Class 10: Joint Geophysical Inversions
Wed, December 1, 2009

- Invert multiple types of data residuals simultaneously
- Apply soft mutual constraints: empirical, physical, statistical
- Deal with data in the same magnitude order

This class introduces high-end research topics for performing joint geophysical inversions with multiple types of data involved, and inverted simultaneously. In particular, joint seismic traveltimes (first arrivals) and gravity inversion.

Joint Inversion Approach

The data to be minimized in a least-square sense consist of:

- Multiple geophysical domains data residuals
- Cross parameter constraints (empirical, physical, statistical; defined as soft constraints)

The objective function for this Joint Inversion problem can be formulated following the theory explained in Tarantola (2003).

We deal with an expanded model vector:

$$m = [m_1 m_2 m_3 \ldots]^T$$

where m1, m2 and m3 are respectively, models of different geophysical domains (for the purposes of the project we would use only seismic-gravity). The objective function for Joint Inversion can be written as (Colombo & De Stefano, 2007):

$$\Phi = r^T V^{-1} r + \lambda_1 m^T L^T L m + \lambda_2 m^T H m + \lambda_3 m^T W m$$

where the first term is the sum of the squared data misfit normalized by their variances (*r=d-G(m)*) or the data residuals, the second term is the model smoothness where Lm approximates the Laplacian of the model integrated over the entire model area. The third term is a regularizing equation that encourages smoothness (i.e. structural or pattern similarity) between models of different nature. The fourth term is introduced to account for the empirical parameter relationships (e.g. Gardner relation or other). Weighting terms are used throughout the inversion process to balance the influence of gravity vs. seismic in the inversion and/or to balance the influence of different cross-parameter regularization terms.

The structural similarity among models is imposed by a cross-gradient function generalized to a 3D case (Gallardo & Meju, 2004):

$$|\mathbf{t}(x,y,z)|^2 = |\nabla m_1(x,y,z) \times \nabla m_2(x,y,z)|^2$$

where m1 and m2 are two models (e.g., velocity and density). The whole objective function is minimized with respect to the multi-parameter model vector (Colombo & De Stefano, 2007).

The detailed objective function will serve the purpose of constraining the “shapes” and the “parameters properties” at the same time.

Seismic Traveltime Inversion

The seismic first-arrival traveltime inversion is to minimize the following objective function:

$$\Phi = r^T V^{-1} r + \lambda_1 m^T L^T L m \qquad (3)$$

Where the first two terms are same with those in equation (1), but here the data residuals *r* are only for traveltimes.

The forward modeling program for calculating 2D or 3D first-arrival traveltimes applies graph theory developed by Moser (1991). Given an arbitrary source point, it shall expand the wavefront by Huygens principle using a graph template. Along the wavefront, find the minimum time to be a new secondary source, and then expand from the new source using the template again. The approach is unconditional stable, while other methods often fail. For example, eikonal solver suffers from caustics when velocity contrast is larger than $\sqrt{2}$. The key issue is how to achieve high-order accuracy in calculation at the same time minimizing computation time. The approach may follow Zhang and Toksöz (1997), in which sorting for determining the minimum time along the wavefront is avoided by using an interval method to store traveltimes.

One of the flexibilities with graph theory is that the wavefront raytracer is valid for any model mesh. Therefore, it may be easy to incorporate multiple types of geophysical inversions with the same grid mesh.

The traveltime tomography process applies the Conjugate Gradient inversion in a parallel fashion. After a set of forward calculations is completed, it stores raypaths locally on each node, and the inversion shall access the raypaths with the shots calculated on that node. Using Conjugate Gradient method does not require an explicit matrix for inversion, and thus it save memory tremendously.

Gravity Forward and Inversion problems

The gravity inversion problem is affected by inherent non-uniqueness. For such data there are infinite possible distributions for density, which can fit the observed data. The inversion of potential field data such as gravity is therefore only possible by introducing variable degrees of prior information and by imposing regularizing equations to the solution such to obtain physically meaningful data.

