Time-dependent, axisymmetric model phrased in R space

- Hydrostatic and gradient balance above PBL
- Moist adiabatic lapse rates on M surfaces above PBL
- Boundary layer quasi-equilibrium
- Deformation-based radial diffusion

Potential Radius:

$$\frac{f}{2}R^2 \equiv M = rV + \frac{f}{2}r^2 \tag{1}$$

Local energy conservation:

$$\frac{1}{r_b^2} - \frac{1}{r_t^2} = \frac{-2\left(T_s - T_t\right)}{f^2 R^3} \frac{\partial s^*}{\partial R} \tag{2}$$

Differentiate in time:

$$\frac{1}{r_b^3}\frac{\partial r_b}{\partial \tau} - \frac{1}{r_t^3}\frac{\partial r_t}{\partial \tau} = \frac{\left(T_s - T_t\right)}{f^2 R^3} \frac{\partial}{\partial R} \frac{\partial s^*}{\partial \tau} \tag{3}$$

Mass continuity:

$$ru = \frac{\partial \psi}{\partial p},$$

$$r\omega = -\frac{\partial \psi}{\partial r}$$

Transform to potential radius coordinates:

$$ru = r\frac{dr}{dt} = r\left[\frac{\partial r}{\partial \tau} + \cancel{\frac{\partial r}{\partial R}\frac{dR}{dt}} + \frac{\partial r}{\partial P}\frac{dP}{dt}\right] = \frac{\partial \psi}{\partial p},$$

$$\rightarrow r\frac{\partial r}{\partial \tau} = \frac{\partial \psi}{\partial p} - r\omega\frac{\partial r}{\partial P} = \frac{\partial \psi}{\partial p} + \frac{\partial \psi}{\partial r}\frac{\partial r}{\partial P} = \frac{\partial \psi}{\partial P}$$

$$\boxed{\frac{\partial r^2}{\partial \tau} = 2\frac{\partial \psi}{\partial P}}$$

Define ψ_0 as streamfunction at top of boundary layer and use simple finite difference in vertical:

$$\frac{\partial r_b^{\,2}}{\partial \tau} \cong 2\left(\psi_0 - \psi\right),$$

$$\frac{\partial r_t^{\,2}}{\partial \tau} \cong 2\psi \qquad (4)$$

PBL flow:

$$\cancel{r\frac{\partial r}{\partial \tau}} + r\frac{\partial r}{\partial R}\frac{dR}{dt} = \frac{\partial \psi}{\partial P}$$

Angular momentum balance:

$$\boxed{V = \frac{f}{2}\frac{R^2 - r^2}{r}}$$

$$\frac{f}{2}\frac{dR^2}{dt} = -gr\frac{\partial \tau_\theta}{\partial P}$$

$$\rightarrow r\frac{\partial r^2}{\partial R^2}\frac{2}{f}g\frac{\partial \tau_\theta}{\partial P} \cong -\frac{\partial \psi}{\partial P},$$

$$\psi_0 = -\frac{2}{f}g\frac{\partial r^2}{\partial R^2}\tau_s = \frac{2}{f}g\frac{\partial r^2}{\partial R^2}\rho_s C_D\,|\,\mathbf{V}\,|\,V$$

Saturation entropy:

$$\frac{\partial s^*}{\partial \tau} = \frac{\Gamma_d}{\Gamma_m}\left[\left(M_u - M_d - w\right)\frac{\partial s_d}{\partial z} + \frac{\dot{Q}_{rad}}{T}\right]$$

Downdraft:

$$M_d = \left(1 - \varepsilon_p\right) M_u$$

Boundary layer entropy:

$$h_s \frac{\partial s}{\partial \tau} = C_k \,|\mathbf{V}|\left(s_0^* - s\right) + C_D \,|\mathbf{V}|^3 - \left(M_u - w_0\right)\left(s - s_m\right) + C_D r \,|\mathbf{V}|\, V \frac{\partial s}{f R \partial R}$$

$$\textit{Used to define } M_{ueq} \textit{ when} > 0; \textit{ otherwise, equation integrated for } s$$

Relaxation equation:

$$\frac{\partial M_u}{\partial \tau} = \frac{M_{ueq} - M_u}{\tau_c}$$

Precipitation efficiency:

$$\varepsilon_p = \frac{s_m - s_{m0}}{s - s_{m0}}$$

Middle troposphere entropy:

$$\frac{\partial s_m}{\partial \tau} = \Lambda M_u \left(s - s_m \right) + \dot{Q}_{rad}$$

Radiation: $\dot{Q}_{rad} \approx -\left(s^* - s^*(t=0)\right)$

Radial diffusion added to equations
for $r_b, r_t, s^*, and\ s_m$

$$D_b = -\frac{1}{R}\frac{\partial}{\partial R}\left[r_b^2 \upsilon_b \frac{\partial}{\partial r_b}\left(\frac{R^2}{r_b^2}\right)\right]$$

$$D_t = -\frac{1}{R}\frac{\partial}{\partial R}\left[r_t^2 \upsilon_t \frac{\partial}{\partial r_t}\left(\frac{R^2}{r_t^2}\right)\right]$$

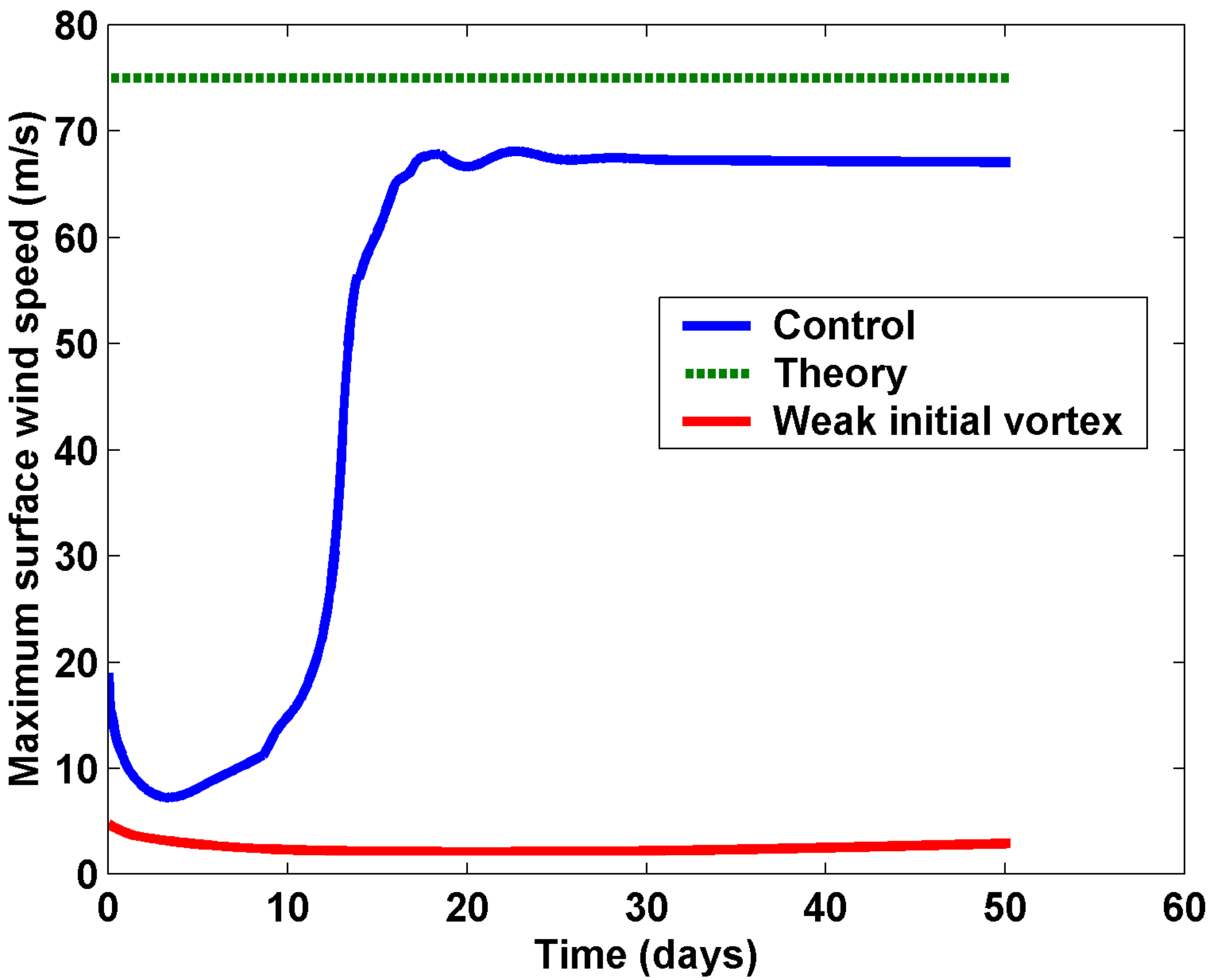
$$D_{s^*} = \frac{\partial}{\partial r_b^2}\left(r_b \upsilon_b \frac{\partial s^*}{\partial r_b}\right) \qquad D_{s_m} = \frac{\partial}{\partial r_m^2}\left(r_m \upsilon_b \frac{\partial s_m}{\partial r_m}\right)$$

$$\upsilon_i = l^2 \left| r_i \frac{\partial}{\partial r_i}\left(\frac{R^2}{r_i^2}\right)\right|$$

