12.215 Modern Navigation

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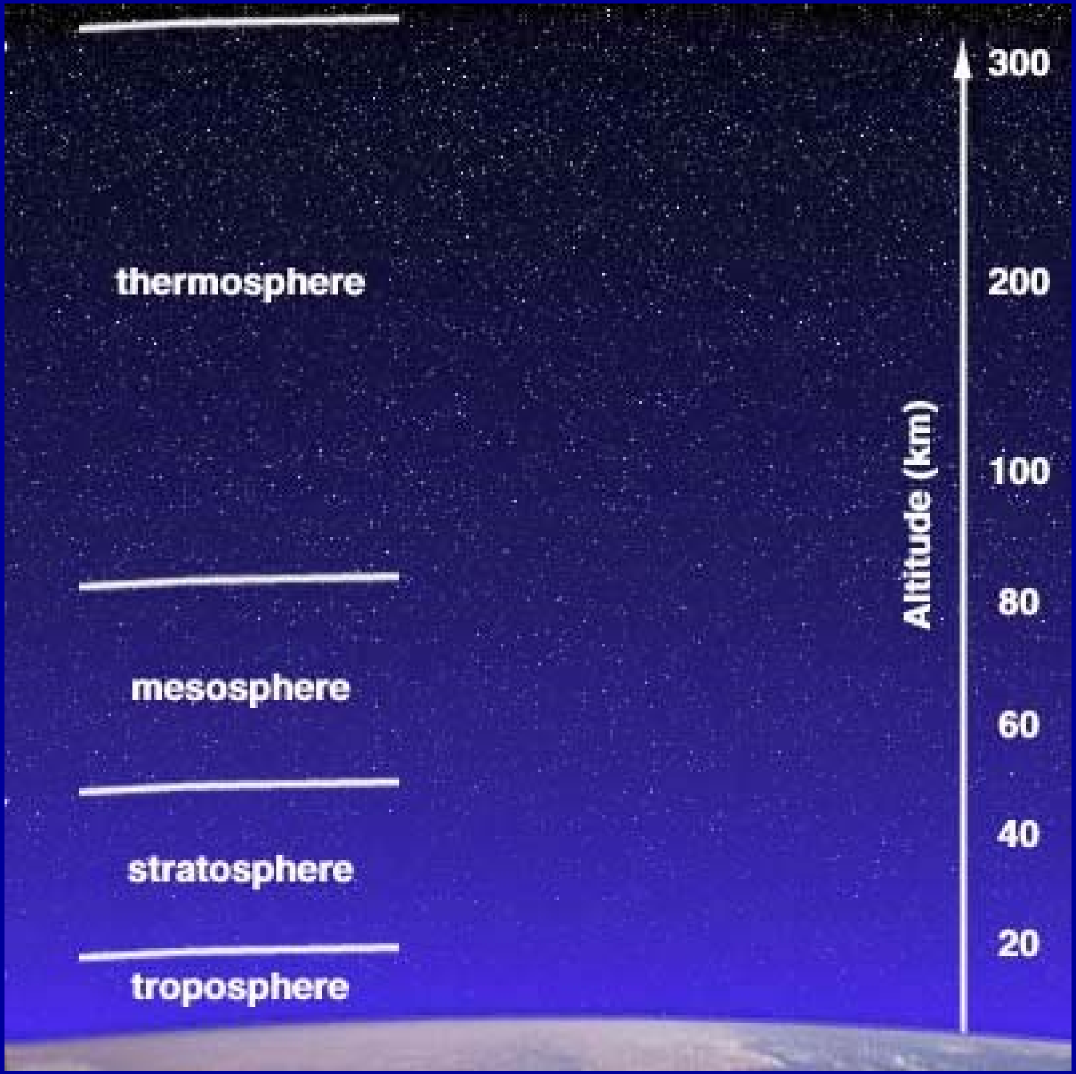
Summary of Last Class

- GPS measurements
  - Basics of pseudorange measurements
  - Phase measurements (allow millimeter level position with GPS and cm in real-time)
  - Examine some GPS data.
- Positioning modes
- Dilution of precision numbers

Today’s Class

- Atmospheric propagation.
  - Basic structure of the atmosphere
  - Refractive index of atmospheric air
  - Accounting for atmospheric delays in precise GPS navigation:
    - Hydrostatic and wet delays
  - Use of GPS for “weather forecasting”

Basic atmospheric structure



Troposphere is where the temperature stops decreasing in the atmosphere. (~10 km altitude)

Troposhere

- Lots of examples of web-based documents about the atmosphere: See for example.

http://www-das.uwyo.edu/~geerts/cwx/notes/chap01/tropo.html

- Tropopause is where temperature stops decreasing. Generally at pressure levels of about 300 mbar but can be as low as 500 mbar.
- Sometimes term “tropospheric delay” used but this is only about 70% of delay.
- Generally by height of 50-100km all of atmospheric delay accounted for.
- Troposphere is where weather system occur and aircraft fly on the tropopause.

