Dynamics of the Zonal-Mean, Time-Mean Tropical Circulation

First consider a hypothetical planet like Earth, but with no continents and no seasons and for which the only friction acting on the atmosphere is at the surface.

This planet has an exact nonlinear equilibrium solution for the flow of the atmosphere, characterized by

1. Every column is in radiative-convective equilibrium,

2. Wind vanishes at planet's surface

3. Horizontal pressure gradients balanced by Coriolis accelerations

Hydrostatic balance:

$$\frac{\partial p}{\partial z} = -\rho g$$

In pressure coordinates:

$$\frac{\partial \phi}{\partial p} = -\alpha,$$

$$where$$

$$\alpha = {}^{1}\!/_{\rho} \equiv specific\ volume,$$

$$\phi = gz \equiv geopotential$$

Geostrophic balance on a sphere:

$$\left(\frac{\partial\phi}{a\partial\theta}\right)_p = -2\Omega\sin\theta u_{rel} - \frac{u_{rel}^2}{a}\tan\theta$$

Can be phrased in terms of absolute angular momentum per unit mass:

$$M = a\cos\theta\left(\Omega a\cos\theta + u\right)$$

$$\rightarrow \quad \frac{\partial\phi}{\partial\theta} = -\sin\theta\left[\frac{M^2 - \Omega^2 a^4\cos^4\theta}{a^2\cos^3\theta}\right]$$

Eliminate ϕ using hydrostatic equation:

$$\frac{1}{a^2}\frac{\tan\theta}{\cos^2\theta}\frac{\partial M^2}{\partial p} = \left(\frac{\partial\alpha}{\partial\theta}\right)_p = \left(\frac{\partial T}{\partial p}\right)_{s*}\left(\frac{\partial s*}{\partial\theta}\right)_p$$

(Thermal wind equation)

Take $s* = s*(\theta)$ (convective neutrality)

Integrate upward from surface ($p=p_0$),
taking u=0 at surface:

$$M^2 = a^2\cos^2\theta\left[\Omega^2 a^2\cos^2\theta - \cot\theta\left(T_s - T\right)\frac{\partial s^*}{\partial\theta}\right]$$

$$\rightarrow \quad u^2 + 2\Omega a\cos\theta\, u + \cot\theta\left(T_s - T\right)\frac{\partial s^*}{\partial\theta} = 0.$$

$$\boxed{u \cong -\frac{\left(T_s - T\right)}{2\Omega\sin\theta}\frac{\partial s^*}{\partial y}}$$

Implies strongest west-east winds where entropy gradient is strongest, weighted toward equator

Two potential problems with this solution:

1. Not enough angular momentum available for required west-east wind,

2. Equilibrium solution may be unstable

How much angular momentum is needed?

At the tropopause,

$$M^2 = a^2 \cos^2\theta \left[\Omega^2 a^2 \cos^2\theta - \cot\theta \left(T_s - T_t\right) \frac{\partial s^*}{\partial \theta} \right]$$

How much angular momentum is available?

$$M_{eq}^2 = \Omega^2 a^4$$

$$\rightarrow \quad \left(T_s - T_t\right) \frac{\partial s^*}{\partial \theta} \geq -\frac{\Omega^2 a^2 \tan\theta \left(1 - \cos^4\theta\right)}{\cos^2\theta}$$

More generally, we require that angular momentum decrease away from the equator:

$$\frac{1}{\sin\theta}\frac{\partial M^2}{\partial\theta} < 0$$

$$\rightarrow \quad 4\Omega^2 a^2 \cos^3\theta + \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left[\cos^2\theta\cot\theta\left(T_s - T_t\right)\frac{\partial s^*}{\partial\theta}\right] > 0.$$

Let $\quad y \equiv \sec^2\theta$

$$\rightarrow \quad 1 + y^3\frac{\partial}{\partial y}\left(\frac{T_s - T_t}{\Omega^2 a^2}\frac{\partial s^*}{\partial y}\right) > 0$$

What happens when this condition is violated?

Overturning circulation must develop

Acts to drive actual entropy distribution back toward criticality

Postulate constant angular momentum at tropopause:

