Lab 2: Strain
Solutions

Fall 2005

1 Particle trajectories

1.1 Body translation

Strain is rigid, non-rotational, and homogeneous.

$$x' = x + A$$
$$y' = y + B$$

1.2 Body rotation

Rigid, rotational, homogeneous.

$$x' = \cos\omega x + \sin\omega y$$
$$y' = -\sin\omega x + \cos\omega y$$

1.3 Simple shear

Non-rigid, homogeneous, rotational (lines originally parallel to y-axis are deflected through an angle ψ).

$$x' = x + y\tan\psi = x + \gamma y$$
$$y' = y$$

1.4 Pure shear

Non-rigid, homogeneous, irrotational (the strain ellipse long axis is parallel to x, the short axis parallel to y. These have the same orientation as the original directions of these lines).

$$x' = kx$$
$$y' = \frac{y}{k}$$

1.5 General homogeneous rotational strain

Although the displacement field appears complex, each originally square shaped element is transformed into an identically shaped and oriented parallelogram. Originally straight and parallel lines change length and orienta-

tion, but remain straight and parallel. Displacement vector gradients are constant, so the coordinate transformations are defined:

$$x' = ax + by$$
$$y' = cx + dy$$

where a, b, c, d are constants.

1.6 General heterogeneous strain

Displacement vector gradients change from point to point. Each element has a different shape after deformation. Initially straight lines become curved. Coordinate transformation equations must be smooth, single-valued non-linear functions of position:

$$x' = f_1(x, y)$$
$$y' = f_2(x, y)$$

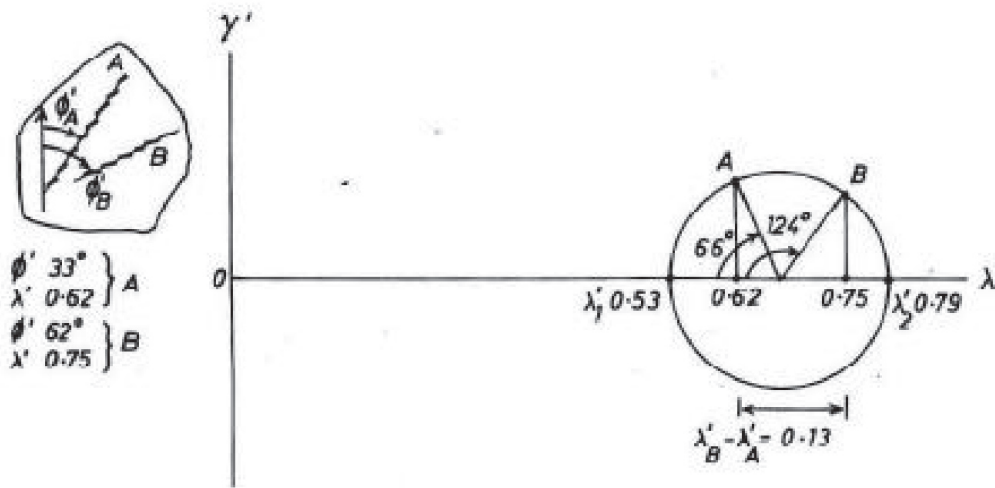# 2 Mohr circles for strain: the belemnites

Figure 1: Mohr circle solution

The construction proceeds by drawing the Mohr circle with points A and B positioned on its circumference using the $2\phi'$ relationships. These points establish the scale along the λ' axis from the measured value of $\lambda'_1 - \lambda'_2$, and is used to fix the position of the origin. Once the origin is found, you can read off the values of λ'_1 and λ'_2.