Model parameterization is performed by means of a mesh of rectangular cells of constant density. The approach followed to solve the gravity inversion problem is similar to that of Li and Oldenburg (1998). Modifications consist of the introduction of limiting functional to the density values imposed during the inversion and the definition of the model covariance matrix.

Gravity Model Parameterization

The most appropriate parameterization of the model is though a mesh of rectangular cells of constant density. The portions of the model outside the investigation area are padded with similar cells to account for border effects. The horizontal boundaries of the inversion area are defined by the location of the outermost measuring points.

The mesh should be adaptive to account for the different resolution achievable by variable spatial sampling of the measurements and the decay of the gravity field versus depth. Three vectors are defined to describe the mesh dimension in the three Cartesian coordinates x, y, z.

A sensitivity analysis is envisaged to determine the effect of the described model parameterizations on simple geometrical shapes such as a sphere. Trade off analysis of the size of the mesh versus the radius of the sphere as well as versus the density contrast and depth of the sphere would provide the guidelines for automatic generation of gravity inversion meshes.

Gravity Forward Problem

The data we are dealing with are the vertical components of the total gravity field measured from the earth surface. These values represent the effects of the combination of an ambient field and the field produced by an anomalous distribution of mass in the subsurface. By numerical processing the ambient field is removed from the data leaving only the anomalous field. Task of the inversion is therefore the reconstruction in the subsurface of the anomalous mass distribution.

The vertical component of the gravity field produced by the density $\delta(x, y, z)$ is given by:

$$F_z(\vec{r}_0) = \gamma \int_V \delta(\vec{r}_0) \frac{z - z_0}{\left|\vec{r} - \vec{r}_0\right|^3} dv$$

where $\vec{r}_0$ is the vector denoting the observation location and $\vec{r}$ is the source location. V represents the volume of the anomalous mass, and γ is the gravitational constant (Li and Oldenburg, 1998). The Cartesian system is right-handed with origin on the earth's surface and z pointing vertically downward.

The gravity forward algorithm is based on the principle of superimposition of the effects. The density model is divided in cells each with the shape of a right rectangular prism. The gravitational field on a particular point of the space is given by the sum of all the gravitational fields due to all the prisms of the model, computed in that point. The exact analytical expression of the gravitational attraction of a right rectangular prism is described by Haaz (1953), Nagy (1966) and Okabe (1979).

Gravity Inverse Problem

The inversion algorithm is based on the iterative least squares minimization of an objective function of the form:

$$\phi\left(\mathbf{m},\mathbf{d}^{obs}\right)=\phi_m(\mathbf{m})+\phi_d\left(\mathbf{d}^{obs}\right)$$

where $\phi_d(\mathbf{d}^{obs})$ is a misfit function and $\phi_m(\mathbf{m})$ is a regularization function, **m** is the vector of model parameters (each component of **m** is the density of a particular cell of the model), $\mathbf{d}^{obs}$ is the vector of observed data (gravity measurements at the station positions). Furthermore,

$$\phi_d\left(\mathbf{d}^{obs}\right)=\left\|\mathbf{W}_d\left(\mathbf{d}-\mathbf{d}^{obs}\right)\right\|^2$$

where **d** is the vector of data, computed by the forward algorithm with the current model and $\mathbf{W}_d$ is the *Cholesky factorization of the inverse data covariance matrix*. If **d** has N components, $\phi_d(\mathbf{d}^{obs})$ is distributed as a $\chi^2(N)$ and $E[\chi^2(N)]=N$ provides a target misfit for the inversion.

The regularization function $\phi_m(\mathbf{m})$ takes the form:

$$\begin{aligned}\phi_m(m)=\ &\alpha_s\int_V w_s\left\{w(z)\left[m(\mathbf{r})-m_0\right]\right\}^2 dv+\\ &\alpha_x\int_v w_x\left\{\frac{\partial w(z)\left[m(\mathbf{r})-m_0\right]}{\partial x}\right\}^2 dv+\\ &\alpha_y\int_v w_y\left\{\frac{\partial w(z)\left[m(\mathbf{r})-m_0\right]}{\partial y}\right\}^2 dv+\\ &\alpha_z\int_v w_z\left\{\frac{\partial w(z)\left[m(\mathbf{r})-m_0\right]}{\partial z}\right\}^2 dv\end{aligned}$$

where α_i are weights which affect the relative importance of the different components of the objective function, w_i are 3D weighting functions and $w(z)$ is a depth weighting function of the form:

$$w(z)=\frac{1}{\left(z+z_0\right)^{\beta/2}}$$

where parameters z_0 and β have to be calibrated. The function $\omega(z)$ is used to counteract the decay of the gravity kernel with depth. If no depth weighting is used, the inversion will tend to concentrate all the mass anomalies near to the surface.