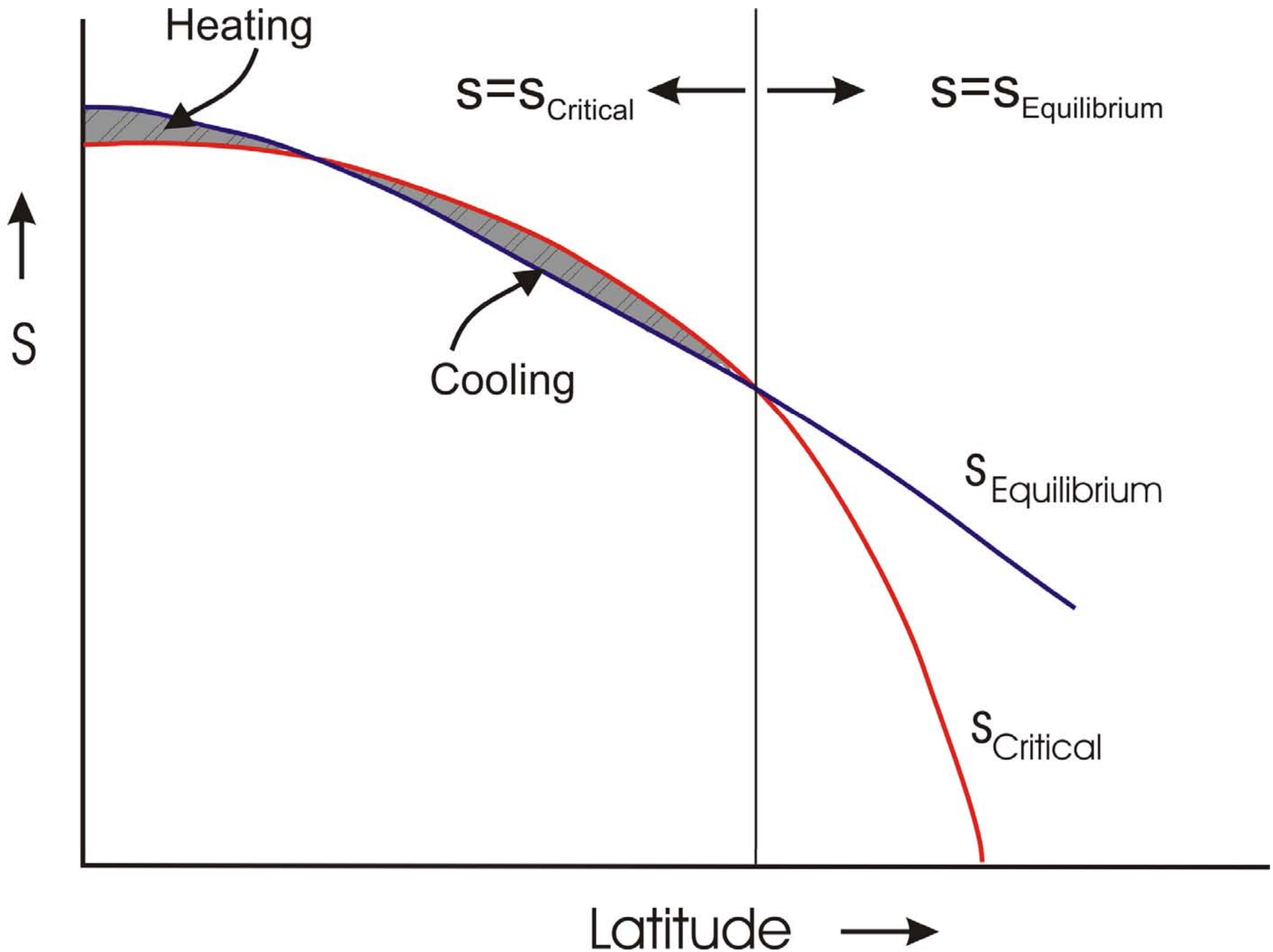
$$M_t = \Omega a^2$$

Integrate thermal wind equations ***down*** from tropopause:

$$M^2 = \Omega^2 a^4 + a^2 \cos^2\theta \cot\theta \left(T - T_t\right)\frac{\partial s^*}{\partial\theta}$$

$$\rightarrow \quad u \simeq \frac{\Omega a}{2}\left(\frac{1}{\cos^3\theta} - \cos\theta\right) + \frac{T - T_t}{2\Omega a \sin\theta}\frac{\partial s^*}{\partial\theta}$$

Note: u at surface always < 0 for supercritical entropy distribution


Heating
S=S$_{Critical}$
S=S$_{Equilibrium}$
S
Cooling
S$_{Equilibrium}$
S$_{Critical}$
Latitude

Critical entropy distribution:

$$\frac{T_s - T_t}{\Omega^2 a^2}\frac{\partial s*}{\partial y} = \frac{1}{2}\left(\frac{1}{y^2} - 1\right)$$

$$\rightarrow \quad \frac{T_s - T_t}{\Omega^2 a^2} s* = \frac{T_s - T_t}{\Omega^2 a^2} s_{eq}^* \left(1 - \frac{1}{2}\left(y + \frac{1}{y}\right)\right)$$

$$= \frac{T_s - T_t}{\Omega^2 a^2} s_{eq}^* \left(1 - \frac{1}{2}\left(\cos^2\theta + \frac{1}{\cos^2\theta}\right)\right)$$

Violation results in large-scale overturning circulation, known as the Hadley Circulation, that transports heat poleward and drives surface entropy gradient back toward its critical value

