Tropical Cyclone Inner Core Dynamics

Assumptions

- Axisymmetric flow
- Gradient and hydrostatic balance above PBL
- Troposphere neutral to slantwise moist convection outside eye
- Moist adiabatic lapse rate in eye above inversion
- Well developed anticyclone at storm top

Local energy balance (from previous lecture):

$$\frac{M}{r_b^2} \cong -\left(T_b - T_o\right)\frac{ds^*}{dM}$$

Definitions:

$$\frac{f}{2}R^2 \equiv M = rV + \frac{f}{2}r^2$$ “potential radius”

$$\chi^* \equiv \left(T_s - T_o\right)\left(s^* - s_a^*\right)$$

$$\chi \equiv \left(T_s - T_o\right)\left(s - s_a\right) \qquad \left(s_a = s_a^*\right)$$

$$\chi_s \equiv \left(T_s - T_t\right)\left(s_{0a}^* - s_a\right) = \text{constant}$$

Scaling:

$$\chi^*, \chi \rightarrow \chi_s\left(\chi^*, \chi\right)$$

$$R, r \rightarrow \frac{\sqrt{\chi_s}}{f}\left(R, r\right)$$

$$\beta \equiv \frac{T_s - T_o}{T_s - T_t}$$

Scaled equations:

$$\frac{1}{r^2} = -\frac{2\beta}{R^3}\frac{\partial \chi^*}{\partial R},$$

$$R^2 = 2rV + r^2 \simeq 2rV \qquad \text{(core)}$$

$$\rightarrow \boxed{V^2 \simeq -\frac{R\beta}{2}\frac{\partial \chi^*}{\partial R}} \qquad (1)$$

Conservation of angular momentum (dimensional):

$$\frac{dM}{dt} = -gr\frac{\partial \tau_\theta}{\partial p}$$

Integrate over depth of PBL:

$$\Delta p_b \frac{dM}{dt} = -gr\tau_{\theta s} \cong -gr\rho_s C_D V^2$$

$$\cong -gr\frac{p_s}{R_d T_s} C_D V^2$$

Scaling for time:

$$t \rightarrow C_D^{-1} \frac{R_d T_s}{g p_s} \Delta p_b \chi_s^{-1/2}\, t$$

Nondimensional angular momentum equation:

$$\frac{dR}{dt} = -r \frac{V^2}{R}$$

$$\text{But} \quad r \simeq \frac{R^2}{2V}$$

$$\rightarrow \boxed{\frac{dR}{dt} \simeq -\frac{1}{2} RV} \qquad (2)$$

Nondimensional PBL entropy equation:

$$\frac{d\chi}{dt} = \frac{C_k}{C_D} V\left(\chi_0^* - \chi\right) + \varepsilon V^3 - F_b, \qquad (3)$$

$$\varepsilon \equiv \frac{T_s - T_t}{T_s}$$

Time derivative in R space:

$$\frac{d}{dt} = \frac{\partial}{\partial \tau} + \dot{R}\frac{\partial}{\partial R} + \omega \frac{\partial}{\partial P}$$

Assume χ well mixed in boundary layer,

use $\dot{R} = -\frac{1}{2} RV$:

$$\frac{\partial \chi}{\partial \tau} = \frac{1}{2} RV \frac{\partial \chi}{\partial R} + \frac{C_k}{C_D} V \left(\chi_0^* - \chi \right) + \varepsilon V^3$$

$$\textit{But} \quad V^2 \simeq -\frac{R\beta}{2}\frac{\partial \chi^*}{\partial R} = -\frac{R\beta}{2}\frac{\partial \chi}{\partial R} \quad \textit{in eyewall}$$

$$\rightarrow \frac{\partial \chi}{\partial \tau} = \left[\frac{C_k}{C_D} V\left(\chi_0^* - \chi\right) - \left(1/\beta - \varepsilon\right) V^3 \right] \quad (4)$$

(Steady state solution:)

$$V^2 = \frac{1}{1/\beta - \varepsilon}\frac{C_k}{C_D}\left(\chi_0^* - \chi\right)$$