There was quite a range of values measured from the diagrams. The attached figure has $\lambda'_A = 0.62$, $\phi'_A = 33$, $\lambda'_B = 0.75$, $\phi'_B = 62$, and the construction yields $\lambda'_1 = 0.53$, $\gamma'_A = 0.12$, $\lambda'_2 = 0.79$, $\gamma'_B = 0.1$. Note that $\gamma' = \frac{\gamma}{\lambda} \Rightarrow \psi = \arctan(\gamma'/\lambda')$. So

$$\psi_A = \arctan(0.12/0.62) \sim 11$$
$$\psi_B = \arctan(0.1/0.75) \sim 8$$

But: ψ is the shear strain, that is, it is the change in angle between two originally perpendicular lines. This is not, in general, the same thing as rotation with respect to some defined frame. What you would need to do is

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relate ϕ' to ϕ[1] , which we did not discuss in lab or class. If you reported a value for ψ, the shear strain, that's ok. The purpose of this note is to make the distinction between rotation and shear strain. It turns out that angles ϕ and ϕ' are related by

$$\tan\phi' = \tan\phi / R$$

$$R = \frac{1 + e_1}{1 + e_2} = \sqrt{\frac{\lambda_1}{\lambda_2}}$$

the above equation is known as Wettstein's equation, and R is the ellipticity of the strain ellipse. In this example, $R = 1.22$, and the changes in angles are 5 and 4 degrees for lines A and B, respectively.

Note also: some of you assumed simple shear when trying to figure out how much each line rotated. This assumption has no justification. It turns out, however, that since simple shear is relatively, well, simple, many geologists assume this when interpreting structural relations in an outcrop. This is a poor habit.

3 Progressive deformation of vein sets

The interpretation of this outcrop depends on the following assumptions: (1) 2-dimensional, plane strain; (2) simple shear; (3) shear plane is parallel to the mylonitic foliation; (4) strain represents the accumulation of identical increments of strain, so that we can relate the infinitesimal strain ellipse to finite strain ellipses for parts of the deforming rock body; (5) the opening, rotation and folding of veins can be related to the accumulation of infinitesimal strain.

As strain accumulates dextrally, extensional veins open up perpendicular to the principal extensional direction of the instantaneous strain ellipse, which lies at 45°to the shear plane. That is, newly formed extensional veins ought to form at -45°to the shear plane. I interpret veins A and C as such newly formed veins. In fact, they are oriented somewhat less than 45°, suggesting either that they have undergone some rotation since formation or that the veins opened in an orientation not quite perpendicular to the presumed principal infinitesimal strain axis (and direction of maximum lengthening).

Once formed, such veins are immediately oriented in the shortening quadrant of the instantaneous strain ellipse. Since all material lines rotate in the sense of shear for simple shear, not only will the veins be shortened, but they will rotate. Eventually, such a vein will rotate out of the instantaneous shortening quadrant and into the instantaneous extensional quadrant. Vein B appears to be an example of such a vein. It likely formed in the same orientation as veins A and C, but shortened as it rotated into its present orientation. Presently, it lies in a quadrant of **instantaneous** extension, but **finite** shortening. That is, it has not spent enough time in the lengthening quadrant such that the lengthening it has suffered exceeds the shortening accumulated during its rotation through the shortening quadrant.

In the figure below, I show strain ellipses for both infinitesimal strain and for finite strain. Note that since the veins are of different ages, they have accumulated different amounts of finite strain. Furthermore, the rock that these veins have formed in has also presumably suffered more finite strain than any of the veins. The finite strain ellipse sketched below is taken to represent the finite strain suffered by vein B. Since I interpret veins A and C as being "newly" formed, the infinitesimal strain ellipses are probably reasonably close representations of the strain for these veins.

If strain were have kept progressing, all existing material lines (veins A, B and C) would have continued to rotate clockwise. Vein B – currently oriented for lengthening – might have boudinaged and eventually accumu-