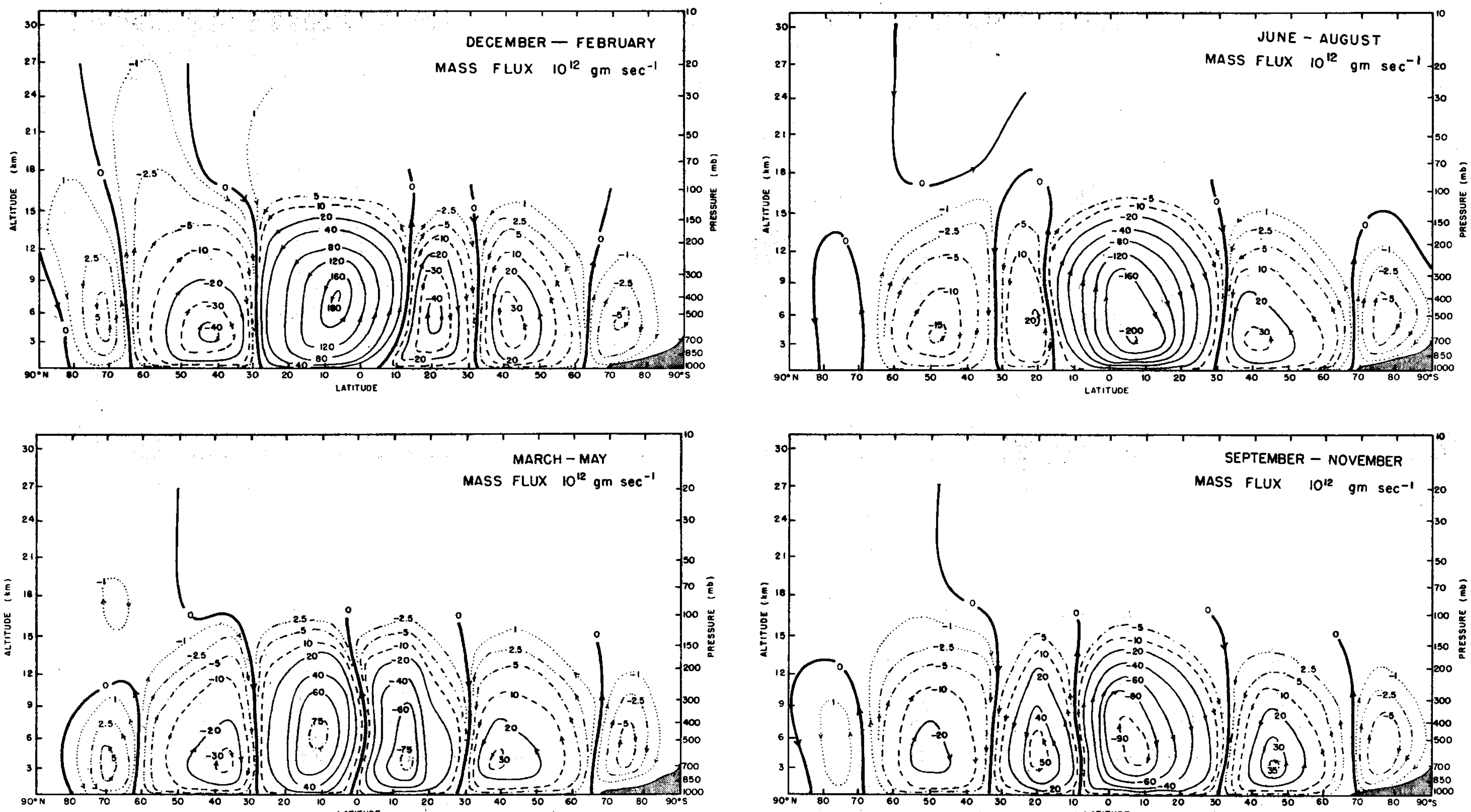


Figure 3.19

From "The General Circulation of the Tropical Atmosphere and Interactions with Extratropical Latitudes - Vol. 1" by MIT Press. Courtesy of MIT Press. Used with permission.