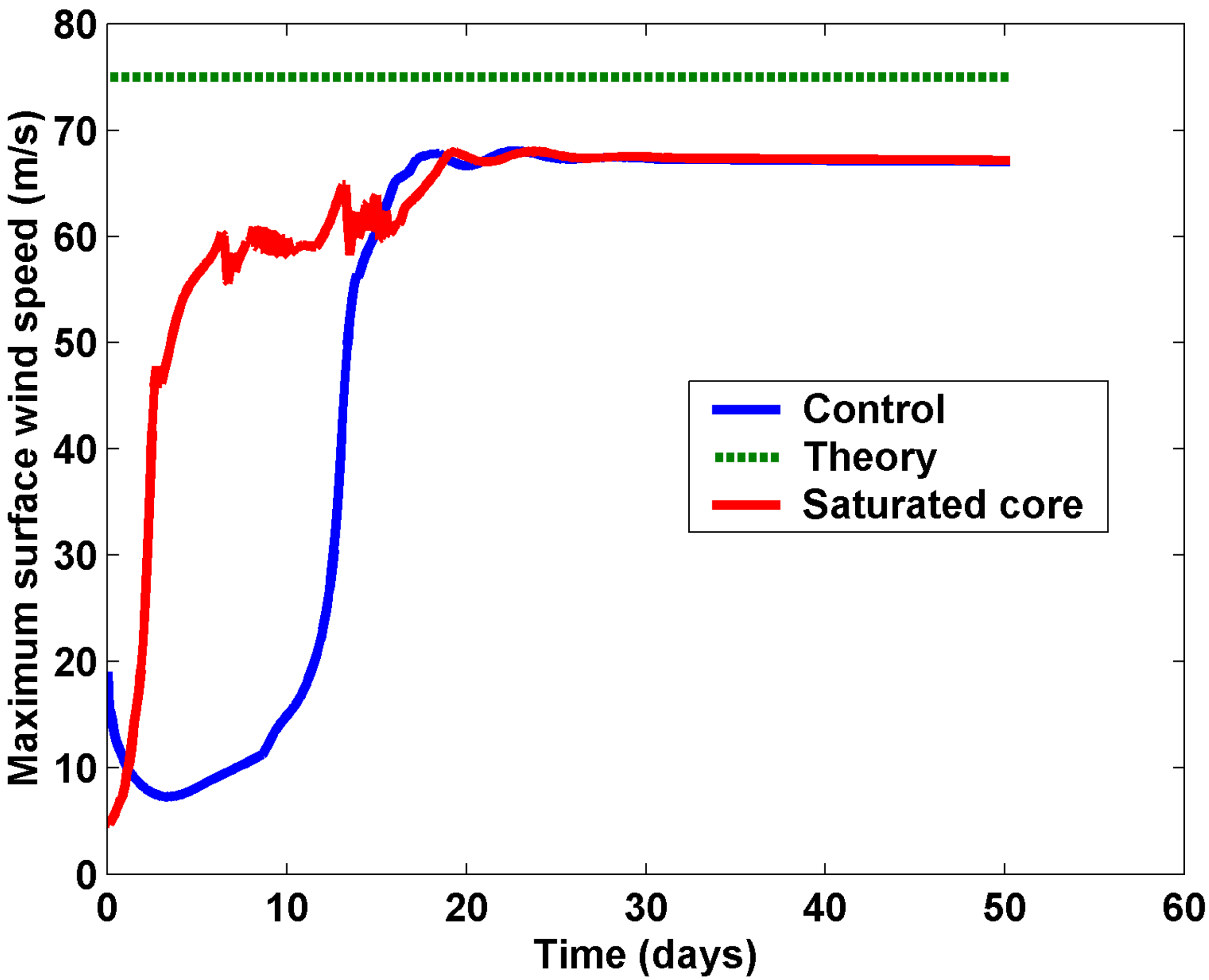
Note that surface saturation entropy depends on pressure, which is calculated from gradient wind balance using *V*

Complete equations summarized in Emanuel (1995), posted on course web page.