Refractivity of air

- Air is made up of specific combination of gases, the most important ones being oxygen and nitrogen.
- Each gas has its own refractive index that depends on pressure and temperature.
- For the main air constituents, the mixing ratio of the constituents is constant and so the refractivity of a packet of air at a specific pressure and temperature can be defined.
- The one exception to this is water vapor which has a very variable mixing ratio.
- Water vapor refractivity also depends on density/temperature due to dipole component.

Refractivity of air

- The refractivity of moist air is given by:

$$N = k_1 \underbrace{\frac{P_d}{T} Z_d^{-1}}_{\substack{\text{Density of} \\ \text{dry air}}} + k_2 \underbrace{\frac{P_w}{T} Z_d^{-1}}_{\substack{\text{Density of} \\ \text{water vapor}}} + k_3 \underbrace{\frac{P_w}{T^2} Z_d^{-1}}_{\substack{\text{Dipole compoent} \\ \text{of water vapor } \rho/\text{T}}}$$

$$k_1 = 77.60 \pm 0.05 \ \ \text{K/mbar}$$

$$k_2 = 70.4 \pm 2.2 \ \ \text{K/mbar}$$

$$k_3 = (3.730 \pm 0.012) \times 10^5 \ \ \text{K}^2/\text{mbar}$$

- For most constituents, refractivity depends on density (ie., number of air molecules). Water vapor dipole terms depends on temperature as well as density

Refractivity in terms of density

- We can write the refractivity in terms of density:

$$N = k_1 \frac{R}{M_d} \rho + \left(\frac{k'_2}{T} + \frac{k_3}{T^2} \right) P_w Z_w^{-1}$$

$$k'_2 = k_2 - k_1 M_w / M_d = 22.1 \pm 2.2 \quad \text{K/mbar}$$

- Density ρ is the density of the air parcel including water vapor. R is universal gas constant, M_d and M_w are molecular weights. Z_w is compressibility (deviation from ideal gas law) See Davis, J. L., T. A. Herring, and I.I. Shapiro, Effects of atmospheric modeling errors on determinations of baseline vectors from VLBI, *J. Geophys. Res.*, *96*, 643–650, 1991.

Integration of Refractivity

- To model the atmospheric delay, we express the atmospheric delay as:

$$D = \int_{atm} n(s)ds - \int_{vac} ds \approx m(\varepsilon)\int_{Z}^{\infty}(n(z)-1)dz = m(\varepsilon)\int_{Z}^{\infty}N(z)\times 10^{-6}dz$$

- Where the *atm* path is along the curved propagation path; *vac* is straight vacuum path, z is height for station height Z and m(ε) is a mapping function. (Extended later for non-azimuthally symmetric atmosphere)
- The final integral is referred to as the ”zenith delay”

Zenith delay

- The zenith delay is determined by the integration of refractivity vertically.
- The atmospheric is very close to hydrostatic equilibrium meaning that surface pressure is given by the vertical integration of density. Since the first term in refractivity depends only on density, its vertical integration will depend only on surface pressure. This integral is called the “zenith hydrostatic delay (ZHD)”. (Often referred to as “dry delay” but this is incorrect because has water vapor contribution).