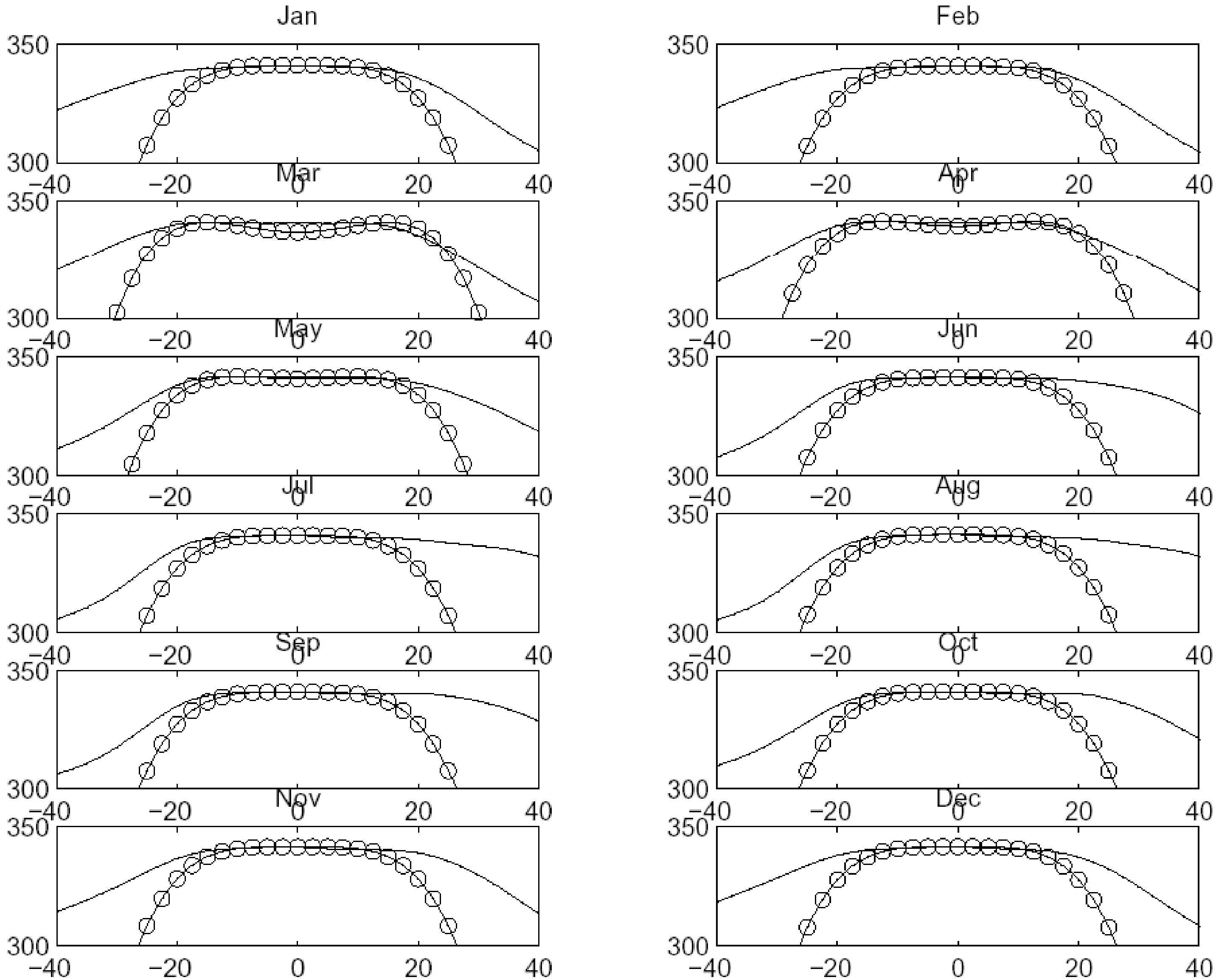
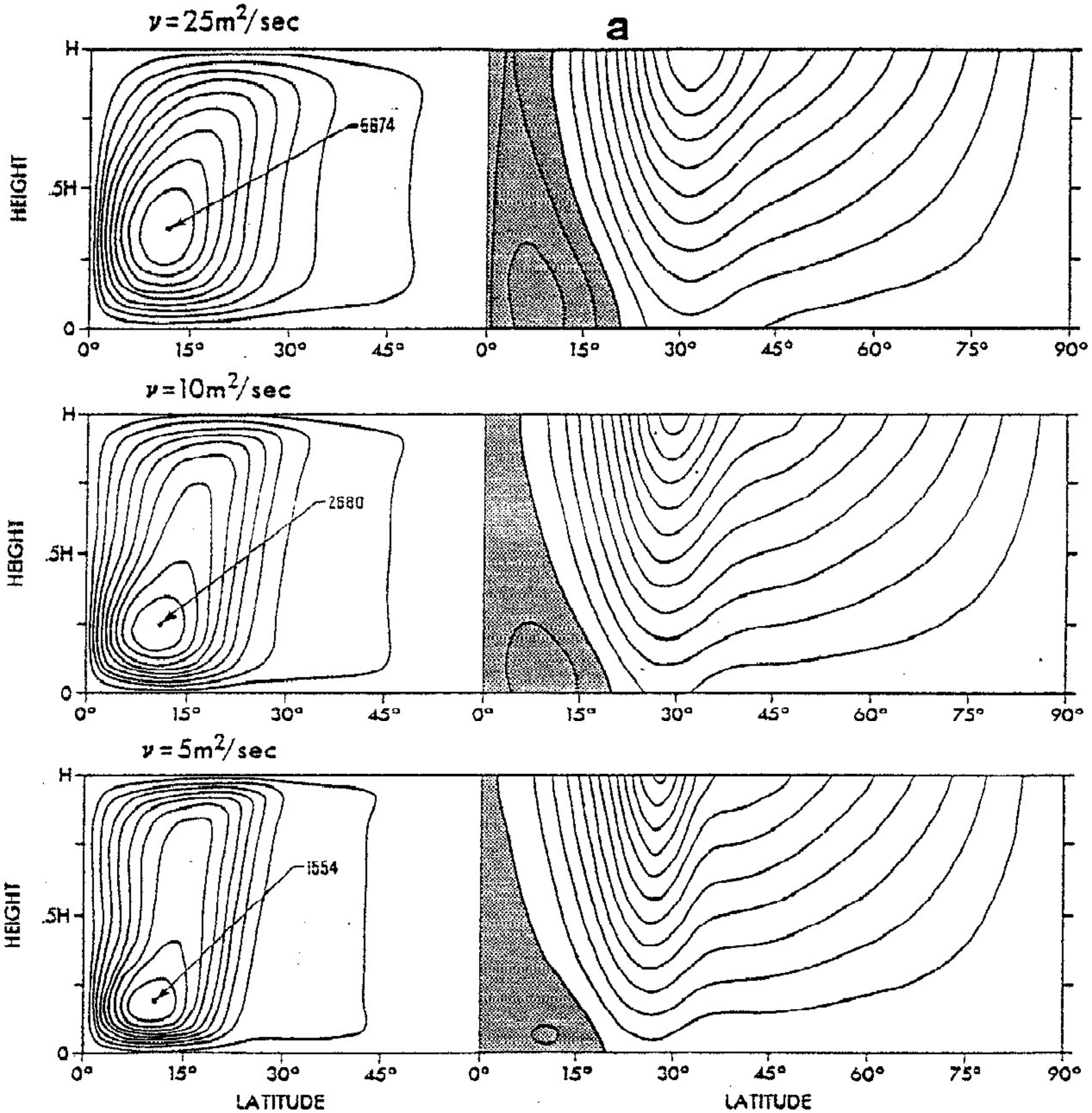