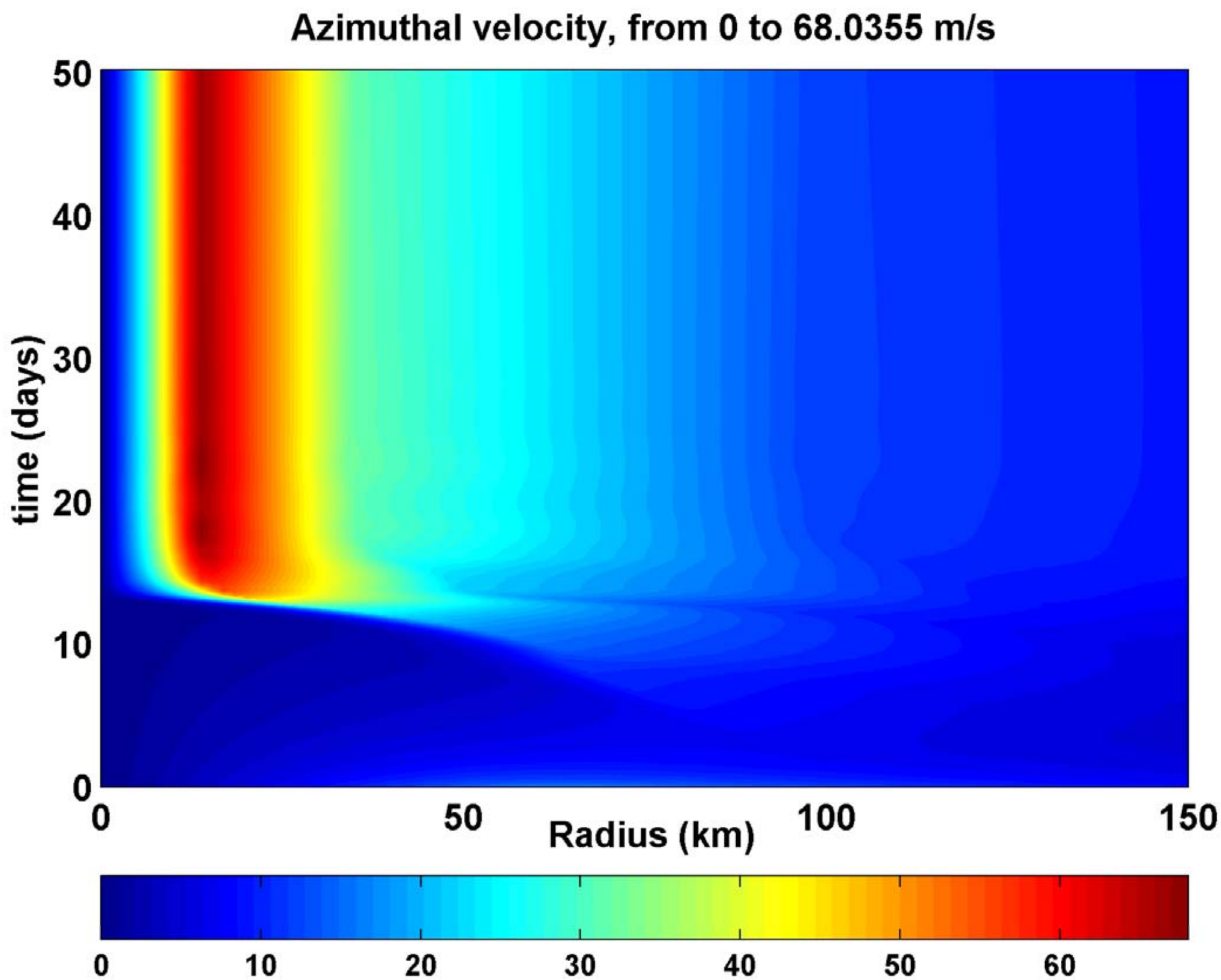
Model behavior

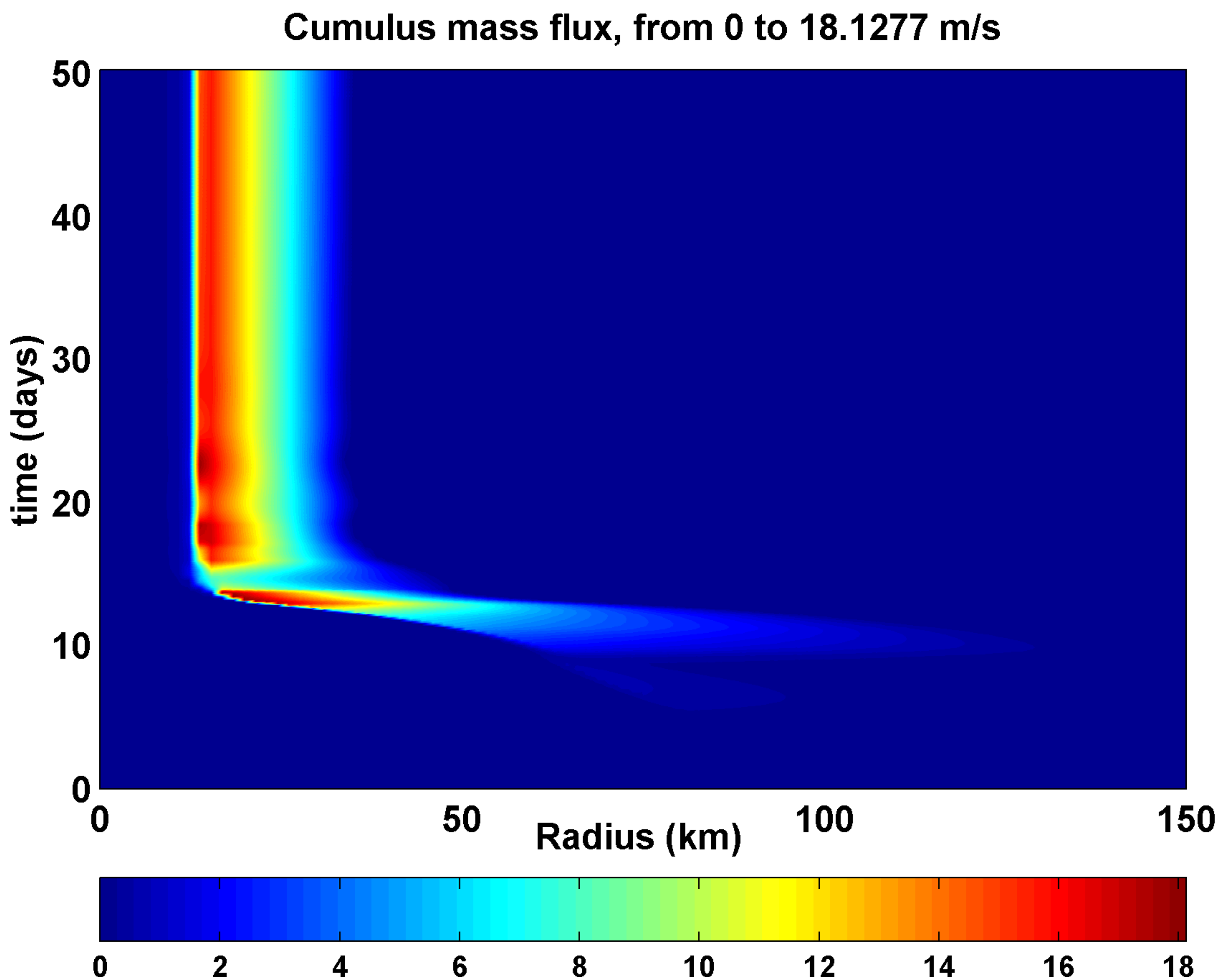


Saturate troposphere inside 100 km in initial state:

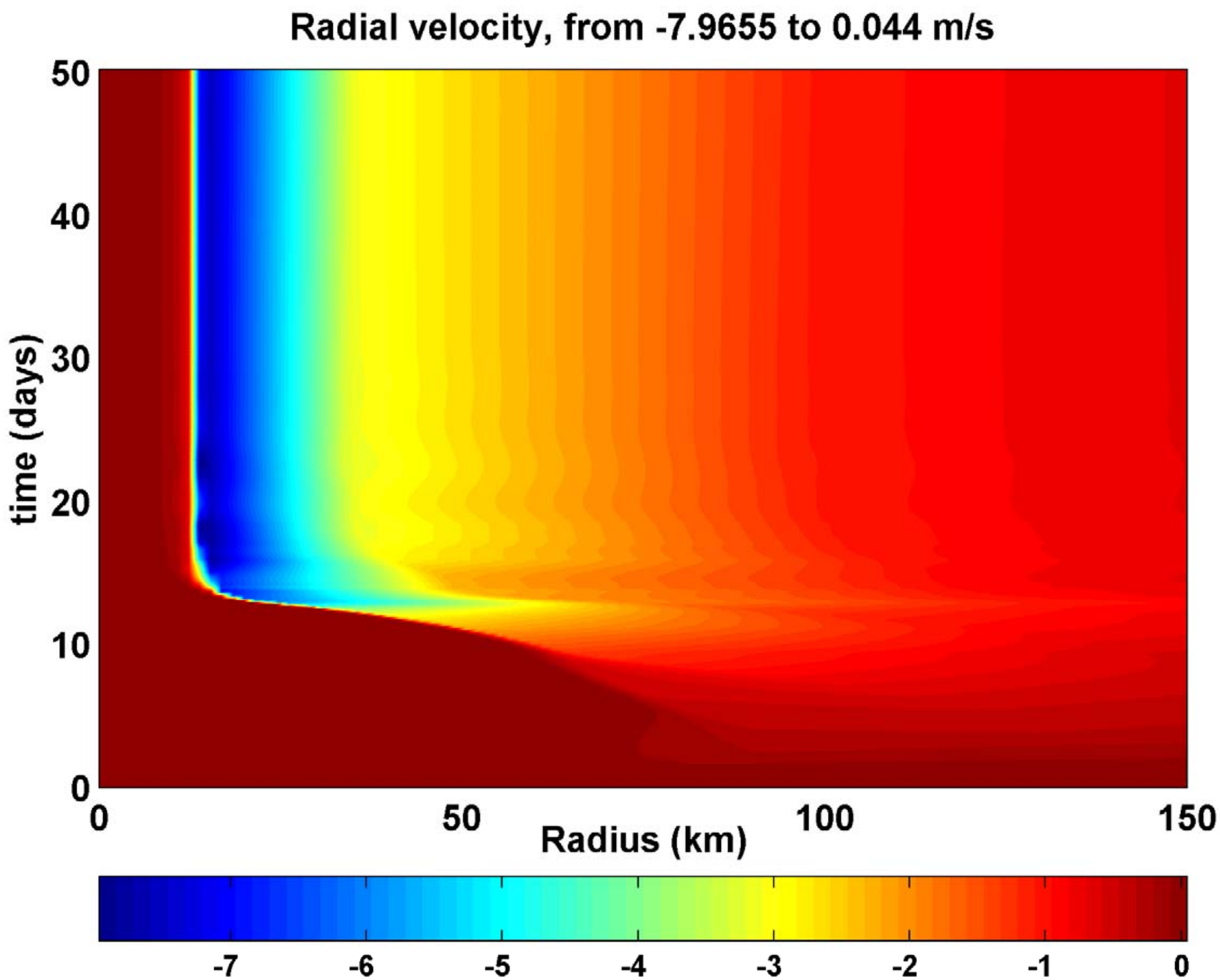


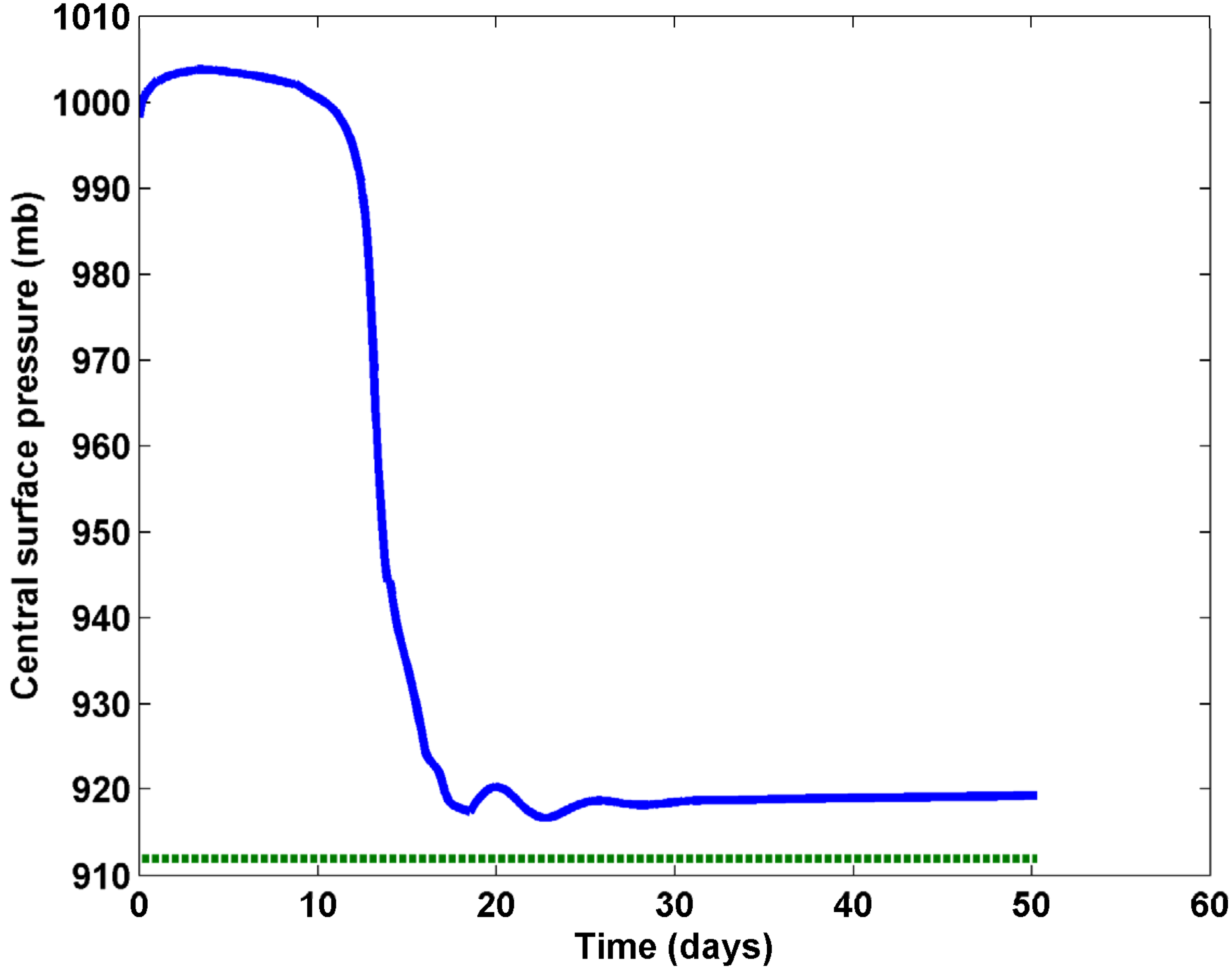
Character of control simulation

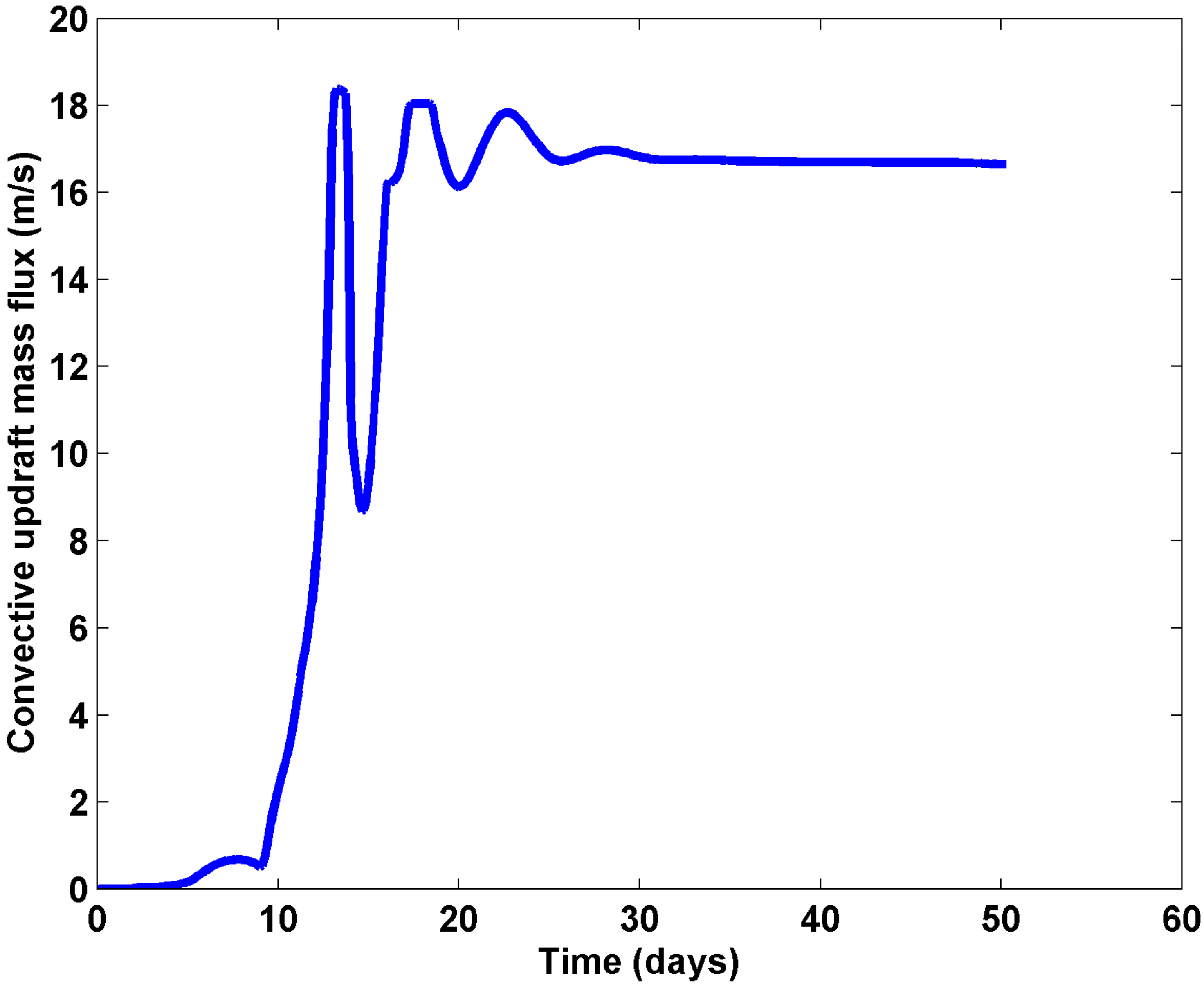




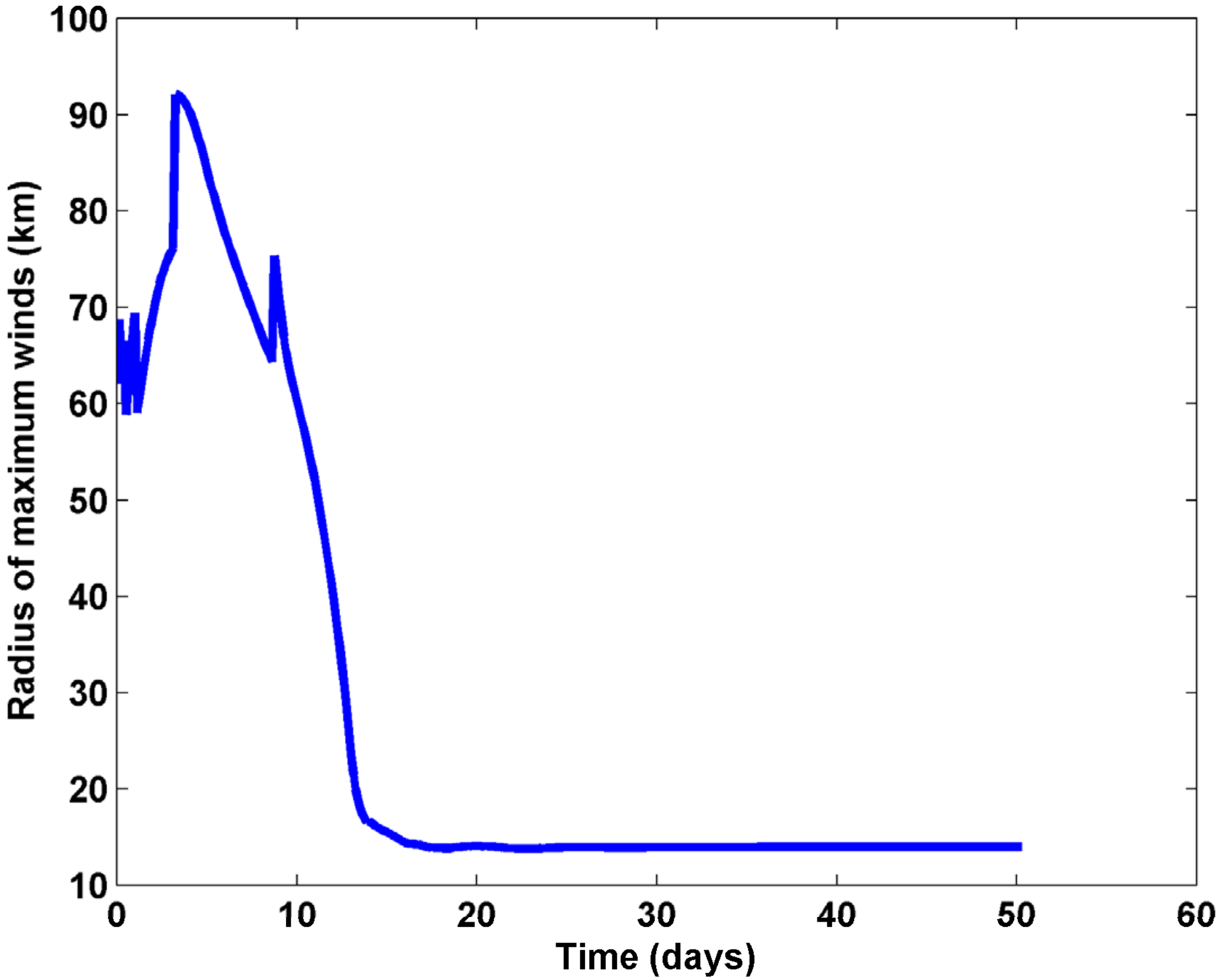
Cumulus mass flux, from 0 to 18.1277 m/s
time (days)
50
40
30
20
10
0
0
50
100
150
Radius (km)
0
2
4
6
8
10
12
14
16
18