Zenith hydrostatic delay

- The Zenith hydrostatic delay is given by:

$$ZHD = 10^{-6} k_1 \frac{R}{M_d} g_m^{-1} P_s \approx 0.00228 \text{ m/mbar}$$

- Where g_m is mean value of gravity in column of air (Davis et al. 1991)
  g_m=9.8062(1-0.00265cos(2ϕ)-3.1x10^{-7}(0.9Z+7300)) ms^{-2}
- Ps is total surface pressure (again water vapor contribution included)
- Since Ps is 1013 mbar at mean sea level; typical ZHD =2.3 meters

Zenith wet delay

- The water vapor delay (second term in refractivity) is not so easily integrated because of distribution of water vapor with height.
- Surface measurements of water vapor pressure (deduced from relative humidity) are not very effective because it can be dry at surface and moist above and visa versa.
- Only effective method is to sense the whole column of water vapor. Can be done with water vapor radiometer (WVR) which infers water vapor delay from thermal emission from water vapor molecules and some laser profiling methods (LIDAR). Both methods are very expensive (200K$ per site)

Zenith wet delay

- In meteorology, the term “Precipitable water” (PW) is used. This is the integral of water vapor density with height and equals the depth of water if all the water vapor precipitated as rain (amount measured on rain gauge).
- If the mean temperature of atmosphere is known, PW can be related to Zenith Wet Delay (ZWD) (See next page)

PW and ZWD

- Relationship:

$$ZWD = 10^{-6}\frac{R}{M_w}(k'_2 + k_3/T_m)PW$$

$$T_m = \frac{\int P_w/T dz}{\int P_w/T^2 dz}$$

- The factor for conversion is ~6.7 mm delay/mm PW
- This relationship is the basis of ground based GPS meteorology where GPS data are used to determine water vapor content of atmosphere.
- ZWD is usually between 0-30cm.

Mapping functions

- Zenith delays discussed so far; how to relate to measurements not at zenith
- Problem has been studied since 1970's.
- In simplest form, for a plain atmosphere, elevation angle dependence would behave as 1/sin(elev). (At the horizon, elev=0 and this form goes to infinity.
- For a spherically symmetric atmosphere, the 1/sin(elev) term is "tempered" by curvature effects.
- Most complete form is "continued fraction representation" (Davis et al., 1991).

Continued fraction mapping function

- Basic form of mapping function was deduced by Marini (1972) and matches the behavior of the atmosphere at near-zenith and low elevation angles. Form is:

$$m(\varepsilon)=\cfrac{1}{\sin(\varepsilon)+\cfrac{a}{\sin(\varepsilon)+\cfrac{b}{\sin(\varepsilon)+\cfrac{c}{\sin(\varepsilon)+\cdots}}}}$$

Truncated version

- When the mapping function is truncated to the finite number of terms then the form is:

$$m(\varepsilon)=\frac{1+\dfrac{a}{1+\dfrac{b}{1+c}}}{\sin(\varepsilon)+\dfrac{a}{\sin(\varepsilon)+\dfrac{b}{\sin(\varepsilon)+c}}}$$

$$\text{when } \varepsilon=90; \quad m(e)=1$$

Davis et al. 1991 solved problem by using tan for second sin

Mapping functions

- Basic problem with forming a mapping function is determining the coefficient a,b, c etc for specific weather conditions.
- There are different parameterizations:
  - Niell mapping function uses a, b,c that are latitude, height and time of year dependent
  - MTT (MIT Temperature) model uses temperature as proxy for atmospheric conditions.
  - Recent Niell work uses height of 500mbar surface (needs to be determined from assimilation models).

Coefficients in mapping function

- The typical values for the coefficients are
- Hydrostatic:
  - a=1.232e-3, b=3.16e-3; c=71.2e-3
- Wet delay
  - a =0.583e-3; b=1.402e-3; c=45.85e-3
- Since coefficients are smaller for wet delay, this mapping function increases more rapidly at low elevation angles.
- At 0 degrees, hydrostatic mapping function is ~36. Total delay ~82 meter