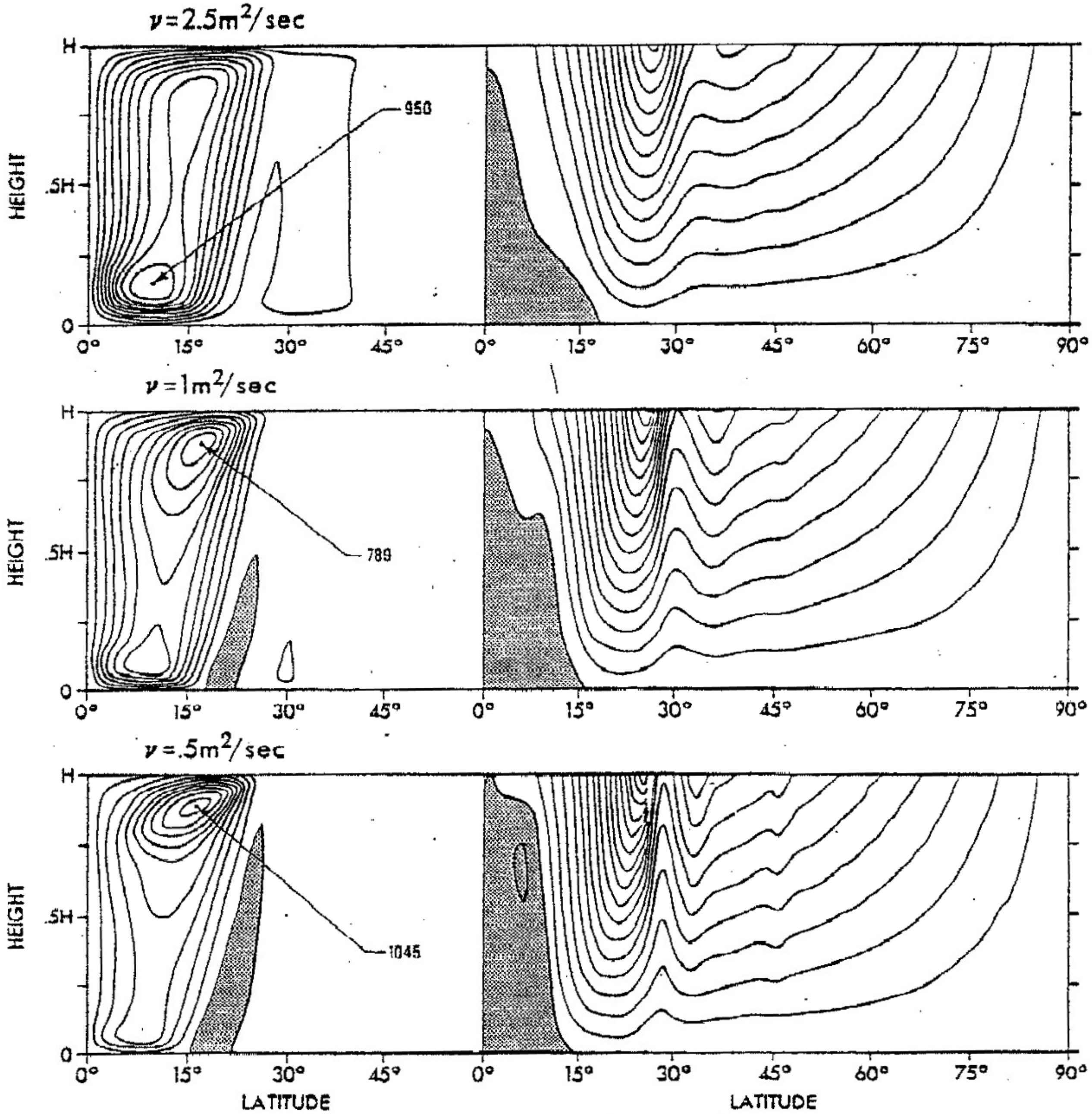