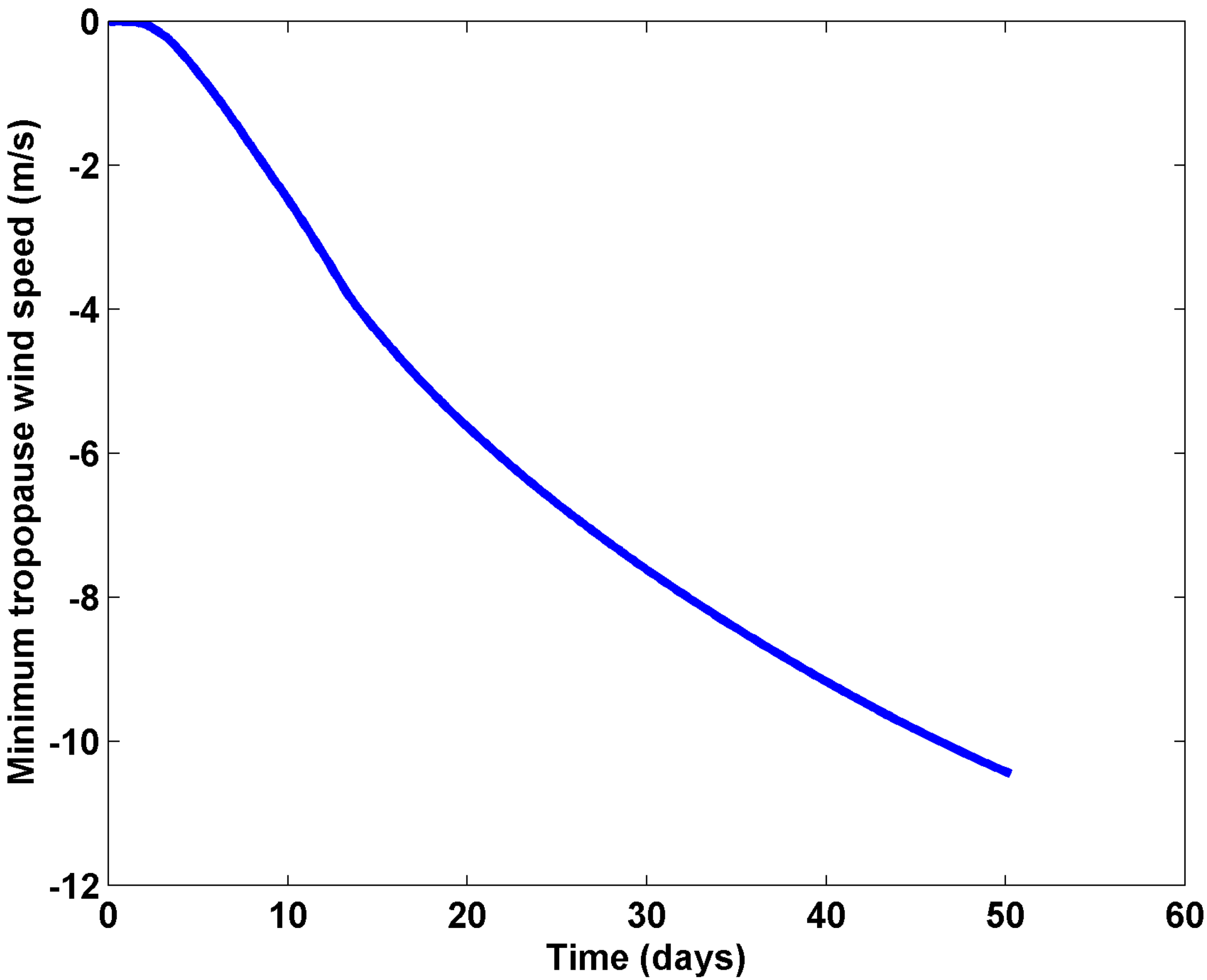

Radial velocity, from -7.9655 to 0.044 m/s
time (days)
50
40
30
20
10
0
0
50
100
150
Radius (km)
-7
-6
-5
-4
-3
-2
-1
0

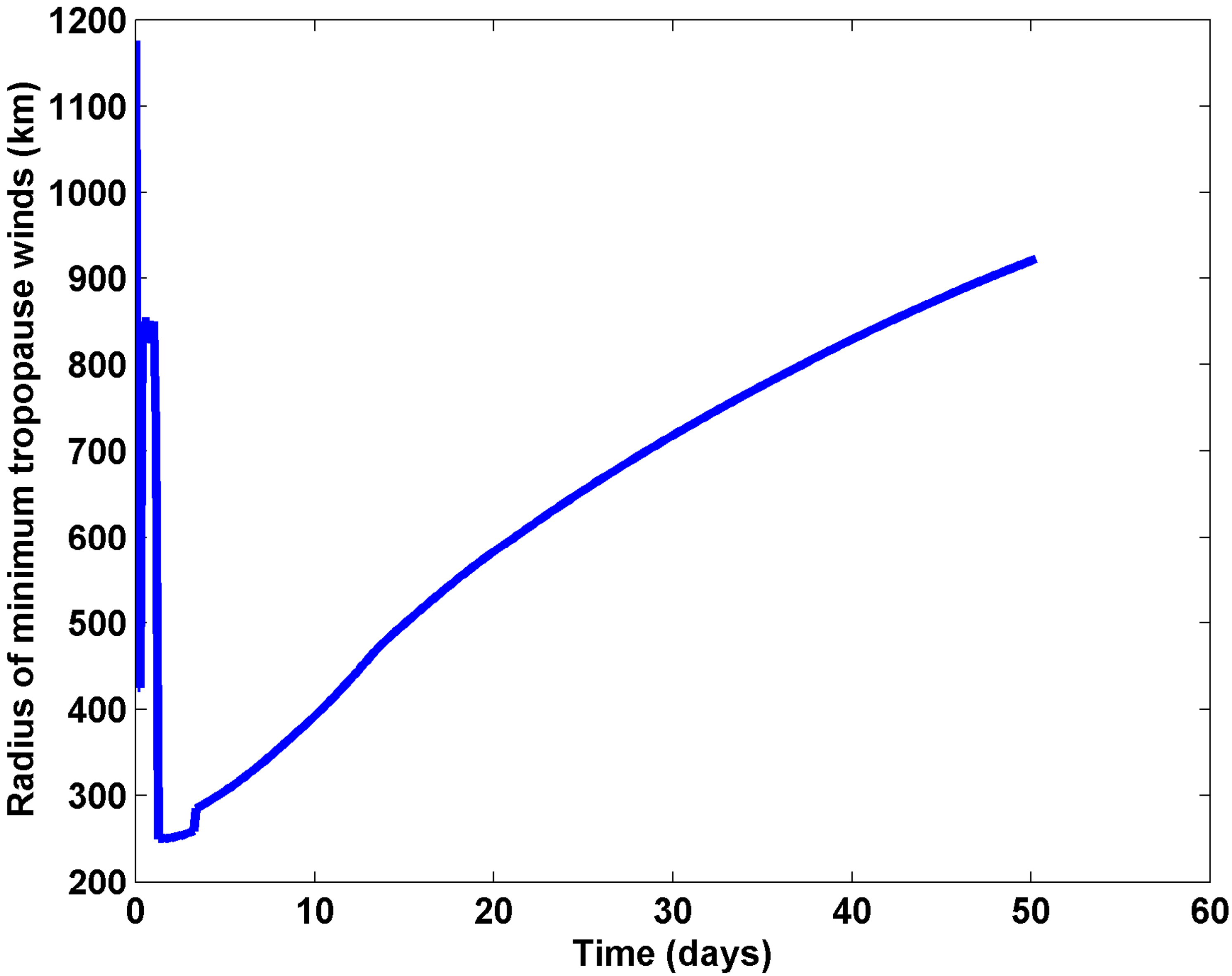

1010
1000
990
980
970
960
950
940
930
920
910
Central surface pressure (mb)
0
10
20
30
40
50
60
Time (days)



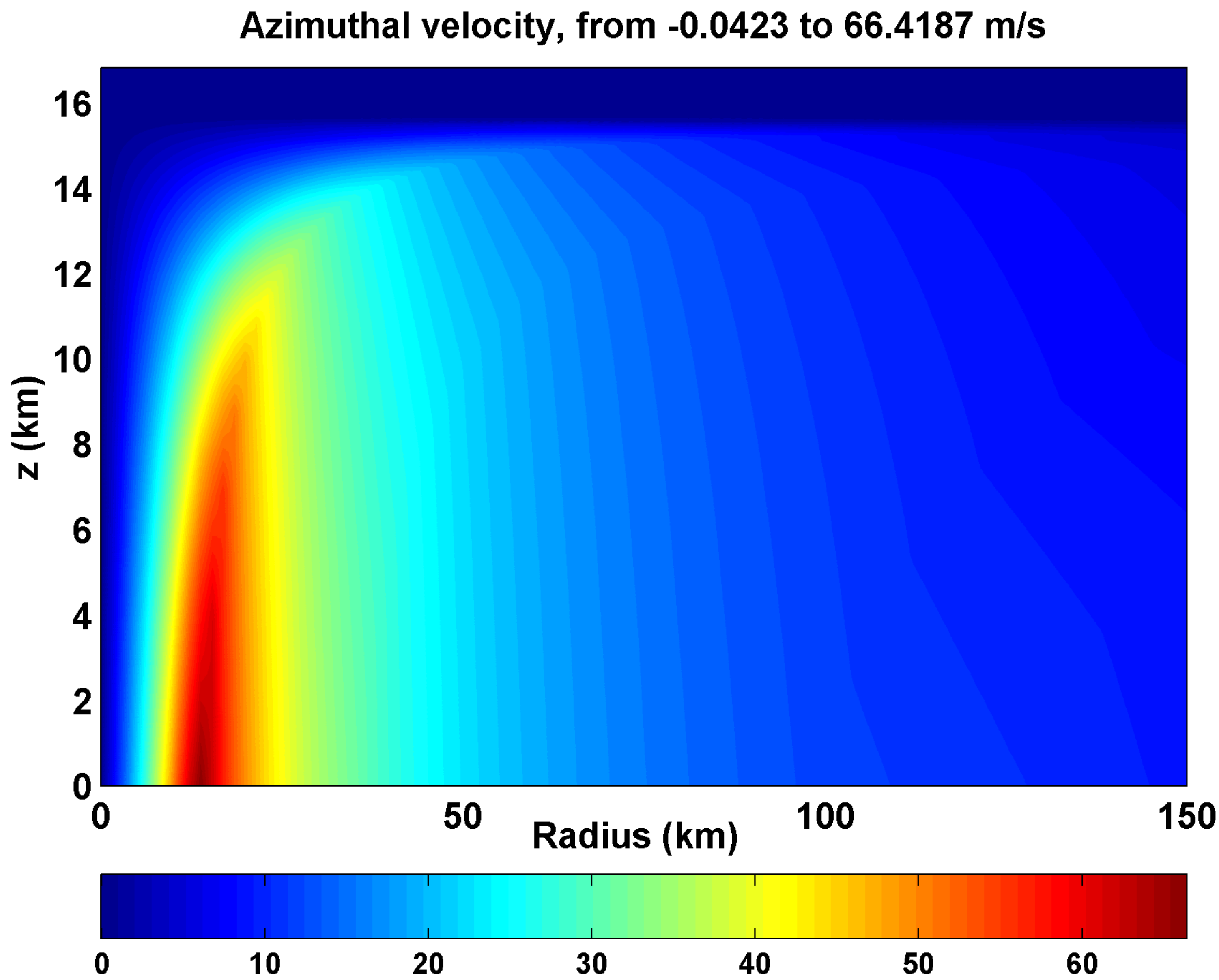
Convective updraft mass flux (m/s)
0
2
4
6
8
10
12
14
16
18
20
0
10
20
30
40
50
60
Time (days)