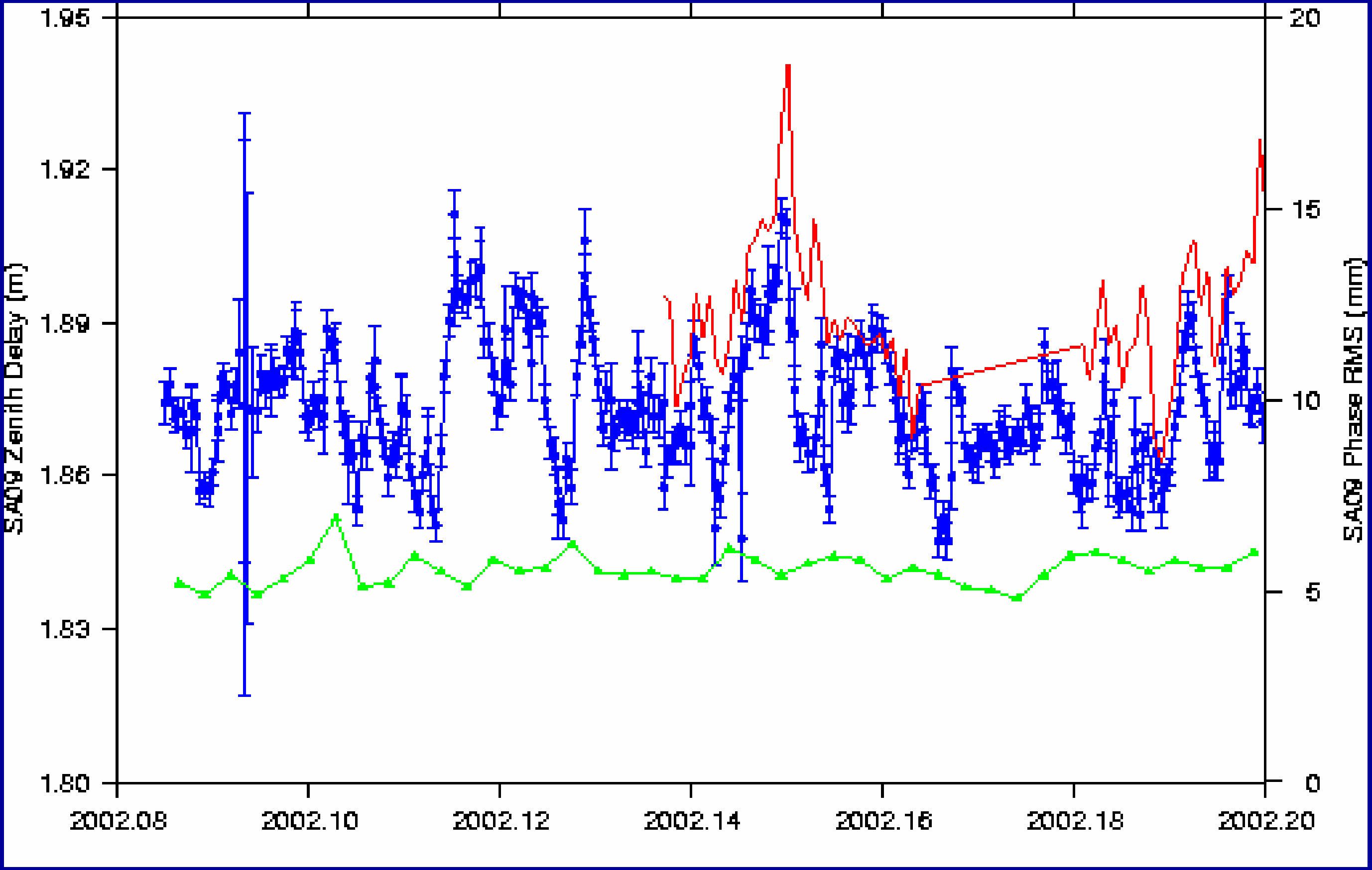
Effects of atmospheric delay

- If atmospheric zenith delay not estimated, then when data is used to 10 degree elevation angle, error in height is ~2.5 times zenith atmospheric delay error (see
  Herring, T. A., Precision of vertical position estimates from very–long–baseline interferometry, J. Geophys. Res., 91, 9177–9182, 1986.
- A simple matlab program can reproduce these results
- Herring Kalman filter paper also discusses effects of process noise value in height estimate uncertainty.

Parameterization of atmospheric delay

- Given the sensitivity of GPS position estimates to atmospheric delay, and that external calibration of the delay is only good to a few centimeters; atmospheric zenith delays and often gradients are estimated high-precision GPS analyses.
- Parameterization is either Kalman filter or coefficients of piece-wise linear functions (GAMIT)

Example using NCEP analysis field



Blue is GPS estimates of delay, red is NCEP calculation

Summary

- Atmospheric delays are one the limiting error sources in GPS
- In high precision applications the atmospheric delay are nearly always estimated:
  - At low elevation angles can be problems with mapping functions
  - Spatial inhomogeneity of atmospheric delay still unsolved problem even with gradient estimates.
  - Estimated delays are being used for weather forecasting if latency <2 hrs.
- Material today:
  - Atmospheric structure
  - Refractive index
  - Methods of incorporating atmospheric effects in GPS