Differentiate (4) with respect to *R* and use $V^2 \simeq -\frac{R\beta}{2}\frac{\partial\chi}{\partial R}$:

$$\frac{\partial}{\partial\tau}\left(\frac{V}{\beta}\right) = \frac{1}{4}\frac{R}{V}\frac{\partial V}{\partial R}\left[3\left(1/\beta - \varepsilon\right)V^2 - \frac{C_k}{C_D}\left(\chi_0^* - \chi\right)\right]$$
$$-\frac{1}{4}V^2\left[2\frac{C_k}{C_D}\frac{1}{\beta} + \frac{R}{\beta^2}\frac{\partial\beta}{\partial R}\right]$$

First two terms: Propagation; Second term: damping;

Third term: Possible amplification

From previously derived dependence of outflow temperature on angular momentum:

$$\frac{\partial \beta}{\partial R} = -\frac{1}{T_s - T_t}\frac{\partial T_o}{\partial R} = -\frac{Ri_c}{r_t^2} R^3 \frac{\beta}{2V^2}$$

Thus development equation becomes

$$\frac{\partial}{\partial \tau}\left(\frac{V}{\beta}\right) = \frac{1}{4}\frac{R}{V}\frac{\partial V}{\partial R}\left[3\left(1/\beta - \varepsilon\right)V^2 - \frac{C_k}{C_D}\left(\chi_0^* - \chi\right)\right]$$

$$-\frac{1}{2}\frac{C_k}{C_D}\frac{1}{\beta}V^2 + \frac{1}{8}\frac{Ri_c}{r_t^2}R^4\beta$$

Note that first term steepens V gradient when $V^2 > \frac{1}{3}V_{max}^2$

$$\frac{\partial \beta}{\partial R} < 0 \quad necessary\ \ for\ amplification$$

V gradient cannot steepen indefinitely:

$$\varsigma = \frac{V}{r} + \frac{\partial V}{\partial r},$$

$$\frac{\partial}{\partial r} = \frac{\partial R}{\partial r}\frac{\partial}{\partial R} = \frac{r}{R}\left(1 + \frac{V}{r} + \frac{\partial V}{\partial r}\right)\frac{\partial}{\partial R} = \frac{r}{R}(1+\varsigma)\frac{\partial}{\partial R}$$

$$\rightarrow \varsigma = \frac{V}{r} + \frac{r}{R}(1+\varsigma)\frac{\partial V}{\partial R}$$