Radius of maximum winds (km)
Time (days)
100
90
80
70
60
50
40
30
20
10
0
10
20
30
40
50
60

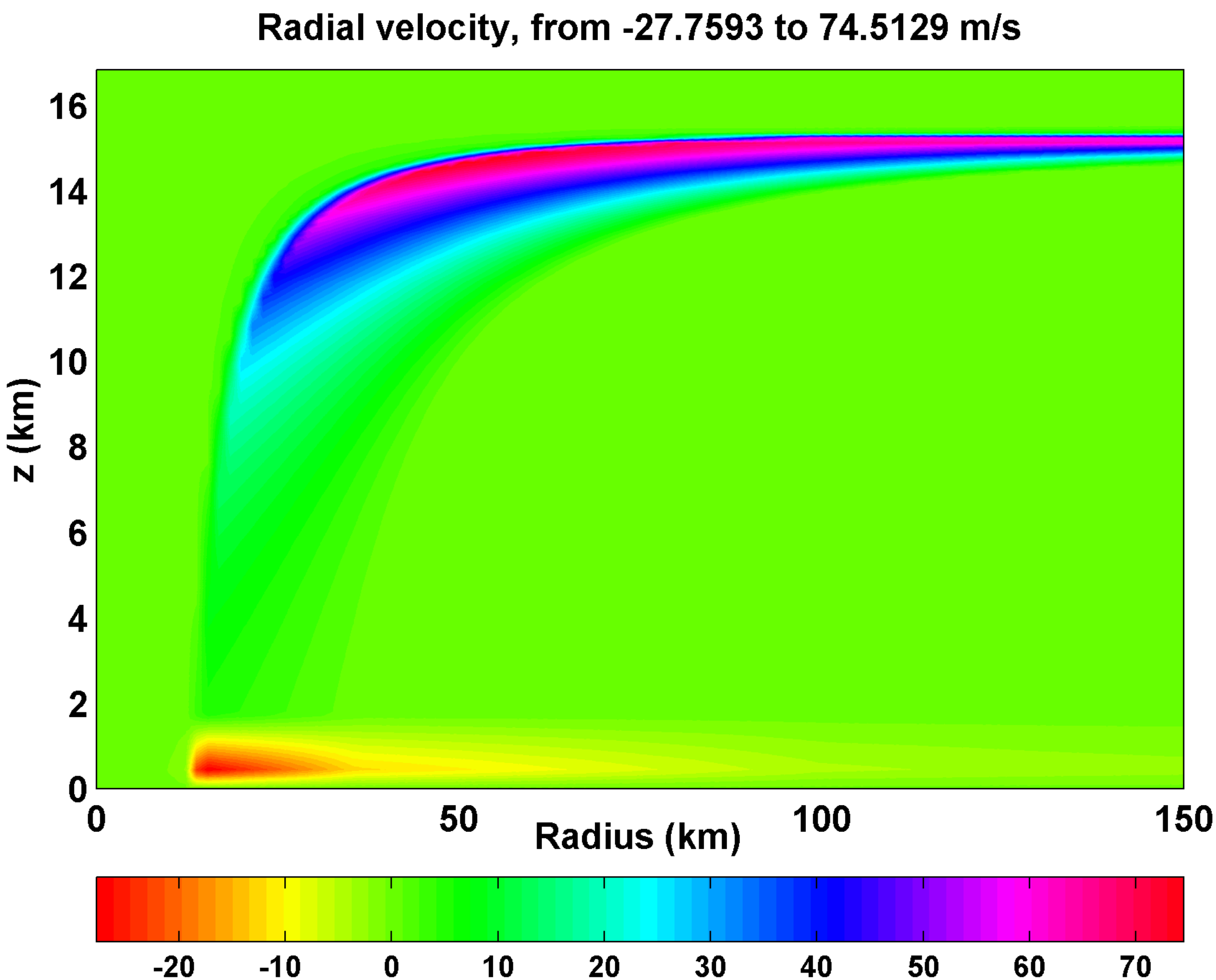

0
-2
-4
-6
-8
-10
-12
0
10
20
30
40
50
60
Time (days)
Minimum tropopause wind speed (m/s)

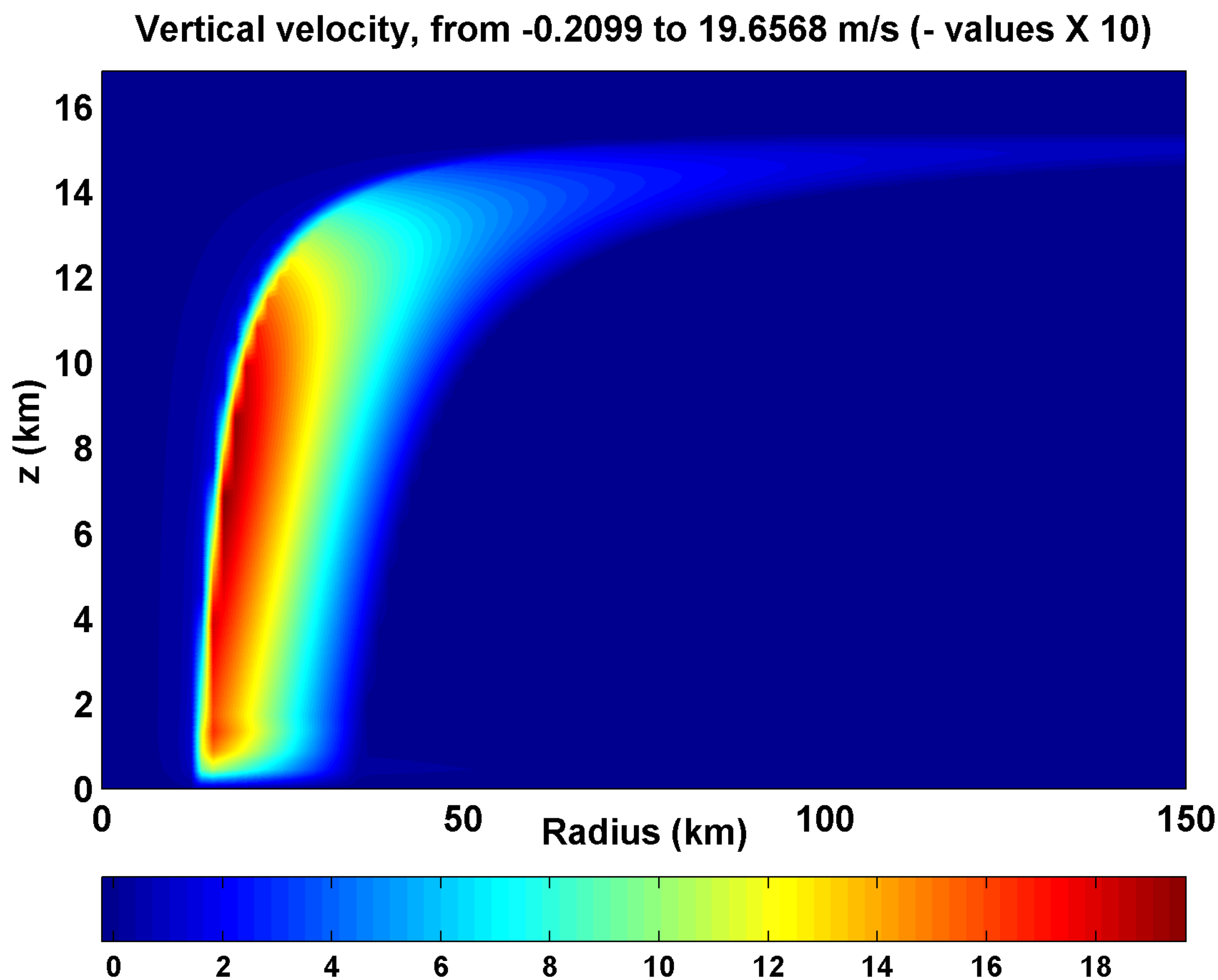


Radius of minimum tropopause winds (km)
Time (days)
1200
1100
1000
900
800
700
600
500
400
300
200
0
10
20
30
40
50
60

