**12.800 Fall 2004: Problem set 2**

1) A two-dimensional steady flow as velocity components

$$u = y \qquad v = x.$$

Show that the streamlines are rectangular hyperbolas

$$x^2 - y^2 = const$$

Sketch the flow pattern, and demonstrate that the flow is irrotational.
[KC04: Chapter 3, exercise 1, page 73]

2) Consider a steady axisymmetric flow of a compressible fluid. The equation of continuity in cylindrical coordinates $(R, \varphi, x)$ is

$$\frac{\partial}{\partial R}(\rho R u_R) + \frac{\partial}{\partial x}(\rho R u_x) = 0.$$

Show how we can define a streamfunction so that the equation of continuity is satisfied automatically.
[KC04: Chapter 3, exercise 2, page 73]

3) The velocity components in an unsteady plane flow are given by

$$u = \frac{x}{1+t} \qquad v = \frac{2y}{2+t}.$$

Describe the path lines and the streamlines. Note that path lines are found by following the motion of each particle, that is, but solving the differential equations

$$\frac{dx}{dt} = u(\underline{x}, t) \qquad \frac{dy}{dt} = v(\underline{x}, t),$$

subject to $\underline{x} = \underline{x}_0$ at $t = 0$.
[KC04: Chapter 3, exercise 6, page 74]

4) A fluid blob has the velocity:

$$\underline{u} = 2xy\underline{i} + 4y^2\underline{j}.$$

Calculate the normal and shear rates of strain, and determine if the fluid is rotating at the point $x = 2$, $y = 1$.

5) Consider a plane Couette flow of a viscous fluid confined between two flat plates at a distance *b* apart. At steady state the velocity distribution is

$$u = \frac{Uy}{b} \qquad v = w = 0$$

where the upper plate at $y = b$ is moving parallel to itself at speed *U*, and the lower plate is held stationary. Find the rate of linear strain, the rate of shear strain, and vorticity. Show that the streamfunction is given by

$$\psi = \frac{Uy^2}{2b} + const$$

[A variant of KC04: Chapter 4, exercise 11, page 127]