Figure 2: Critical (open circles) and observed (solid line) distributions of θ_{es} for every month at the 600 mb level as a function of latitude

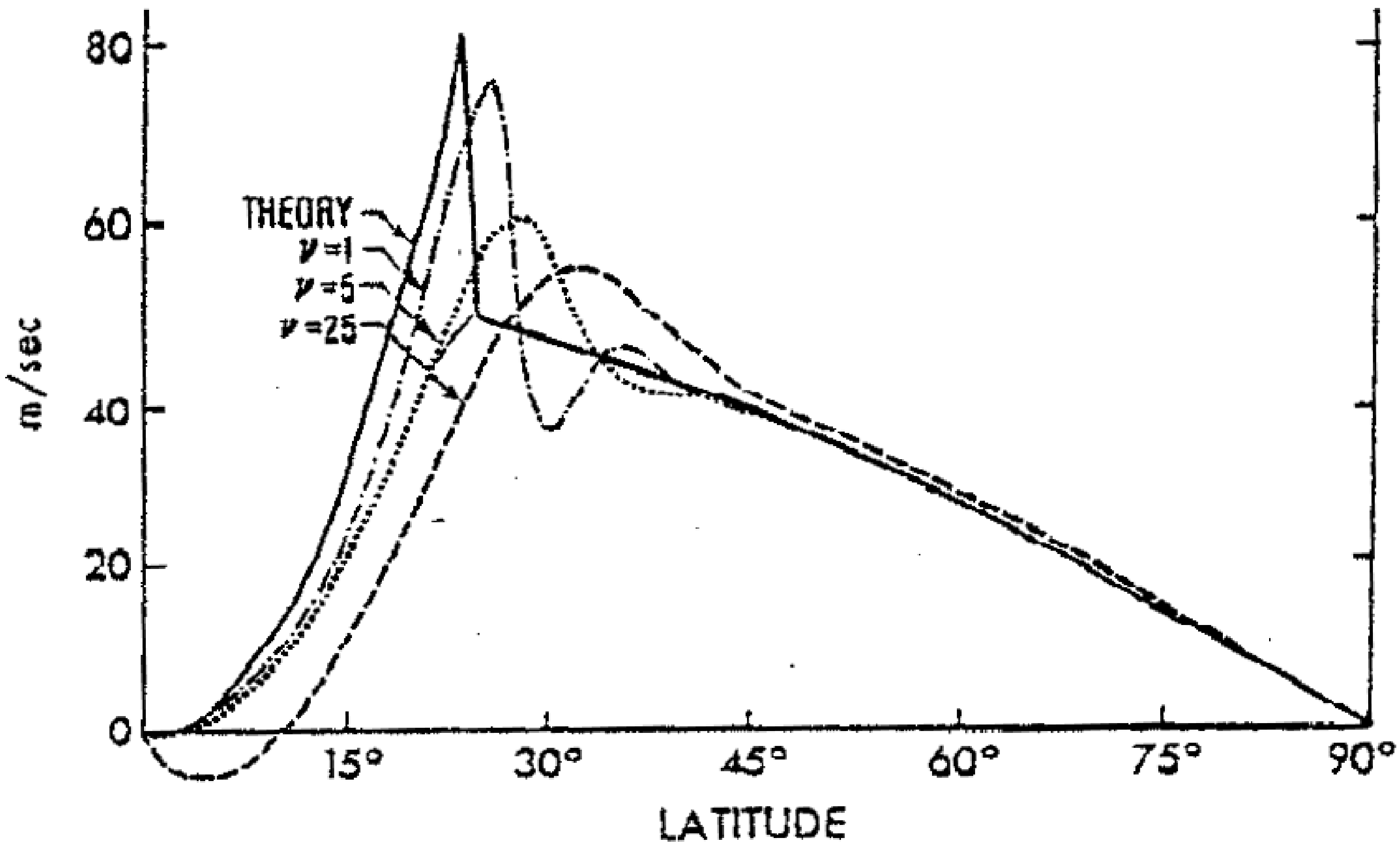
Simulations by Held and Hou (1980)



