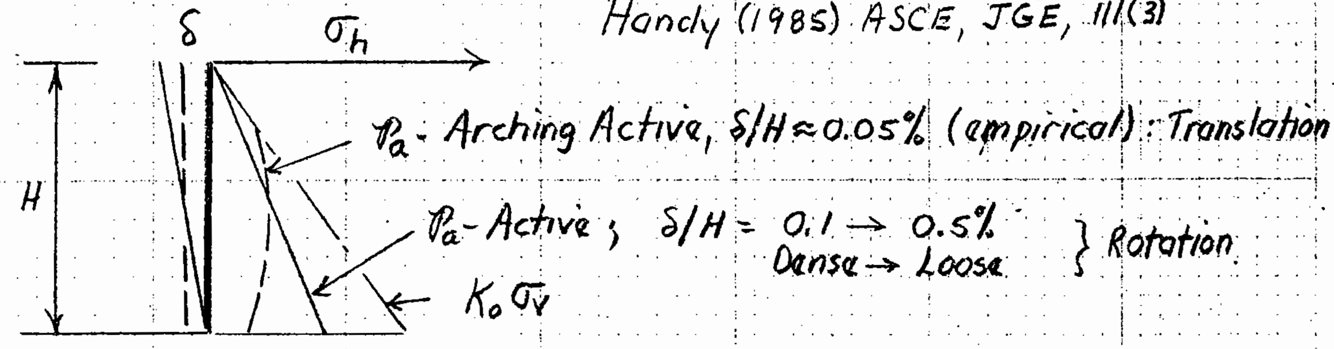
NOTES: $P_A = \frac{1}{2} p_a L$ at H/3 for γ'

$\Delta P_A = \Delta p_a L$ at H/2 for q_s

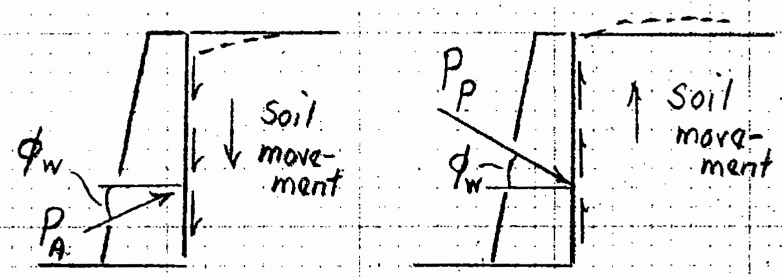
4 EARTH PRESSURES ON RETAINING WALLS

4.1 Movament of Vartical Wall on Active Distribution

Handy (1985) ASCE, JGE, 111(3)



4.2 Wall Friction (Negates Rankine States of Stress, $\theta \neq 45 \pm \phi'/2$)



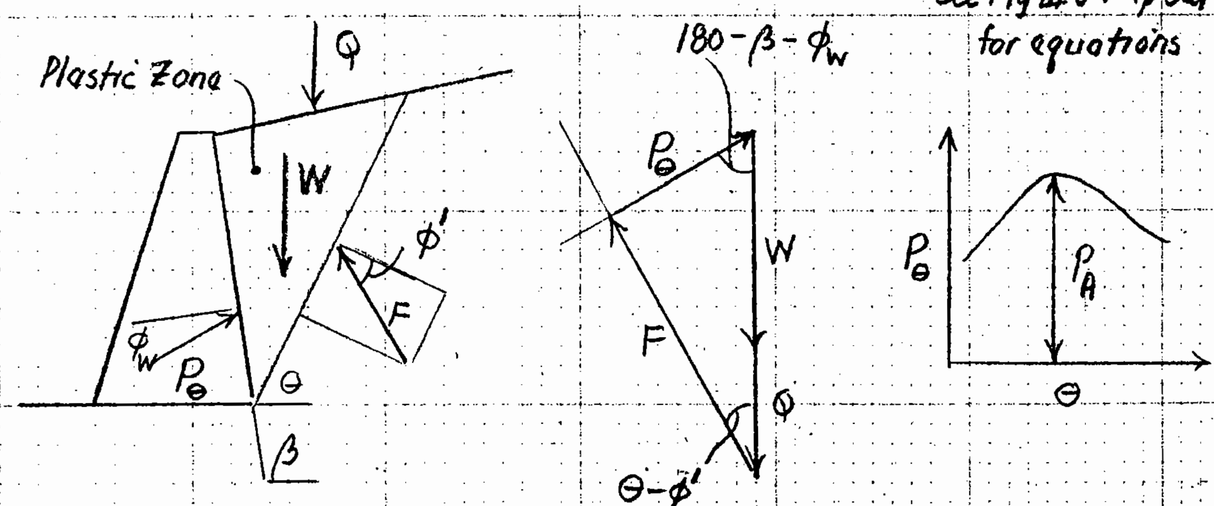
Usually assume $\phi_w = \phi'_{cv} = \phi'_{cs}$ for "rough" wall