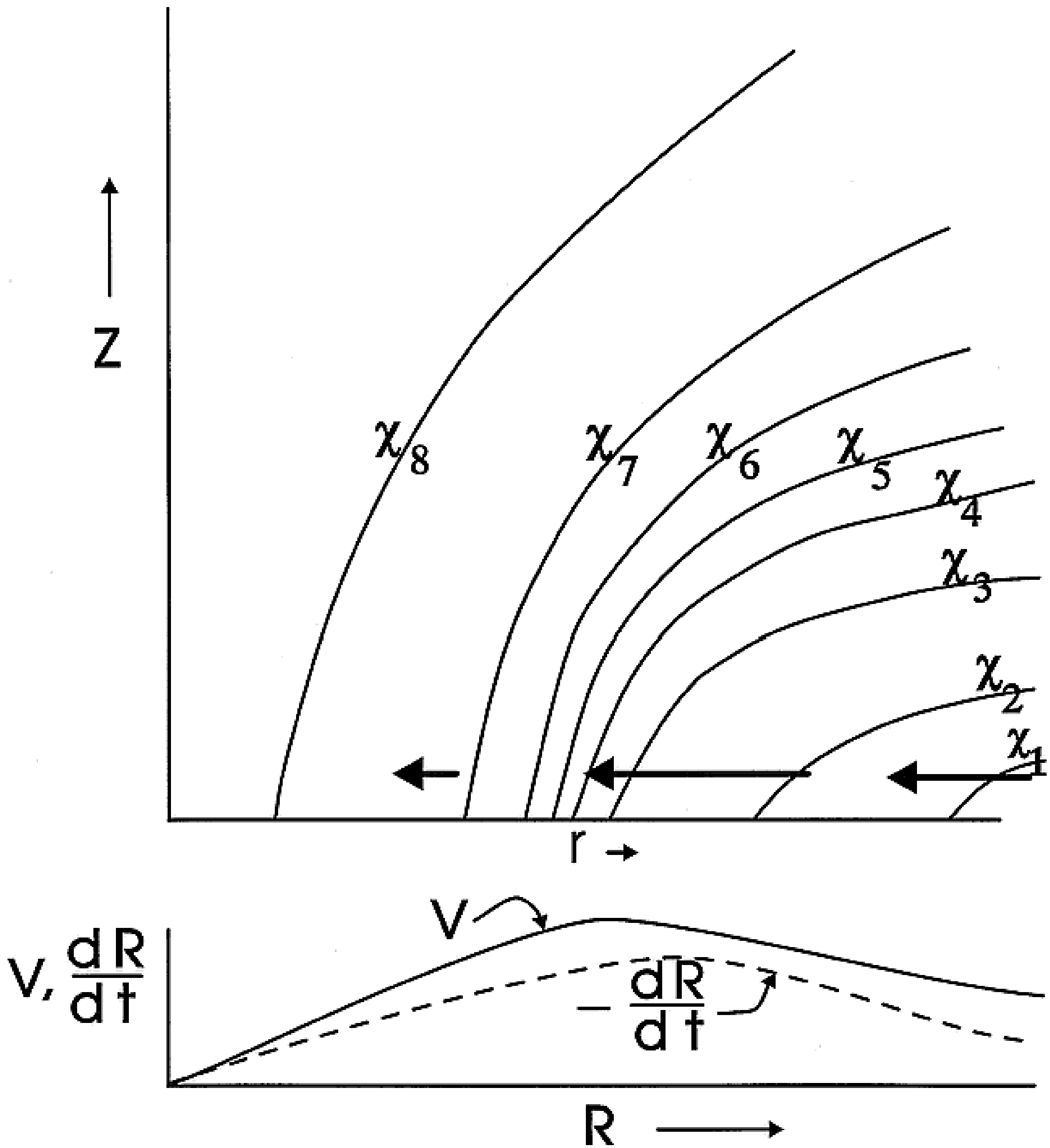
$$\varsigma = \frac{\frac{V}{r} + \frac{r}{R}\frac{\partial V}{\partial R}}{1 - \frac{r}{R}\frac{\partial V}{\partial R}}$$

$$\varsigma \to \infty \quad when \quad \frac{\partial V}{\partial R} \to \frac{R}{r} \cong \frac{2V}{R}$$

Eyewall undergoes frontal collapse!

This can only be prevented by 3-D eddies

$$\dot{R} \simeq -\frac{1}{2} RV$$



Simplified amplification model:

$$\frac{\partial \chi}{\partial \tau} = \left[\frac{C_k}{C_D} V \left(\chi_0^* - \chi \right) - \left(1 / \beta - \varepsilon \right) V^3 \right],$$

$$\chi_0^* = 1 - AP = 1 + A \left(\chi + \frac{1}{2} V^2 \right)$$

$$V^2 = -\frac{R\beta}{2} \frac{\partial \chi^*}{\partial R}$$

$$\frac{\partial \beta}{\partial R} = -\frac{1}{2} R^3 \beta \frac{Ri_c}{r_t^2} \frac{1}{V^2}$$

$$Enforce \quad \chi \leq \chi^*$$

How to handle frontal collapse in model?