The matricial form of $\phi(\mathbf{m},\mathbf{d}^{obs})$, discretization of $\phi_m(\mathbf{m})$ and calibration of $w(z)$ are all explained in detail in Li & Oldenburg (1996) and (1998).

In addition to the algorithm of Li & Oldenburg, the model covariance matrix (Tarantola, 2003), can be introduced in the linearized system. This allows setting constraints when prior knowledge of correlation between cells is available. In this matrix the principal diagonal contains variances of cell densities, while the off-diagonal elements are filled with co-variances between model parameters. If a prior knowledge of the model is available, the user can set the variances of cells and their correlation coefficients with other cells of the model (e.g. using a model mask: a reference pattern, a refracted migrated image, etc.). Co-variances are then calculated between selected cells and the off diagonal elements of the model covariance matrix are filled-in.

The linearized system of equations to be inverted in the least squares sense takes the form:

$$\begin{bmatrix} \mathbf{W}_{\phi} \\ \mathbf{W}_{\mathbf{M}} \\ \mathbf{W}_{d}\mathbf{G} \end{bmatrix} \Delta\mathbf{m} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{W}_{d}\Delta\mathbf{d} \end{bmatrix}$$

kg/m3

where $\mathbf{W}_{\phi}$ is the matricial form of $\phi_m(\mathbf{m})$, $\mathbf{W}_M$ is the *Cholesky factorization of the inverse model covariance matrix*, $\Delta\mathbf{m} = (\mathbf{m}\text{-}\mathbf{m}_{pri})$ (“pri” stands for “prior model” or “initial model”) and $\Delta\mathbf{d} = (\mathbf{d}_{obs}\text{-}\mathbf{d}_{cal})$ ($\mathbf{d}_{cal}$ is the data vector computed from the prior model with the forward algorithm). **G** is the *forward matrix* and

$$\mathbf{d} = \mathbf{G} \cdot \mathbf{m}$$

is the matricial expression of the forward process.

With this formulation negative or large values of density can arise from the inversion process. For this reason parameter functionals varying only between user assigned density values should be introduced in the inverse problem. Density can then be recovered by reverse transformation after the inversion.

The above concepts are demonstrated numerically and displayed by Figure 1 to

Figure 6.

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Figure 1. Cross-section of synthetic model 3D (background: δ=0kg/m^3, cube: δ=1000kg/m^3).

Figure 2. Gravity field produced by the synthetic model in Figure 1 plus noise.

Figure 3. Inversion results if no constraints are imposed to the inversion.

Figure 4. Inversion results after imposing the w(z) weighting function to counteract the natural decay of the gravity field.

Figure 5. Inversion results after imposing the w(z) weighting function and imposing upper and lowed bounds to the density values from the inversion.

Figure 6. Inversion results obtained after imposing the cross-correlation of the model cells. The density values were uniformly set to 200kg/m^3 in the starting model.

Joint Inversion workflows

The Joint Inversion workflow can be represented as per Figure 7 (below). The workflow is extended to the inclusion of an additional geophysical method (other) and to the inversion of the post-migration domain residuals (i.e. CIG: Common Image Gather residuals). The two described extensions of the workflow can be considered as future additional developments after the Seismic travel-time / Gravity JI.

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Figure 7. General Joint Inversion Workflow with extension to: 1) an additional methodology (i.e. horizontal integration); 2) include tomography of image gathers.

Joint Seismic and Gravity Inversion Examples

Numerical example (Colombo et al., 2009):

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Real-Data Examples:

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With seismic refraction statics applied:

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With statics from joint seismic and gravity inversion:

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