12.520 Problem Set 4   **Due 10/25/06**

1) (90%) Consider a dry, noncohesive critical wedge, as discussed in class, as a model of snow in front of a snow plow. We will consider 4 cases: i) $\phi = 30°$, $\phi_b = 10°$; ii) $\phi = 30°$, $\phi_b = 30°$; iii) $\phi = 60°$, $\phi_b = 60°$; and iv) $\phi = 60°$, $\phi_b = 30°$.

a) List the important assumptions in the critical wedge model that we developed in class and comment on whether they might be appropriate for this example (i.e., snowplows as oppesed to mountain belts).

For the remainder of the problem, assume that the critical wedge model is OK (think of it as sand vs teflon beads, puched by a bulldozer, if that makes you happier.)

b) On the same graph, plot $\Phi(\psi)$ vs ψ both for $\phi = 30°$ and for $\phi = 60°$.

c) Plot stability diagrams (Figure 11, p 10 of notes) for cases i) - iv), all on the same graph.

d) On the plot you made for b), sketch the constructions and give values for ψ_0, ψ_b, α and $\alpha + \beta$ for the points approximately corresponding to points 1, 4, 5 & 8 in Figure 9 (p 10) of the notes.

Also, sketch the wedge geometries and orientations of the principal axes and slip planes for these points.

Armed with these plots, you can carry out some thought experiments. Assume that snow is a Coulomb material falling on a level surface ($\beta = 0$). A snowplow pushes the snow into a critical Coulomb wedge.

e) Assume $\phi = \phi_b = 30°$ ("Fairy dust" snow). What slope does the wedge develop?

f) Assume $\phi = \phi_b = 60°$ ("Sierra cement" snow). What slope does the wedge develop?

g) Suppose initially $\phi = \phi_b = 30°$. The plow builds a wedge of snow, then stops for a (hot) afternoon, during which time its properties change to $\phi = \phi_b = 60°$. What happens to the wedge? What happens when the plow starts moving again?

h) Suppose the opposite - initially $\phi = \phi_b = 60°$, but cold dry weather causes it to change to $\phi = \phi_b = 30°$. What happens?

i) Suppose $\phi = \phi_b = 60°$ and the plow builds a wedge. It then hits a patch of ice, $\phi_b = 30°$. What happens?

j) Suppose the opposite. The plow starts on ice, $\phi = 60°$; $\phi_b = 30°$, then moves on to snow, $\phi_b = 60°$. What happens?
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k) For a given ϕ, β , and $\phi_b < \phi$, how does the wedge decide which branch of the solution plot to lie on? What happens to minor pertubations in slope near the critical value(s)? Do they grow or die out?

l) Using these thought experiments, and others that might come to mind (e.g., building a wedge of snow by dumping from above using a conveyor belt), critically assess the critical wedge model.

Does increasing the basal friction always increase the taper, whereas increasing the strength of the wedge always decrease it?
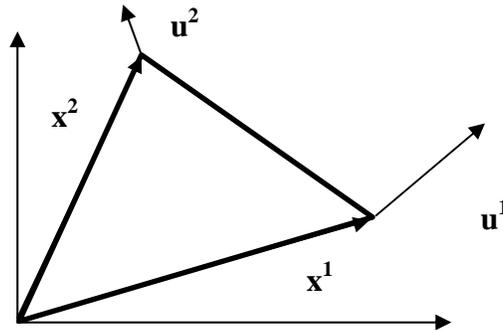
Can you give any insight into the paradoxes mentioned in class, with high ϕ giving steep sand piles, but gentle snow wedges?

2) (10%) In class we discussed the 2-D deformation gradient tensor, using a triangle of GPS sites. In the next problem set, we will calculate the deformation gradient tensor over a large network of GPS sites by dividing the large network into a number of triangular subnetworks, each consisting of 3 GPS sites, assuming that the deformation gradient tensor is constant within each triangle. In preparation, you will need to derive the relationships between the local deformation gradient tensor, d_{ij}, and the relative displacements, u_i, and coordinates, x_j of the 3 sites.

For simplicity, place one site (site 0) at the origin. Site 1 has coordinates (x_1^1, x_2^1) and displacements (u_1^1, u_2^1) relative to the origin. Site 2 has coordinates (x_1^2, x_2^2) and displacements (u_1^2, u_2^2) relative to the origin.

a) Write out the set of linear equations relating d_{ij} to $x_1^1, x_2^1, x_1^2, x_2^2, u_1^1, u_2^1, u_1^2$, and u_2^2.

b) Is this set of equations underdetermined, determined, or overdetermined?

2) (20%) Just NE of Los Angeles, the San Andreas fault trends approximately N65°W - S65°E. To within observational error, the displacement gradient there is observed to be (each year):

$$\begin{bmatrix} 0.15 & 0.24 \\ 0.00 & -0.15 \end{bmatrix}$$

where x_1 is East, x_2 is North, and the units are 10^{-6} strain.

Write the (two dimensional), strain tensor, the rotation tensor, and the areal dilatation.

What are the directions of maximum principal compression and extension?

Is this what you expect, if the San Andreas is a strike-slip fault?

3) (20%) Turcotte & Schubert, Problem 2-25. (Beware of typo in eq 2.132 - derive this equation yourself.)

4) (20%) Turcotte & Schubert, Problem 2-27.

4) (40%) Consider the homogeneous deformation:

$x' = ax + by$ $x = (dx' - by')/(ad - bc)$

$y' = cx + dy$ $y = (ay' - cx')/(ad - bc)$

a) Write expressions for the displacement vector **u** that takes $(x,y) \rightarrow (x',y')$, both in terms of (x,y) and x', y'.

b) Write expressions for E_{ij} (Lagrangian strain tensor), e_{ij} (Eulerian strain tensor), Ω_{ij} (Lagrangian rotation tensor), ω_{ij} (Eulerian rotation tensor).

These can be applied incrementally over a time t, $0 \le t \le 1$, by setting

$a \rightarrow 1 + t(a - 1)$, $b \rightarrow bt$, $c \rightarrow ct$, $d \rightarrow 1 + t(d - 1)$.

c) Write expressions for $E_{ij(t)}$ and $\dot{\varepsilon}_{ij}(t)$, where $\dot{\varepsilon}_{ij}(t)$ is the instantaneous Cauchy strain rate tensor.

Does

$$E_{ij}(t{=}1) = \int_0^1 \dot{\varepsilon}_{ij}(t)\, dt$$

Why or why not?

Consider the 2 following finite strains:

     1) $x' = x + 1.5y$ 2) $x' = 2x$

        $y' = y$ $y' = y/2$

d) What special strains do they represent?

e) Write $E_{ij}(t{=}1)$, $e_{ij}(t{=}1)$, $\Omega_{ij}(t{=}1)$, $\omega_{ij}(t{=}1)$, and $\varepsilon_{ij}(t{=}1)$ for these cases.

f) Calculate the principal axes for $E_{ij}(t{=}1)$, $\dot{\varepsilon}_{ij}(t{=}1)$, and $e_{ij}(t{=}1)$ for both cases.

g) In their final stage, pure and simple shears can be simply related by a rotation. Yet pure shear has 2 equivalent directions, while simple shear has only 1.

  Materials such as ice or olivine develop preferred fabrics when subjected to simple shear. They recrystallize under deviatoric *stress* with easy glide at 45° to the maximum

compressive *stress*. Since the stresses for pure and simple shear are equivalent, how can pure shear lead to 2 preferred directions, while simple shear leads to only 1?