Three methods:

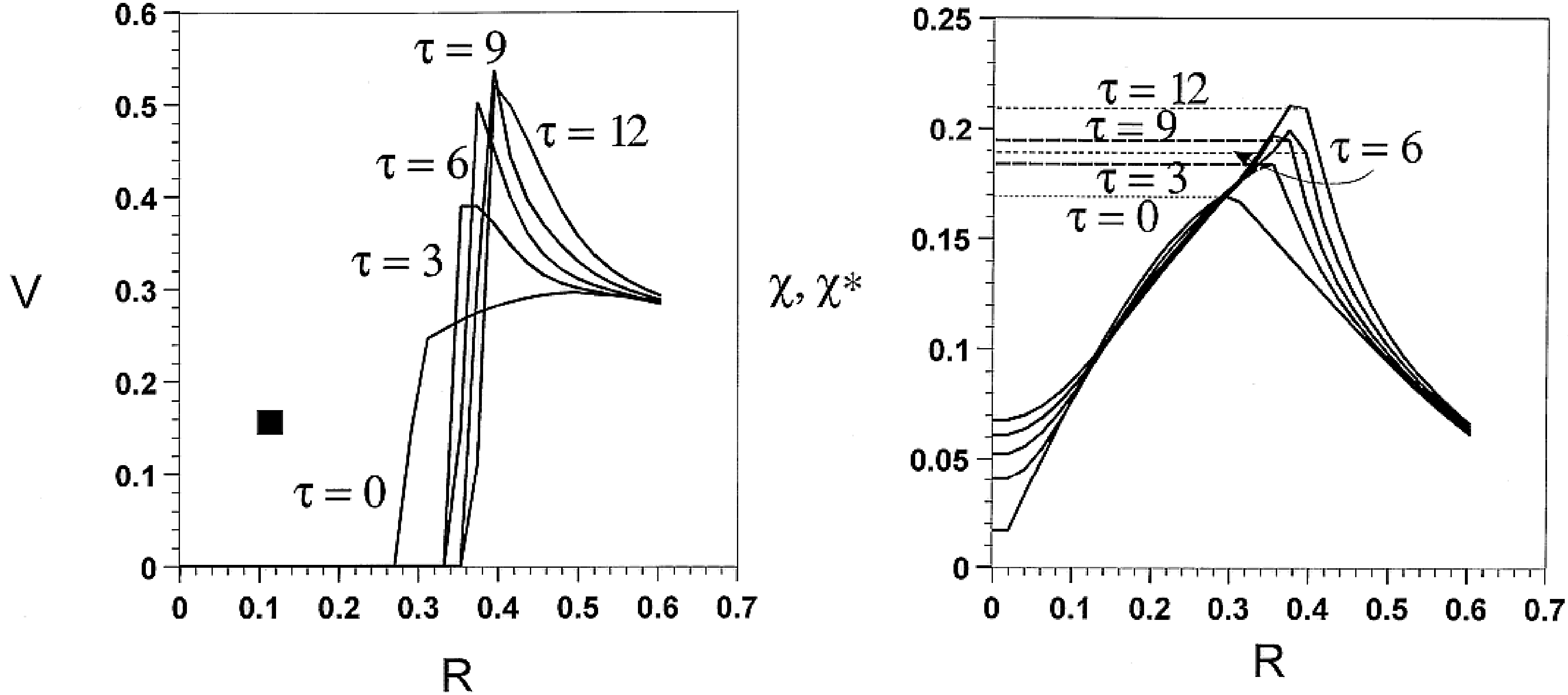
1: Zero diffusion model:

For $r < r_m$:

