Exercises

7.1 Calculate u_M (the distribution of u associated with constant M in Equation 7.22) as a function of ϕ for $u_M(0) = 0$.

7.2 Hide's theorem states that in a symmetric circulation a maximum in angular momentum can only exist at the surface in a region of surface easterlies. Moreover, the absolute maximum of angular momentum will be $M = \Omega a^2$, the value associated with zero relative flow at the equator. Using a similar derivation, what can be said about the existence of a minimum in angular momentum in a symmetric circulation?

7.3 Will the simplified approach to calculating zonal wind given by Equations 7.29 and 7.46 necessarily yield adherence to Hide's theorem?

7.4 Derive the equations for the simple model of asymmetric Hadley circulation.

7.5 Try to incorporate the possibility that the angular momentum in the upper, outward flow in the Hadley cells is characteristic of a broad upward flowing region as in Figure 7.6.

This exercise is fairly open ended, but it can be approached relatively simply. Needless to say, I don't expect a truly analytic solution, but one can get fairly far with relatively simple calculations. Let's focus on the simpler, symmetric case. Let ϕ_m be the latitude separating rising streamlines from descending stream lines. Let the upper poleward branch of the Hadley circulation consist in a well mixed bundle of streamlines, whose angular momentum is characteristic of the angular momentum of the streamlines that have risen up to the latitude in consideration: i.e., for $\phi < \phi_m$,

$$M_b(\phi) \approx \frac{\int_0^\phi M(\phi)\cos\phi d\phi}{\int_0^\phi \cos\phi d\phi}$$

where $M_b(\phi)$ is the angular momentum of the upper bundle of streamlines, and $M(\phi)$ is the angular momentum of a streamline originating at the surface at latitude ϕ. For simplicity, we assume that there are equal updrafts per unit area throughout the upwelling region. For $\phi > \phi_m$,

$$M_b(\phi) \approx \frac{\int\limits_0^{\phi_m} M(\phi)\cos\phi d\phi}{\int\limits_0^{\phi_m} \cos\phi d\phi}.$$

The above allow us to calculate $U(H)$ and $\bar{\Theta}$ as in the original argument, and to evaluate ϕ_H and $\bar{\Theta}(0)$. For purposes of estimation, we might choose $\phi_m \approx 0.5\phi_H$. One might have to do a bit of iteration in order to get everything to work out. At this stage, you might want to see how well you are replicating Figure 7.6, and discuss remaining differences.

The asymmetric case is treated similarly, but is more complicated because the changes wrought by considering the bundle of streamlines are much greater so that iteration becomes more cumbersome. However, there is not too much trouble in estimating how much the easterlies will be reduced at the equator, and how the surface winds will be changed.

As you play with the problem in the above fashion, you will develop a better idea of how to do the problem correctly,' but the above should suffice for the homework.