Handy (1985): $\delta/H = 0.02\% \rightarrow$ full wall friction

4.3 Coulomb Method (Active) [Force Equilibrium]

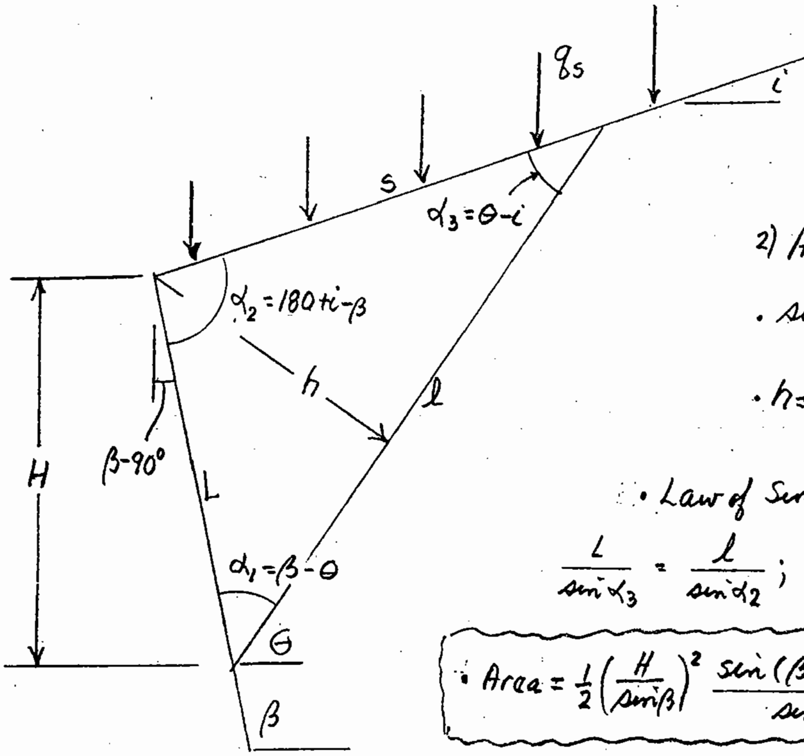
- P_A acts at ϕ_w to ⊥ to wall with planar failure surface
- Trial & error to find $\theta \rightarrow$ maximum P_θ (opposite for Passive)

See Fig III 3-1 (p 8a) for equations



NOTE: For Passive, reverse directions of ϕ_w & ϕ'; get min. $P \rightarrow P_p$

Equations to Obtain W and Q



1) $L = \dfrac{H}{\cos(\beta-90)} = \dfrac{H}{\sin\beta}$

2) $Area = \frac{1}{2}\, h \cdot l$

- $\sin\alpha_1 = \dfrac{h}{L} \rightarrow h = L \sin\alpha_1 =$
- $h = \dfrac{H}{\sin\beta}\sin(\beta-\theta)$

∴ • Law of Sines

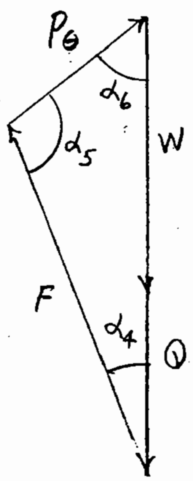
$$\frac{L}{\sin\alpha_3} = \frac{l}{\sin\alpha_2};\quad l = L\,\frac{\sin\alpha_2}{\sin\alpha_3} = \frac{H}{\sin\beta}\,\frac{\sin(\beta-i)}{\sin(\theta-i)}$$

$$\therefore Area = \frac{1}{2}\left(\frac{H}{\sin\beta}\right)^2 \frac{\sin(\beta-i)\sin(\beta-\theta)}{\sin(\theta-i)}$$