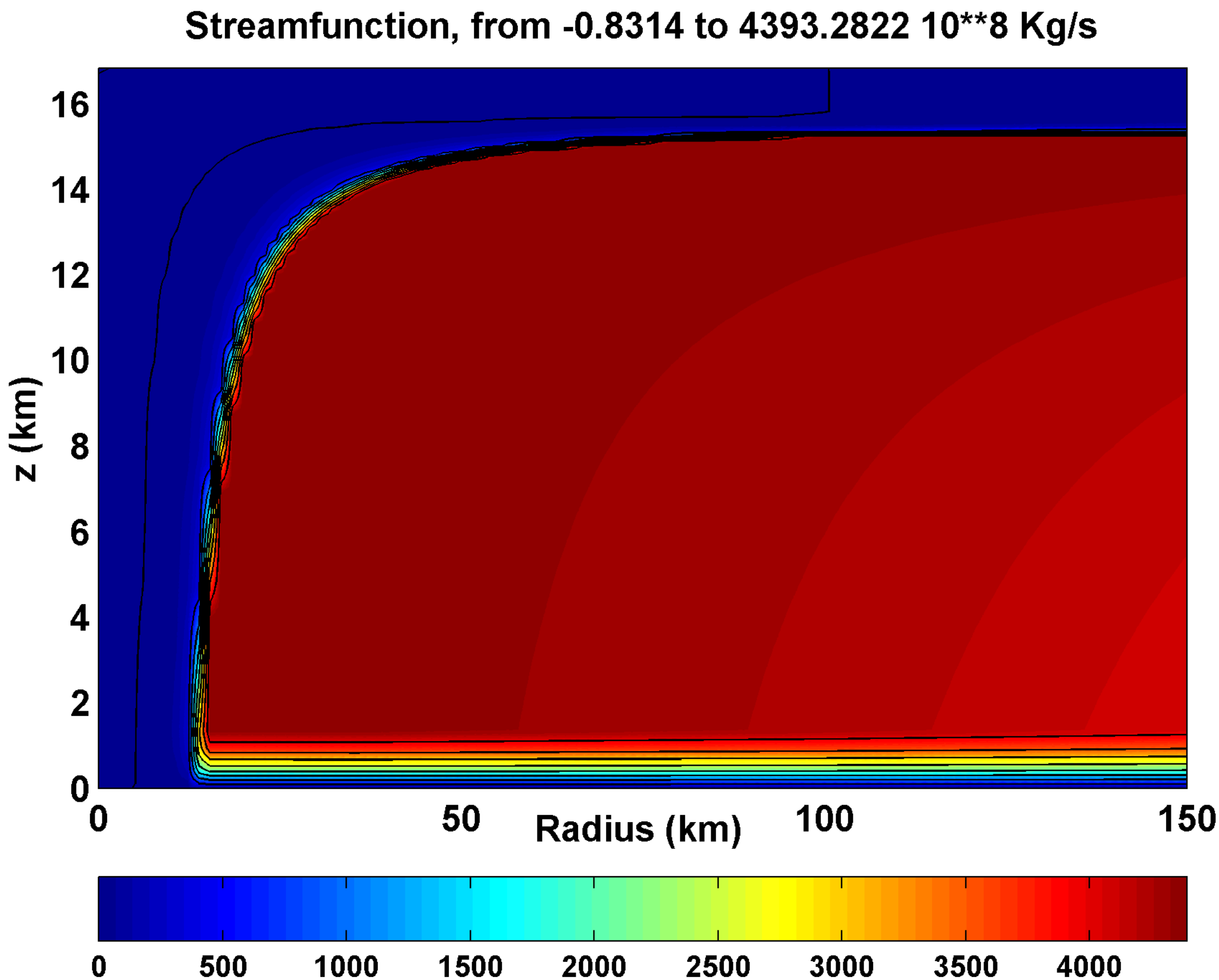


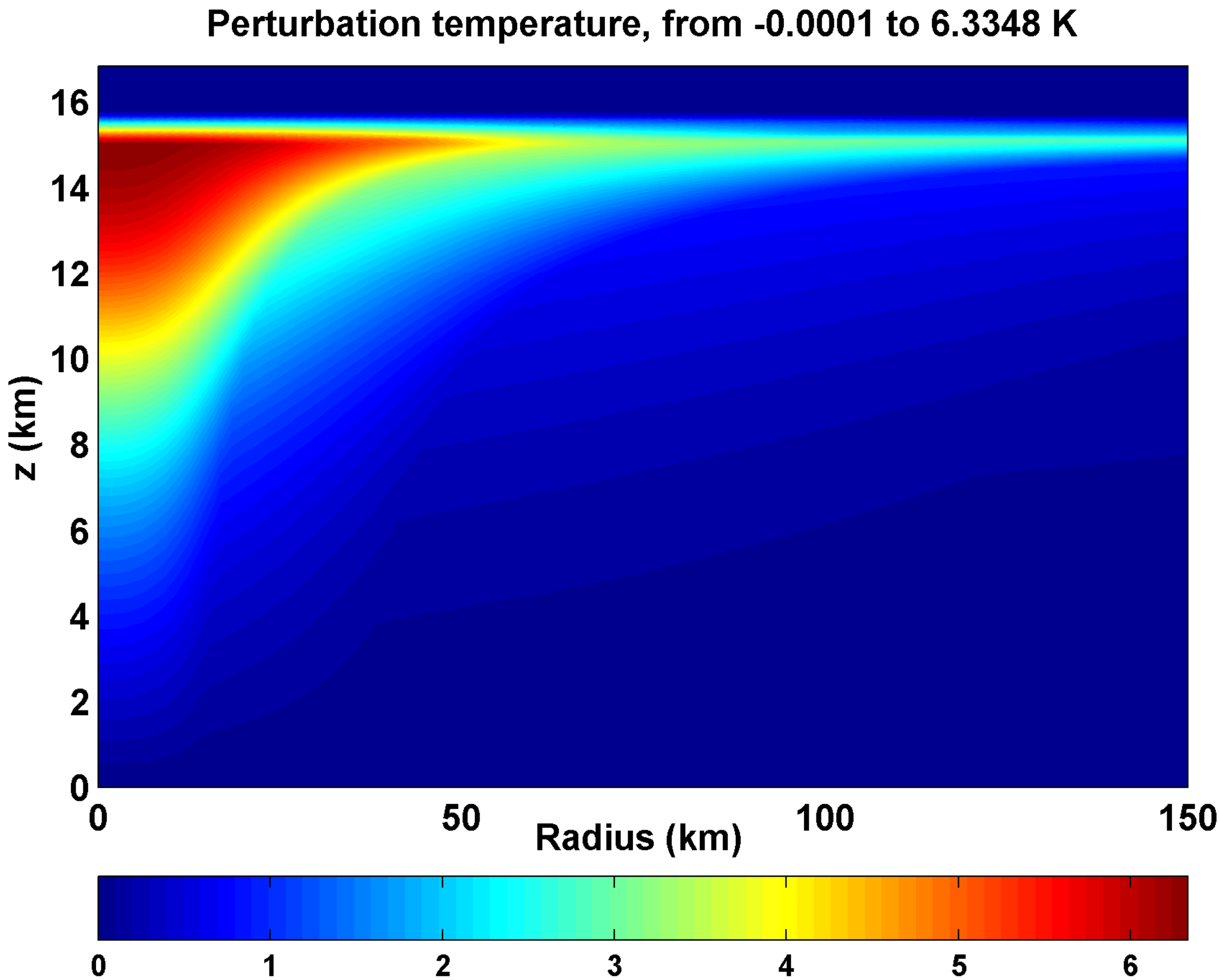
Azimuthal velocity, from -0.0423 to 66.4187 m/s
z (km)
Radius (km)
0
2
4
6
8
10
12
14
16
0
50
100
150
0
10
20
30
40
50
60

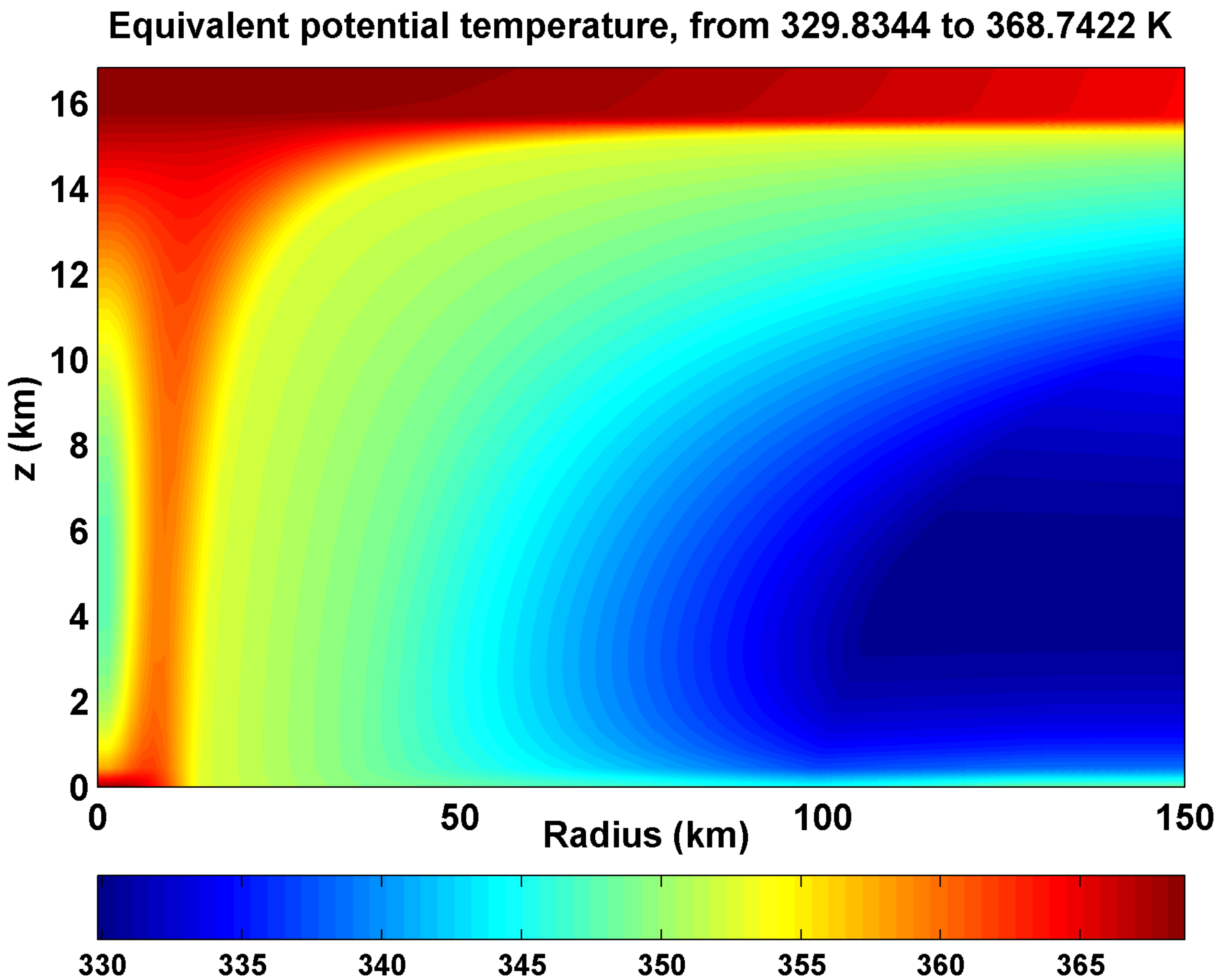

Radial velocity, from -27.7593 to 74.5129 m/s
z (km)
0
2
4
6
8
10
12
14
16
Radius (km)
0
50
100
150
-20
-10
0
10
20
30
40
50
60
70

