12.520 Lecture Notes 9

Mohr's Circle for Strain

Lecture 5 explained a simple and convenient way to find the stress on an arbitrary plane given the stress tensor σ_{ij}. The technique involved writing equations for how the shear stress and normal stress on the $\hat{x}_i$ plane vary when the coordinate system is rotated to $\hat{x}_i'$. These equations plotted as Mohr's circle in stress space (σ,τ) and gave the tractions on plane x_i' at angle θ to the most compressive principle stress.

Since strain is a second-order tensor like stress, the same technique can be applied. Equations for the normal strain and shear strain on a plane at angle θ to the most compressive principle strain may be derived in the same way the equations for stress were derived. Consider the following transformation of coordinates:

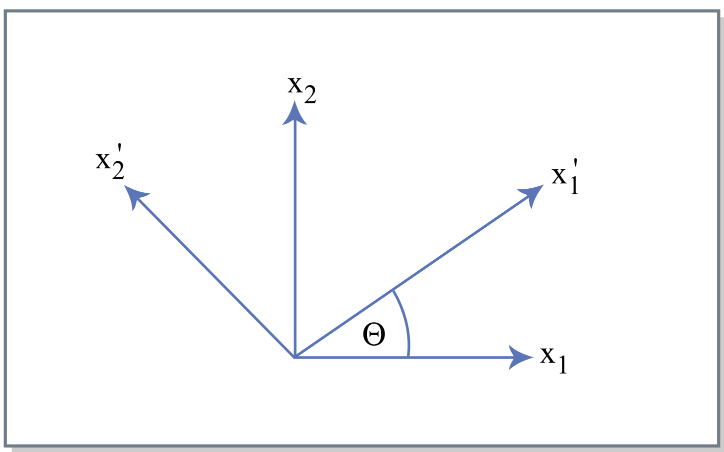

Figure 9.1
Figure by MIT OCW.

The strain tensor ε in the $\hat{x}_i$ coordinate system is transformed to the strain tensor ε' in the $\hat{x}_i'$ coordinate system by the equation

$$\underline{\underline{\varepsilon}}' = \alpha \underline{\underline{\varepsilon}} \alpha^T$$

where the double underbars denote second-rank tensors, α represents the transformation matrix, and the superscript T denotes the transpose of matrix α. See Lecture 5 for the derivation of this equation. Since the coordinate system is rotated about the $\hat{x}_3$ axis, the transformation matrix is

$$\alpha = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The equations for the normal strain and shear strain on the x_1' plane in the new coordinate system are

$$\varepsilon_{11}' = \frac{\varepsilon_{11} + \varepsilon_{22}}{2} + \frac{\varepsilon_{11} - \varepsilon_{22}}{2}\cos 2\theta + \varepsilon_{12}\sin 2\theta$$
$$\varepsilon_{22}' = \frac{\varepsilon_{11} + \varepsilon_{22}}{2} + \frac{\varepsilon_{11} - \varepsilon_{22}}{2}\cos 2\theta - \varepsilon_{12}\sin 2\theta$$
$$\varepsilon_{12}' = \frac{-(\varepsilon_{11} - \varepsilon_{22})}{2}\sin 2\theta + \varepsilon_{12}\cos 2\theta$$

The derivation for these equations follows the derivation for the Mohr's circle equations of stress in Lecture 5.