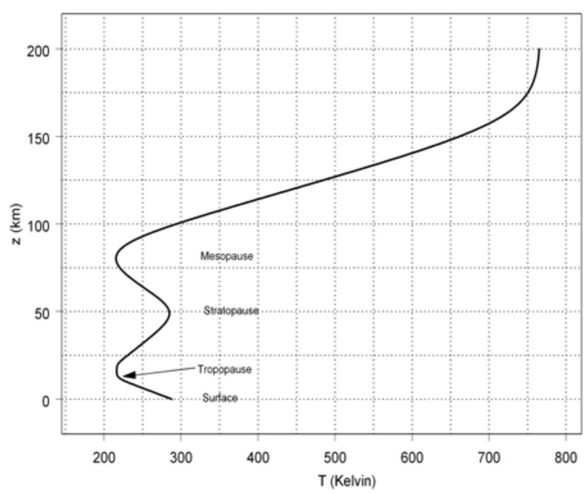
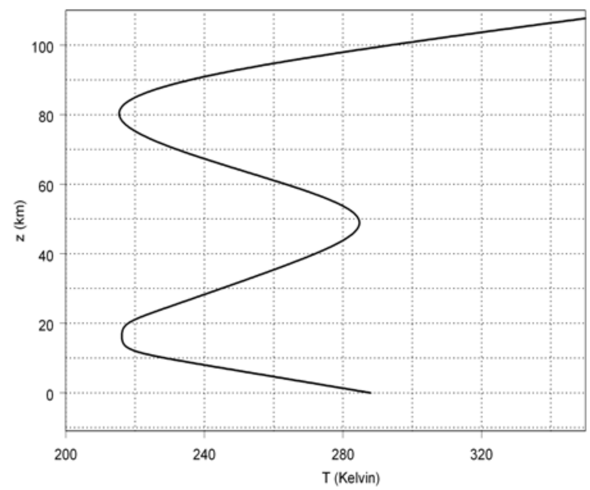
Exercises

1. Equation 8.56, when Q is constant and $J = 0$, has two solutions proportional to e^{iQz} and e^{-iQz}, respectively. One corresponds to an upward propagating wave, while the other corresponds to a downward propagating wave. Requiring that the solution consist only in the former as $z \to \infty$ constitutes the radiation condition. Show how to determine which solution is which.

2. The following figure shows a characteristic profile for temperature for the atmosphere up to the thermosphere. The second figure shows the same profile, but only up to about 100 km. Use Equation 8.57 to obtain an analytic approximation to this temperature profile.





3. Equation 8.56 can be viewed as an equation for the perturbation vertical velocity in a boussinesq fluid. For Q^2 constant, analytic solutions are trivially obtained. Let $Q = 2\pi/15$km, $w_B = 1$, and $J = 0$. Let $z_T = 60$km. For the upper boundary condition, consider both a rigid lid where $w_T = 0$, and an unbounded fluid with a radiation condition where $w \propto e^{iQz}$ or $dw/dz = iQw$. Obtain analytic solutions for these cases, and run the numerical model as well. Determine what happens when resolution is inadequate in each case, and determine how much resolution is sufficient.

4. Repeat the above with a rigid lid. From the analytic solution determine the values of Q which produce resonance. Use the numerical model to examine the impact of resolution on resonance.

5. Repeat Problem 2 with $w_B = 0$, the radiation condition at z_T, and $J = exp(-((z - 30\ km)^2/(12\ km)^2)$. Vary Q so that the ratio of the vertical wavelength to 12 km varies from about 0.2 to 5. Try to get an analytic solution, but even if you can't, run the numerical model (with adequate resolution) in order to see what happens to the wave amplitude at z_T.