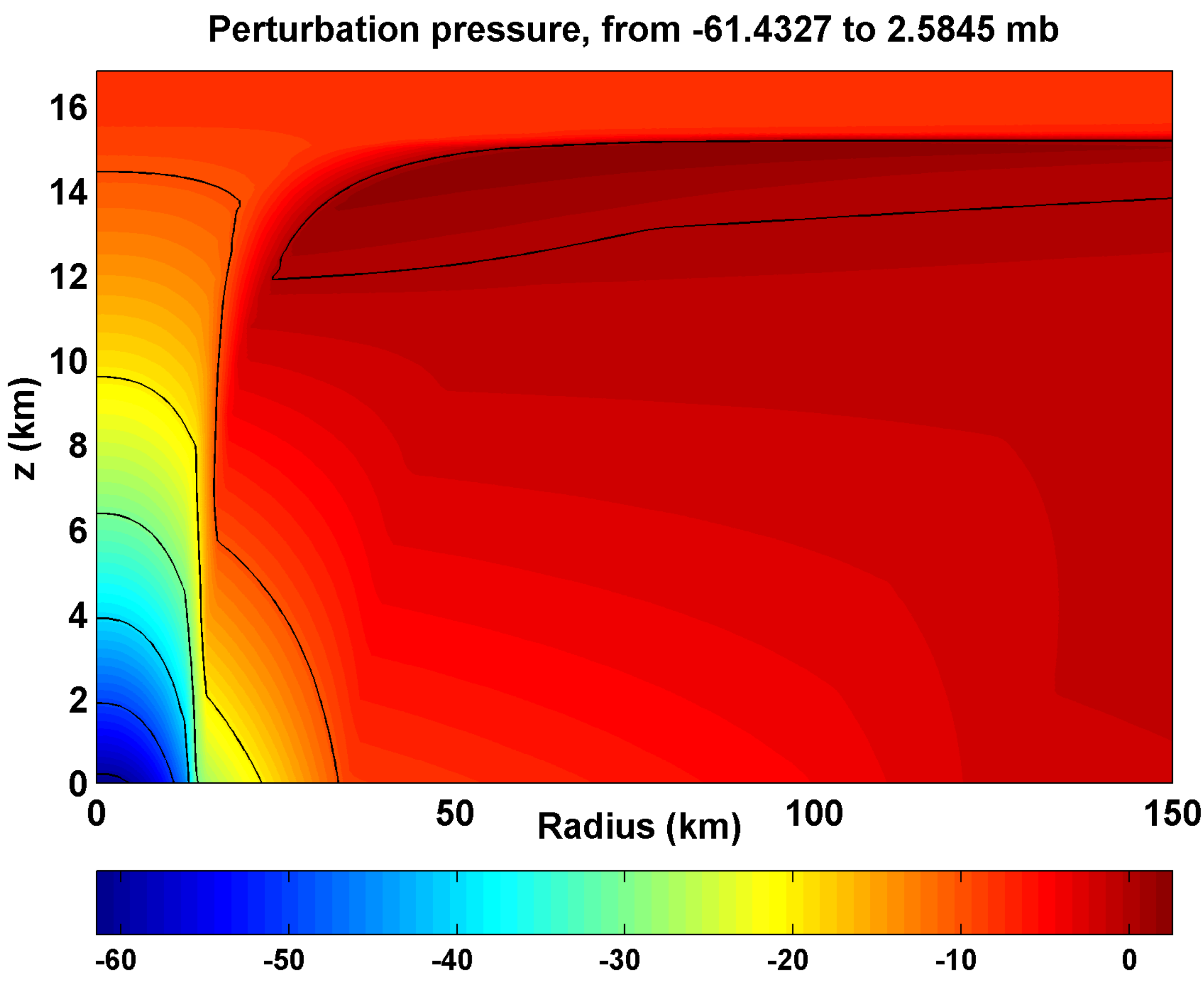


Vertical velocity, from -0.2099 to 19.6568 m/s (- values X 10)
z (km)
0
2
4
6
8
10
12
14
16
0
50
100
150
Radius (km)
0
2
4
6
8
10
12
14
16
18

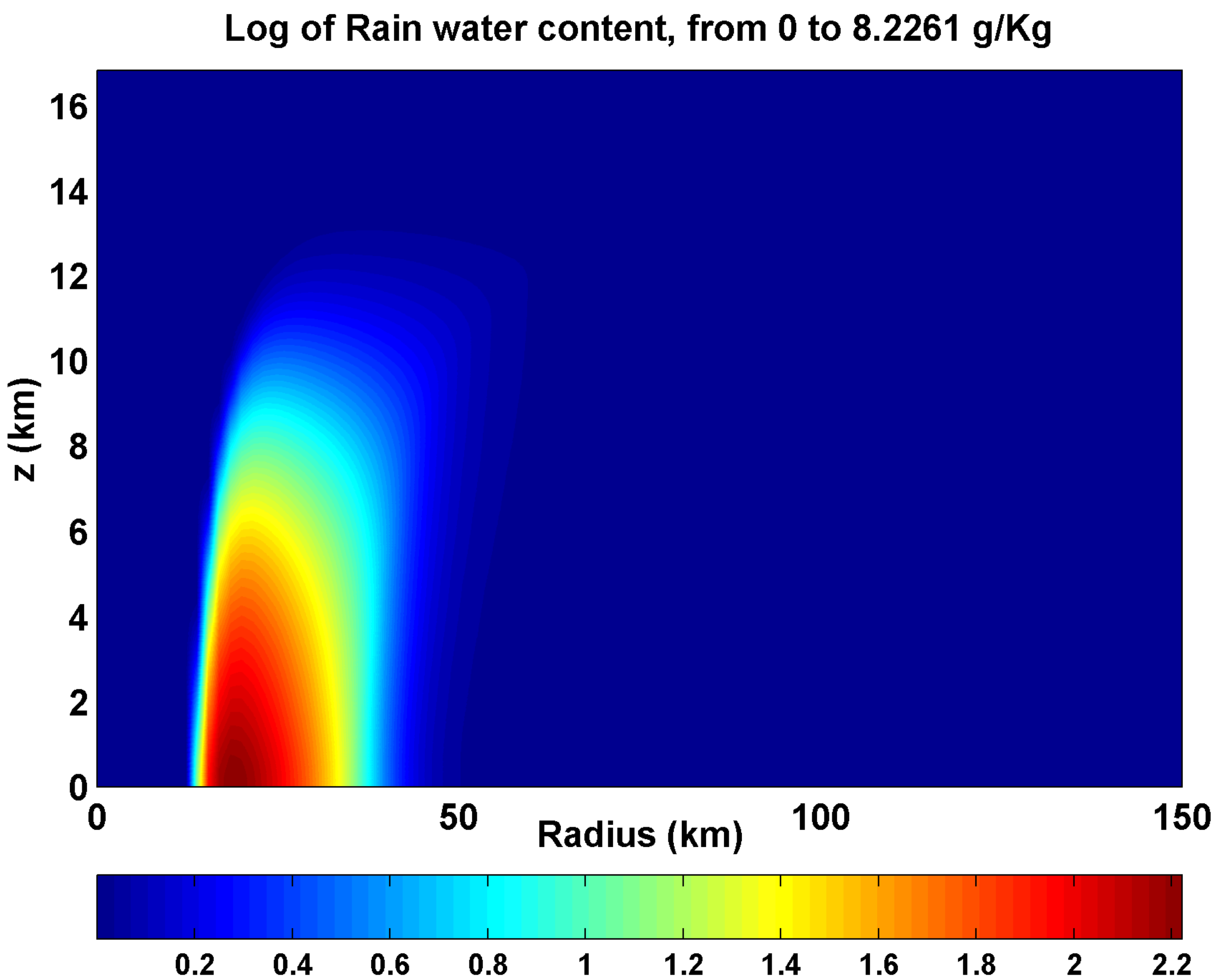

Streamfunction, from -0.8314 to 4393.2822 10**8 Kg/s
z (km)
0
2
4
6
8
10
12
14
16
Radius (km)
0
50
100
150
0
500
1000
1500
2000
2500
3000
3500
4000


Perturbation temperature, from -0.0001 to 6.3348 K
16
14
12
10
8
6
4
2
0
z (km)
0
50
100
150
Radius (km)
0
1
2
3
4
5
6


Equivalent potential temperature, from 329.8344 to 368.7422 K
z (km)
0
2
4
6
8
10
12
14
16
Radius (km)
0
50
100
150
330
335
340
345
350
355
360
365



Perturbation pressure, from -61.4327 to 2.5845 mb
16
14
12
10
8
6
4
2
0
z (km)
0
50
100
150
Radius (km)
-60
-50
-40
-30
-20
-10
0



Log of Rain water content, from 0 to 8.2261 g/Kg
z (km)
16
14
12
10
8
6
4
2
0
0
50
100
150
Radius (km)
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2