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[1]These are the angles the line makes with the principal strain axes after and before deformation. In the derivation of the Mohr circle relationships, the undeformed angles with respect to where the finite strain axes end up. That is, this works for an increment of strain or a finite deformation, but progressive deformation with rotating strain axes presents some complications

lated net lengthening. Veins A and C would begin to fold and buckle. Any newly formed vein (denoted as D in the bottom of the figure) would open up in an orientation close to -45°to the foliation.
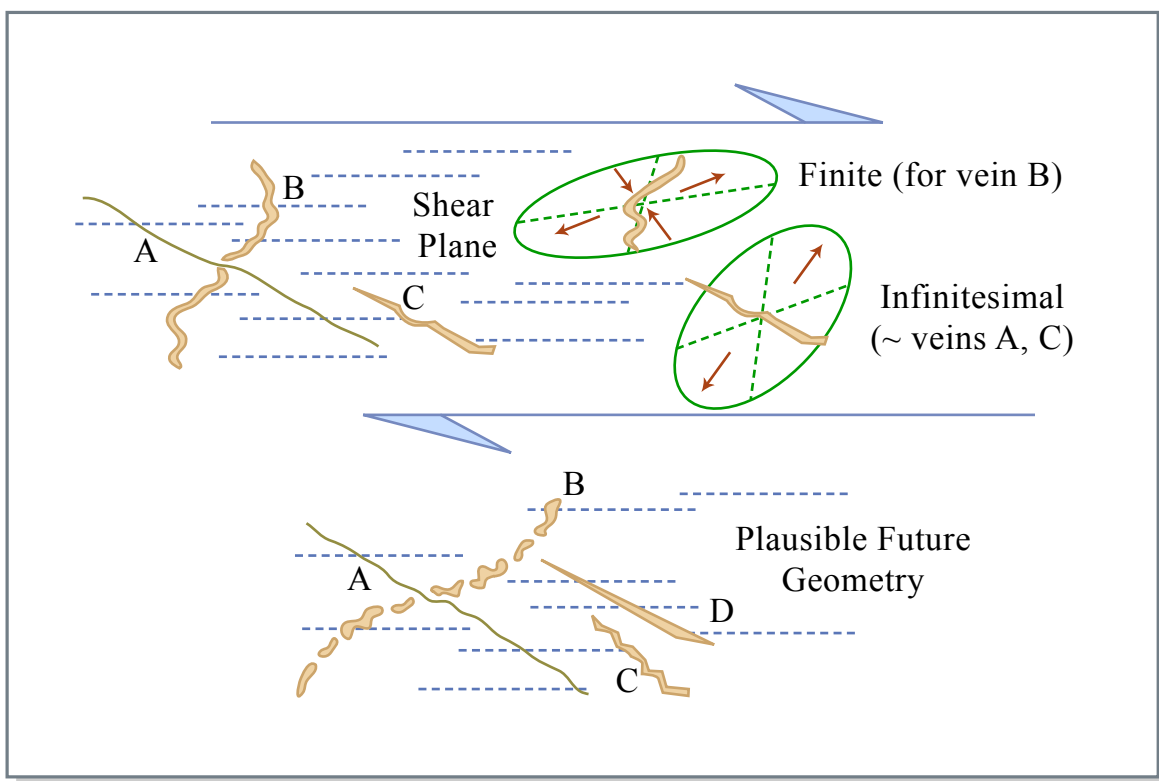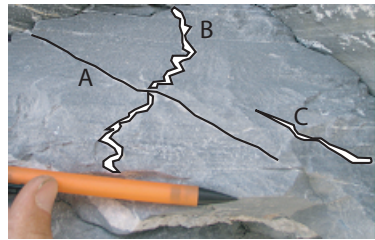

Figure by MIT OCW.

Figure 2: **Top**: photograph, with veins A, B, C labeled. **Middle**: sketch of veins. Shear plane is assumed to be parallel to the mylonitic foliation, and represented as dashed lines. Since veins open up during progressive deformation, vein B has accumulated more strain than A or C. Finite strain ellipse for vein B shown; dashed lines represent boundaries between quadrants of finite shortening and finite lengthening. Strains of veins A and C are probably closest to the infinitesimal strain ellipse; dashed lines represent boundaries between quadrants of instantaneous shortening and lengthening. Inferred sense of shear shown. **Bottom**: Sketch of plausible history had strain kept accumulating: vein B begins to boudinage, veins A and C begin to fold; new vein D opens up.