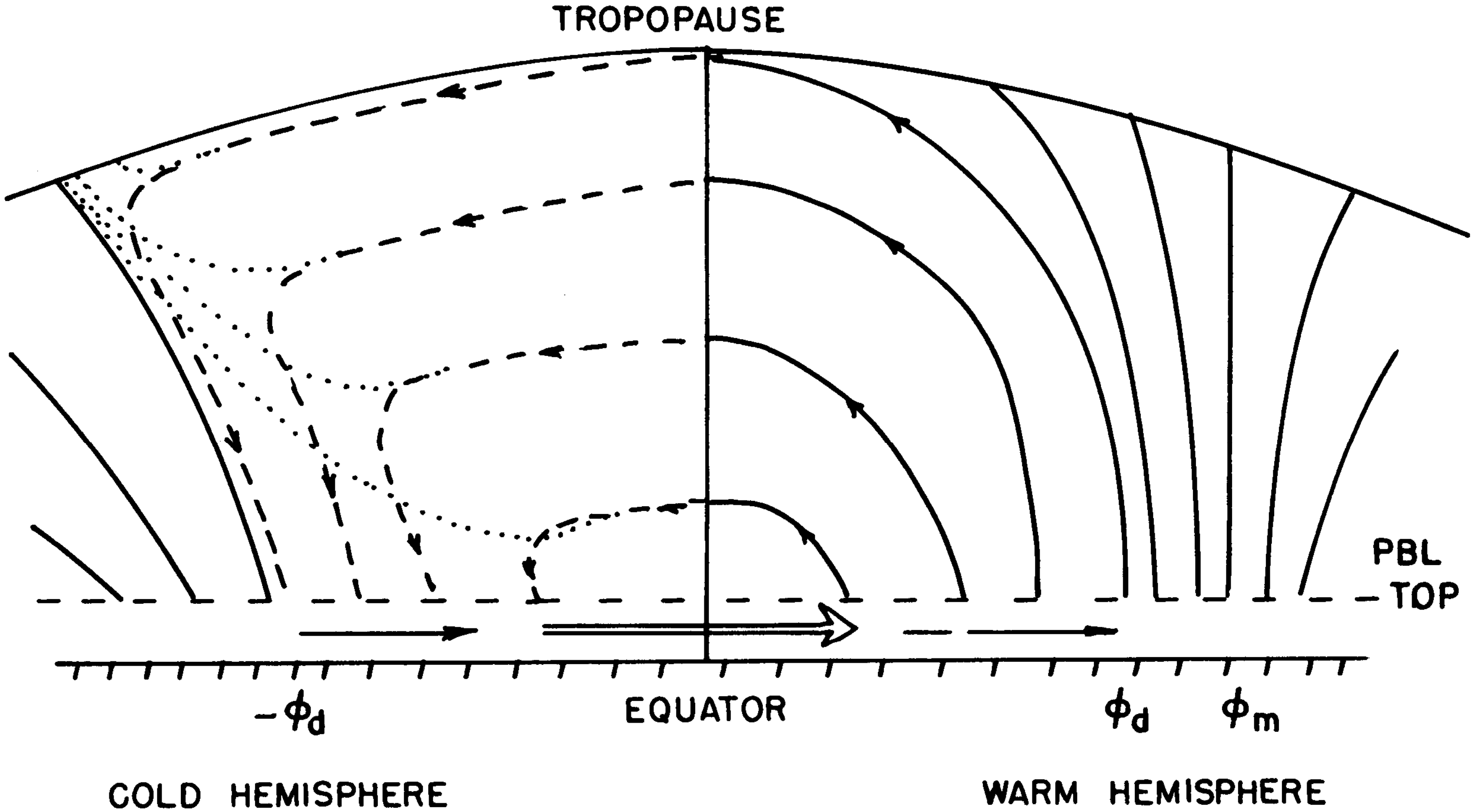


ν=2.5m²/sec
950
ν=1m²/sec
789
ν=.5m²/sec
1045
H
.5H
0
HEIGHT
0° 15° 30° 45°
0° 15° 30° 45° 60° 75° 90°
LATITUDE