3) $W = \gamma \cdot Area$

4) $Q = q_s \cdot s\ ;\ \dfrac{s}{\sin(\beta-\theta)} = \dfrac{L}{\sin(\theta-i)} \rightarrow$ $s = \dfrac{H}{\sin\beta}\,\dfrac{\sin(\beta-\theta)}{\sin(\theta-i)}$

Equations to Obtain P_θ



- $\alpha_4 = \theta - \phi$
- $\alpha_5 = \beta + \phi + \phi_w - \theta$
- $\alpha_6 = 180 - \beta - \phi_w$

• Law of Sines

$$\frac{P_\theta}{\sin\alpha_4} = \frac{W+Q}{\sin\alpha_5}$$

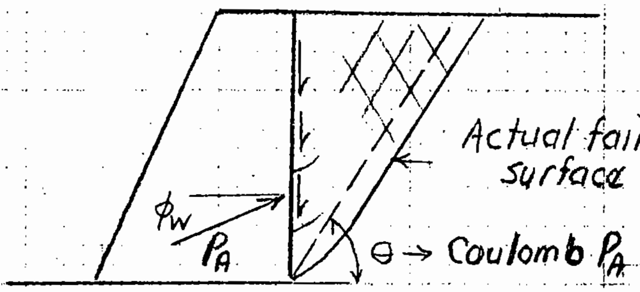
$$P_\theta = (W+Q)\cdot K_\theta$$

$$K_\theta = \frac{\sin(\theta-\phi)}{\sin(\beta+\phi+\phi_w-\theta)}$$

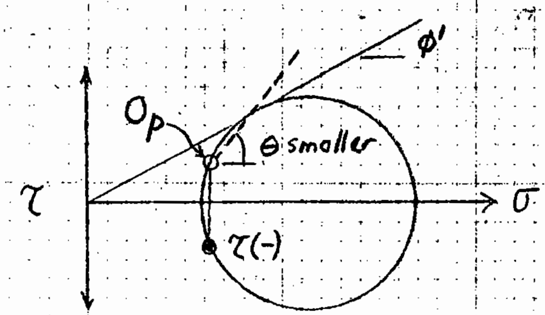
Fig III 3-1 Derivation of Equations for Coulomb P_θ (Active).

4.4 Coulomb vs. Reality

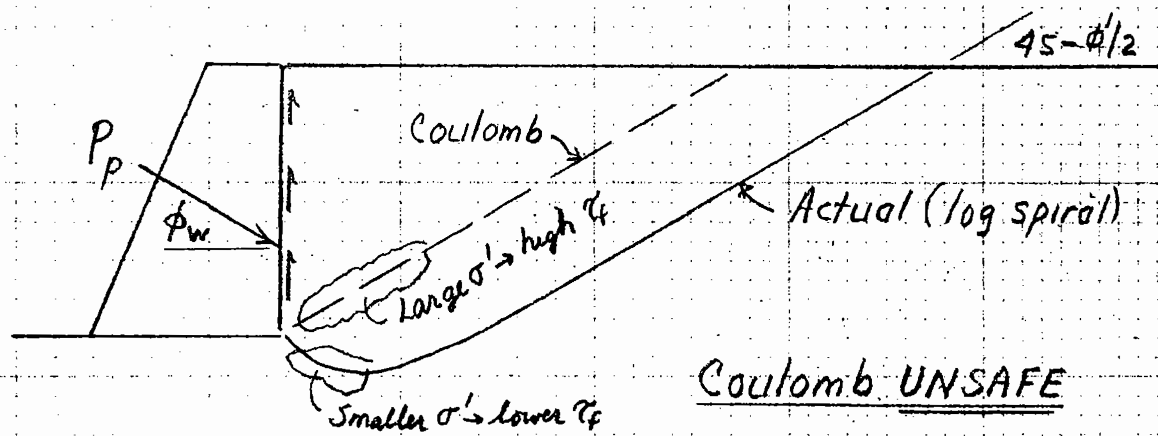
(1) Active case (Fig. 13.16)



- Deviation is relatively small
- Can use Rankine P_A, but inclined at ϕ_w



(2) Passive case



Coulomb UNSAFE

Example $P_p = \frac{1}{2}\gamma' H^2 K_p$

ϕ'	ϕ_w	K_p Coulomb	K_p Actual		Rankine K_p
35°	30°	16	10	(+60%)	$3.7 = \tan^2(62.5°)$
30°	10°	4.4	4.1	(+7%)	3.0
		(Fig. 32.1 of T & P 1967)	(Fig. 13.20)		↑ Conservative