$$V = 0$$

$$\chi = \chi^* = constant$$

Does not prevent frontal collapse


V
R
τ = 0
τ = 3
τ = 6
τ = 9
τ = 12
χ, χ*
R
τ = 12
τ = 9
τ = 6
τ = 3
τ = 0

2. Minimum diffusion model: Just enough radial diffusion to prevent failure of coordinate transformation.

From expression for vertical component of vorticity, we enforce

$$\frac{\partial V}{\partial R} \leq \frac{2V}{R}$$

Inside the outermost radius, R_{crit}, where this is violated, we take

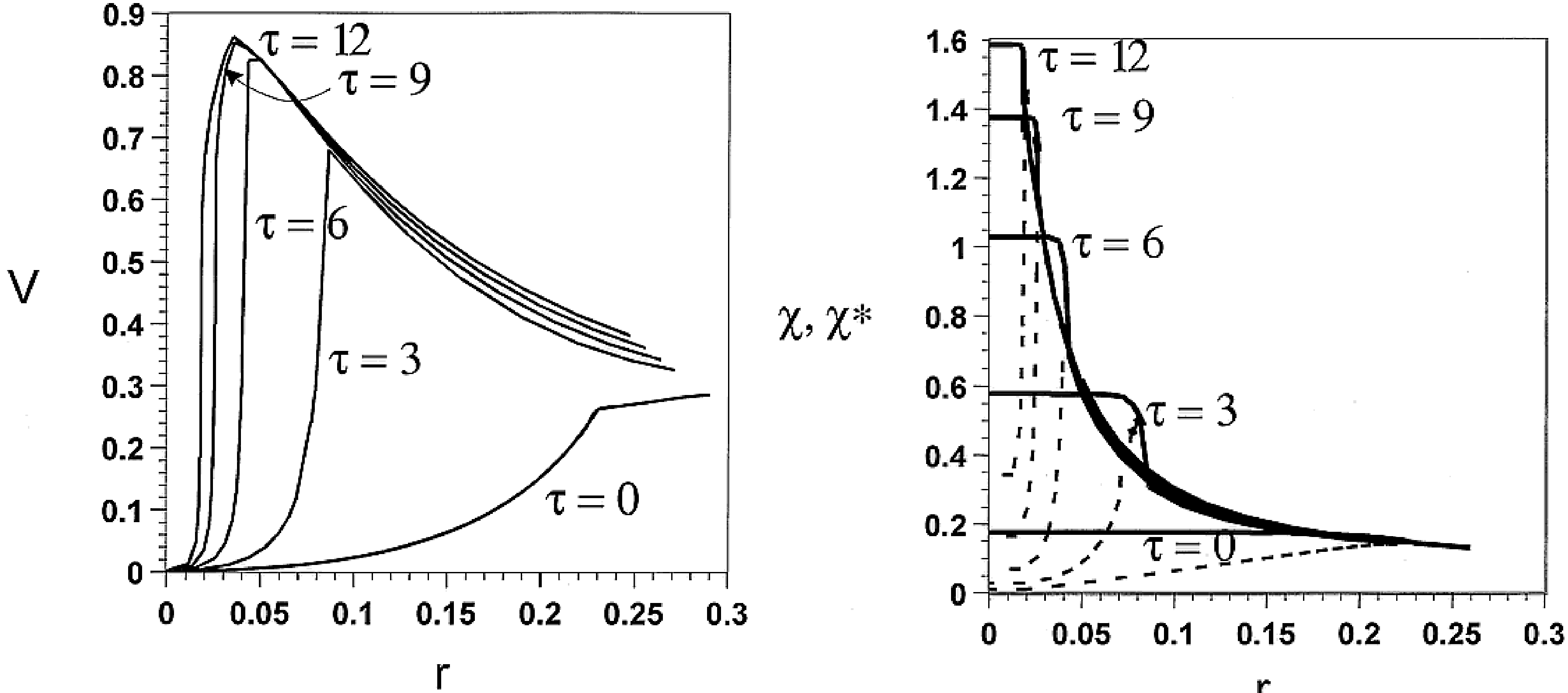
$$\frac{\partial V}{\partial R} = \frac{2V}{R}$$

$$\rightarrow V = V_{crit}\left(\frac{R}{R_{crit}}\right)^2$$

By integration of $$\frac{\partial \chi^*}{\partial R} = -\frac{2V^2}{R}$$

$$\chi^* = \chi_{crit} + \frac{1}{2}V_{crit}^2\left(1-\left(\frac{R}{R_{crit}}\right)^4\right) \quad for\ R < R_{crit},$$