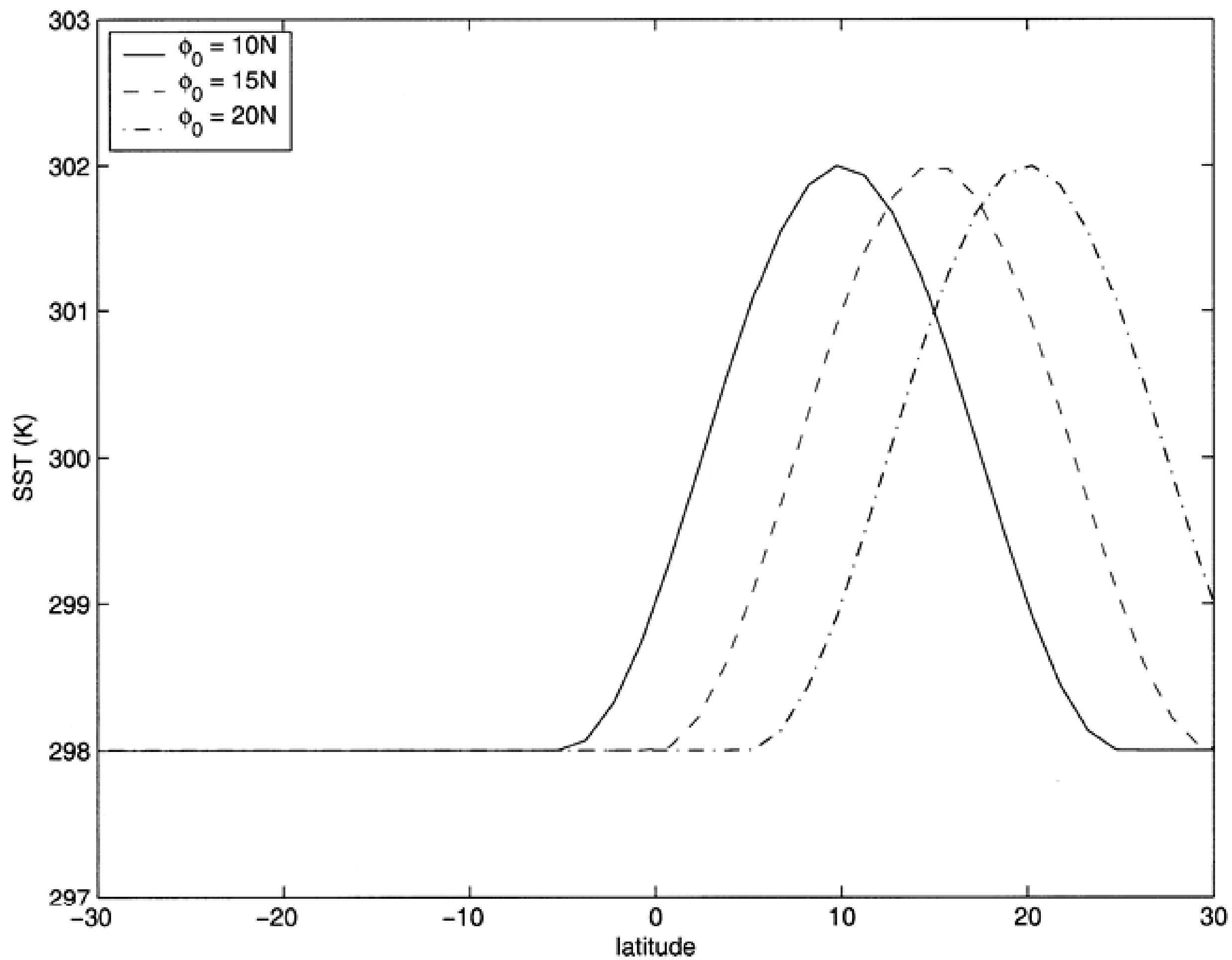
Zonal wind at tropopause



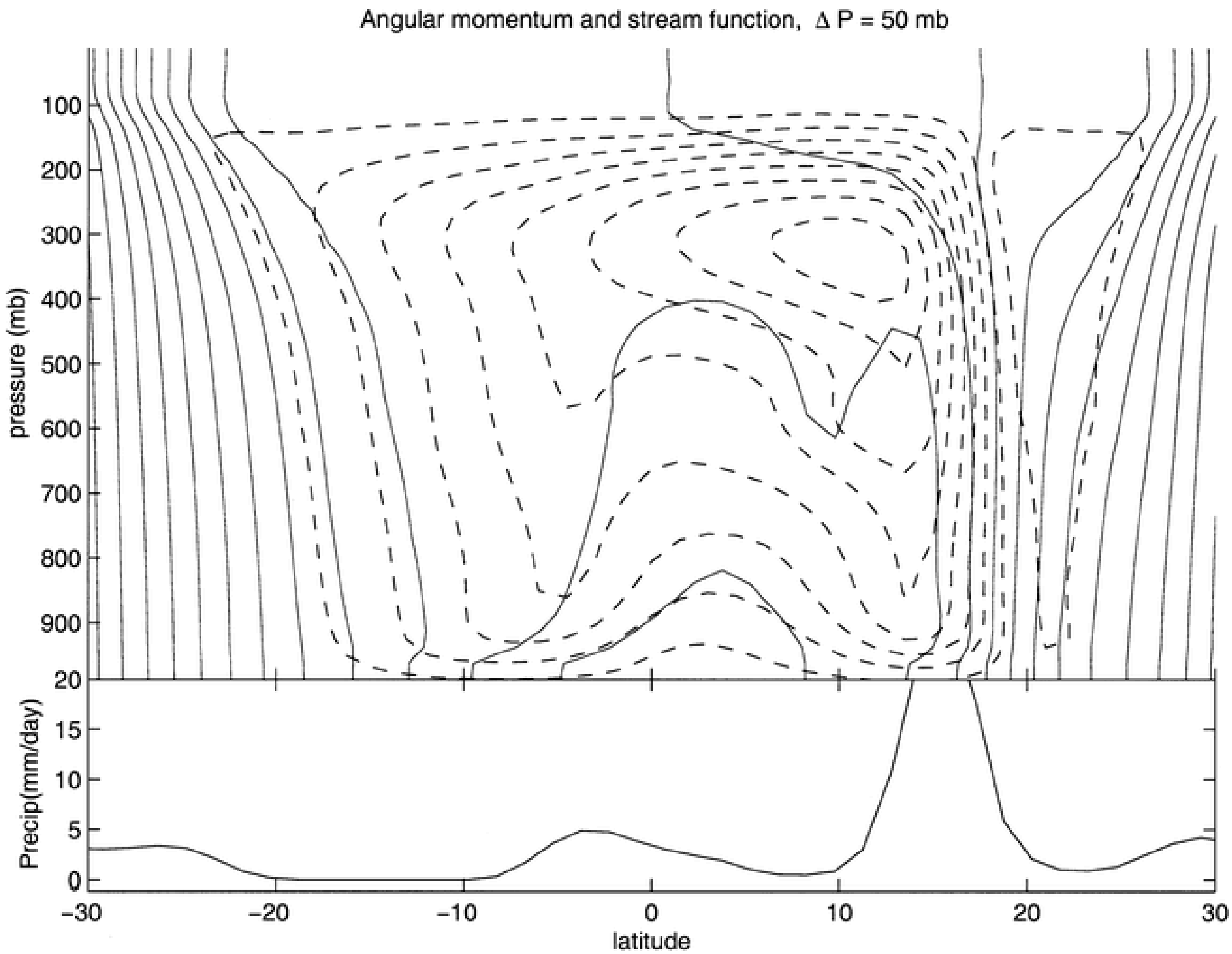
The cross-equatorial Hadley Circulation



Numerical simulation using a 2-D primitive equation model with parameterized convection (Pauluis, 2004)



Imposed sea surface temperature


Angular momentum and stream function, Δ P = 50 mb
pressure (mb)
100
200
300
400
500
600
700
800
900
20
Precip(mm/day)
15
10
5
0
−30
−20
−10
0
10
20
30
latitude

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