14. The Multitaper Idea

Spectral analysis has been used for well over 100 years, and its statistics are generally well understood. It is thus surprising that a new technique appeared not very long ago. The methodology, usually known as the *multitaper* method, is associated primarily with David Thompson of Bell Labs, and is discussed in detail by Percival and Walden (1993). It is probably the best default methodology. There are many details, but the basic idea is not hard to understand.

Thompson's approach can be thought of as a reaction to the normal tapering done to a time series before Fourier transforming. As we have seen, one often begins by tapering x_m before Fourier transforming it so as to suppress the leakage from one part of the spectrum to another. As Thompson has noted however, this method is equivalent to discarding the data far from the center of the time series (setting it to small values or zero), and any statistical estimation procedure which literally throws away data is

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unlikely to be a very sensible one—real information is being discarded. Suppose instead we construct a series of tapers, call them $w_m^{(i)}, 1 \leq i \leq P$ in such as way that

$$\sum_{m=0}^{N-1} w_m^{(i)} w_m^{(j)} = \delta_{ij} \tag{14.1}$$

that is, they are orthogonal tapers. The first one $w_m^{(1)}$ may well look much like an ordinary taper going to zero at the ends. Suppose we generate P new time series

$$y_m^{(i)} = x_m w_m^{(i)} \tag{14.2}$$

Because of the orthogonality of the $w_m^{(i)}$, there will be a tendency for

$$\sum_{m=0}^{N-1} y_m^{(i)} y_m^{(j)} \approx 0, \tag{14.3}$$

that is, to be uncorrelated. The periodograms $\left|\alpha_k^{(i)}\right|^2$ will thus also tend to be nearly uncorrelated and if the underlying process is near-Gaussian, will therefore be nearly independent. We therefore estimate

$$\tilde{\Phi}^{2P}(s) = \frac{1}{P} \sum_{i}^{P} \left|\alpha_k^{(i)}\right|^2 \tag{14.4}$$

from these nearly independent periodograms.

Thompson showed that there was an optimal choice of the tapers $w_m^{(i)}$ and that it is the set of prolate spheroidal wavefunctions (Fig. 21). For the demonstration that this is the best choice, and for a discussion of how to compute them, see Percival and Walden (1993) and the references there to Thompson's papers. (Note that the prolate spheroidal wave functions are numerically tricky to calculate, and approximating sinusoids do nearly as well; see McCoy et al., 1998, who also discuss a special problem with the estimates near zero frequency)