$$\chi^* = \chi \qquad for\ R \geq R_{crit}$$



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τ = 9
τ = 6
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r
χ, χ*
τ = 12
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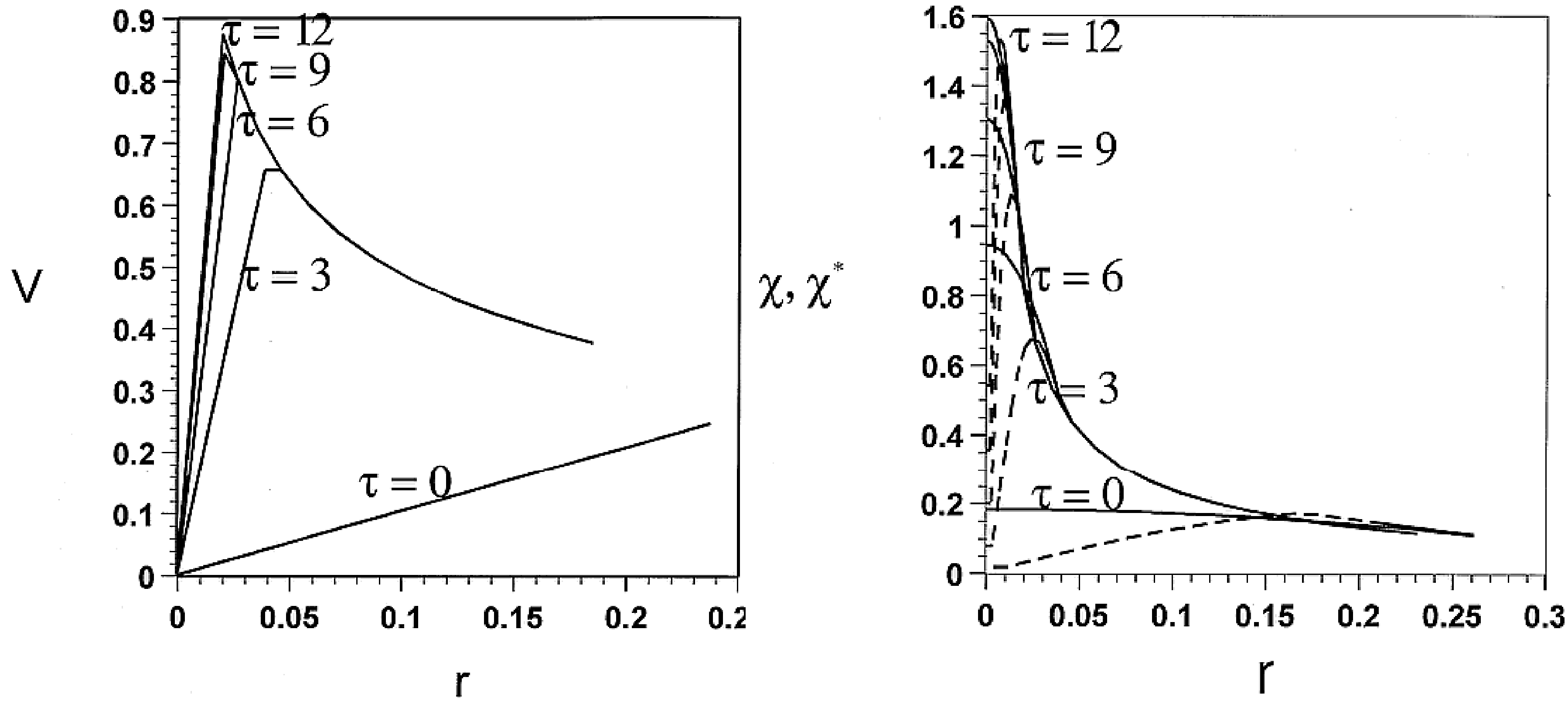
τ = 6
τ = 3
τ = 0
r

3. Maximum diffusion model: 3-D turbulence perfectly efficient in establishing constant angular velocity inside r_m:

$$V = V_{max}\frac{r}{r_m} = V_{max}\frac{R}{R_m}$$

$$\chi^* = \chi_m + V_{max}^2\left[1 - \left(\frac{R}{R_m}\right)^2\right] \quad for\ R < R_m,$$

$$\chi^* = \chi \qquad\qquad for\ R \geq R_m$$



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